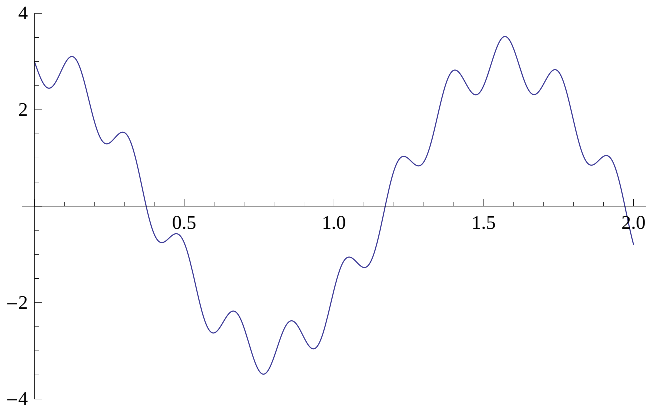
Paquetes de Onda

Ondas

In[564]:=
```
A[x_] := x^2 / 125 - Sin[35 * x] / 2 + 3 * Cos[4 * x]
Plot[ A[x], {x, 0, 2},
 PlotRange → {-4, 4}
]
```

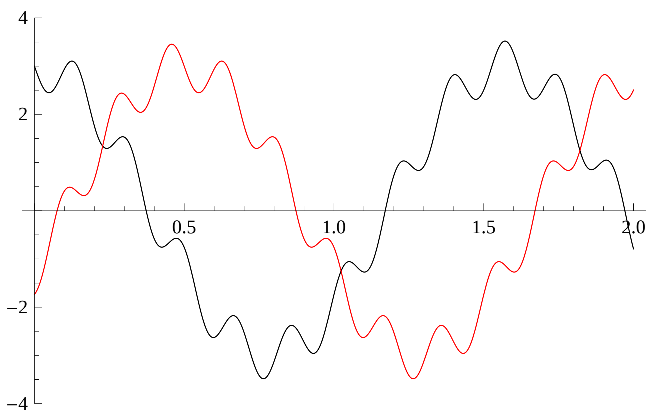
Out[565]=



In[566]:=
```
v = 0.5
t = 1;
Plot[ {A[x], A[x - v * t]}, {x, 0, 2},
 PlotRange → {-4, 4},
 PlotStyle → {RGBColor[0, 0, 0], RGBColor[1, 0, 0] }
]
```

Out[566]= 0.5

Out[568]=



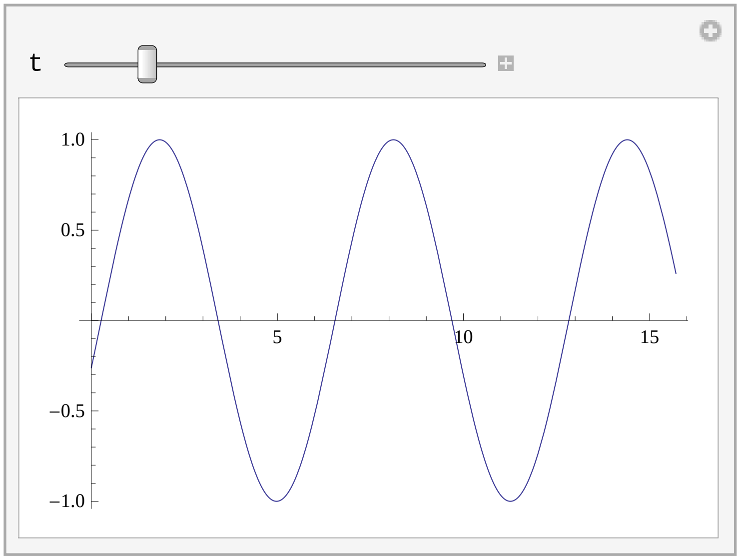
Dibujo de Ondas Armonicas

In[607]:=
```
Onda[x_, t_] := Sin[x - v * t];
```

In[485]:=
```
v = 2;
T = 2 * π / v;
Nf = 4;
Δ t = 2 * T / Nf;
```

In[570]:=
```
Manipulate[Plot[Onda[x, t], {x, 0, 5 π}], {{t, "t"}, 0, T}]
```

Out[570]=



Ejercicios :

1. Hallar las velocidades de avance de la onda (grupo o fase?)
2. Que pasa si en lugar de Sin[x] se pone Cos[x] ?

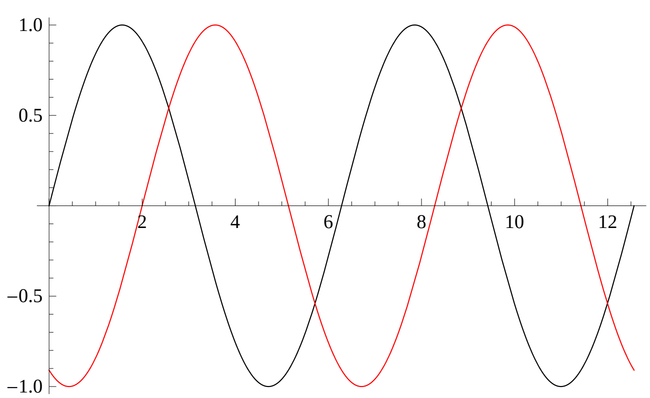
Construccion de paquetes de ondas

In[597]:=
```
Clear[Onda];
Onda[k_, ω_, x_, t_] := Sin[k * x - ω * t];
```

In[599]:=
```
Δ k = k / 10 ;
Δω = ω / 10;
```

In[601]:=
```
k = 1;
ω = 2;
Plot[ {Onda[k, ω, x, 0], Onda[k, ω, x, 1] }, {x, 0, 4 π},
 PlotStyle → {RGBColor[0, 0, 0], RGBColor[1, 0, 0] }
]
```

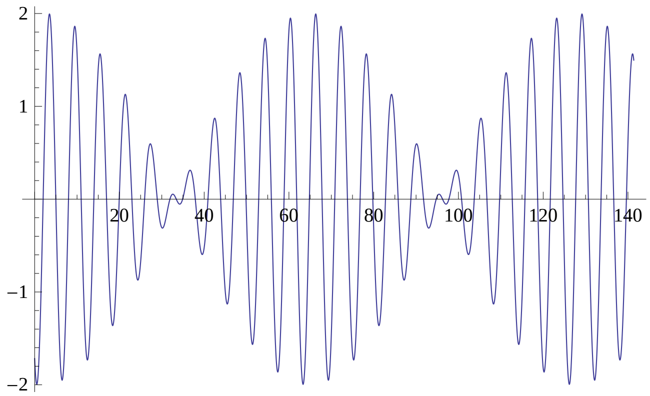
Out[603]=



In[578]:=
```
Δ k = k / 10 ;
Δω = ω / 10;
```

In[604]:=
```
Plot[Onda[k, ω, x, 1] + Onda[k + Δk, ω + Δω, x, 1], {x, 0, 45 π}]
```

Out[604]=

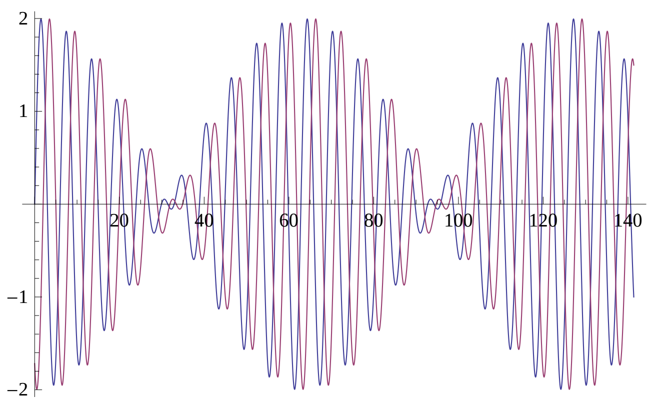


Ejercicios

1. Hallar las velocidades de avance de la onda (grupo o fase?)
2. Construir un paquete de ondas sumando muchas ondas
3. Avanza el paquete? A que velocidad?

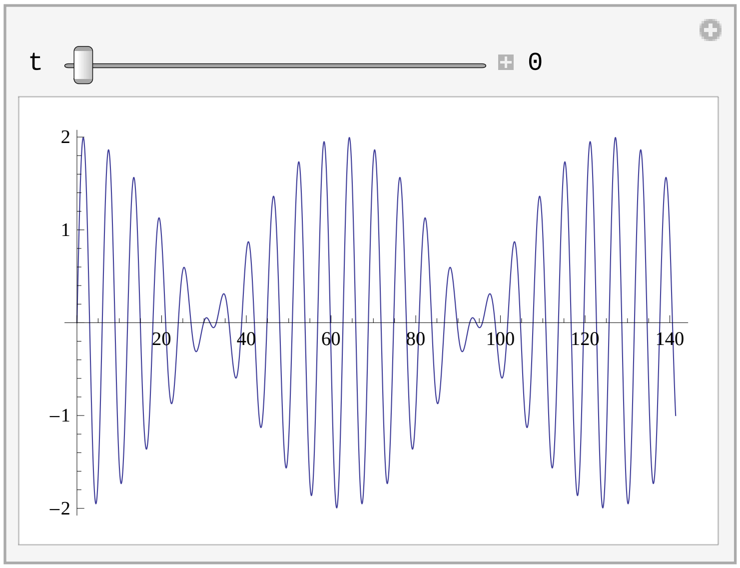
```
Plot[{Onda[k, ω, x, 0] + Onda[k + Δk, ω + Δω, x, 0],
  Onda[k, ω, x, 1] + Onda[k + Δk, ω + Δω, x, 1]}, {x, 0, 45 π}]
```

Out[605]=



In[606]:=
```
Manipulate[Plot[ Onda[k, ω, x, t] + Onda[k + Δ k, ω + Δω, x, t] , {x, 0, 45 π} ],
 {{t, 0, "t"}, 0, T, Δ t/ 100, Appearance → "Labeled"}]
```

Out[606]=



In[504]:=
Paquete de Ondas

In[583]:=
```
ClearAll[k, ω, x, t, wavepacket]
```

In[584]:=
```
nondas = 10;
k = 1;
ω = 2;
Δ k = k / nondas ;
Δω = ω / nondas;
```

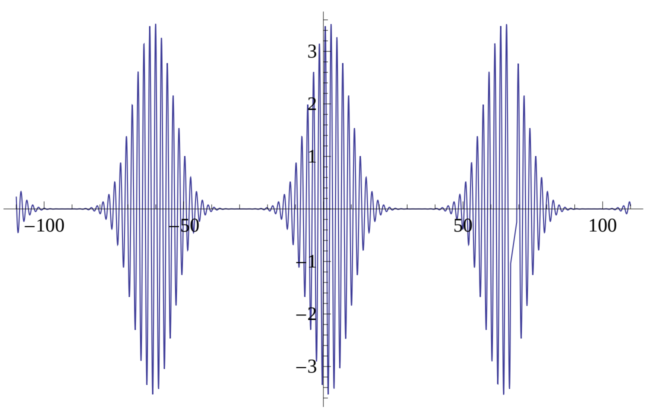
In[589]:=
```
δ k= 0.2;
```

In[590]:=
```
Onda[k_, ω _, x_, t_] := Exp[I (k * x - ω * t)];
```

In[591]:=
```
wavepacket[k_, ω _, x_, t_] :=
 Sum[Exp[- (i Δ k/ δ k) ^2] Onda[k + i Δ k, ω + i Δω, x, t], {i, -1 nondas, 1 nondas, 1}]
```

In[514]:=
```
Plot[Re[wavepacket[3, 2, x, 1]], {x, -35 π, 35 π}, PlotRange → All]
```

Out[514]=



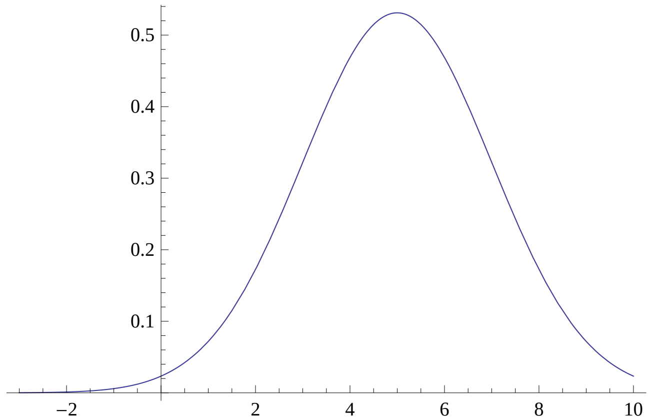
Paquete de Ondas Gaussiano

In[516]:= $G[x\_] := \frac{1}{\sqrt{\sigma\sqrt{\pi}}}\, e^{-\frac{(x-a)^2}{2\sigma^2}}$

```
σ = 2;
a = 5;
```

```
Plot[ G[x] , {x, -3, 10}]
```

Out[519]=



Ejercicios

1. Comprobar que G^2 esta normalizado
3. Que significan σ y a

Transformacion Fourier

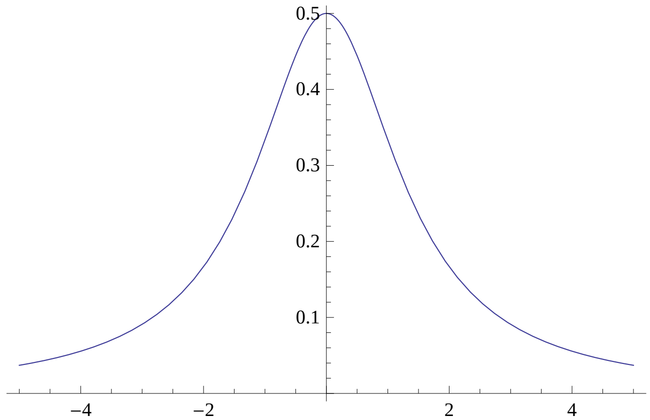
In[524]:=
```
Clear[x, k, Yx, Fk];
Yx[x_] = 1 / (x^2 + 2);
```

In[526]:=
```
Yk[k_] = Integrate[ Yx[x] * Exp[I k x], {x, -Infinity , Infinity}]
```

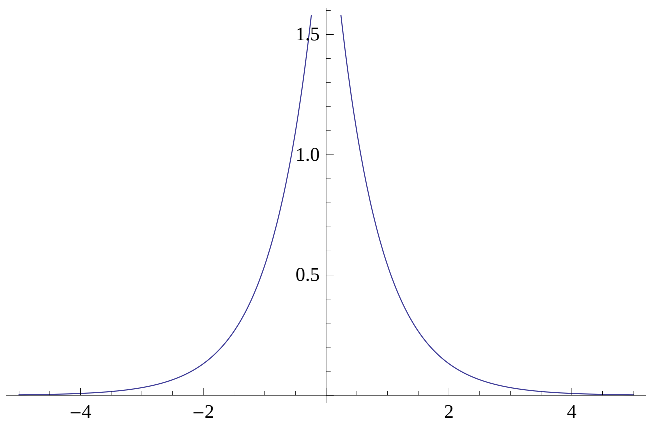
Out[526]= $\text{If}\left[k \in \text{Reals}, \frac{e^{-\sqrt{2}\,\text{Abs}[k]}\,\pi}{\sqrt{2}}, \text{Integrate}\left[\frac{e^{i k x}}{2 + x^2}, \{x, -\infty, \infty\}, \text{Assumptions} \to k \notin \text{Reals}\right]\right]$

In[527]:=
```
Plot[ Yx[x] , {x, -5, 5}]
Plot[ Yk[k] , {k, -5, 5}]
```

Out[527]=



Out[528]=

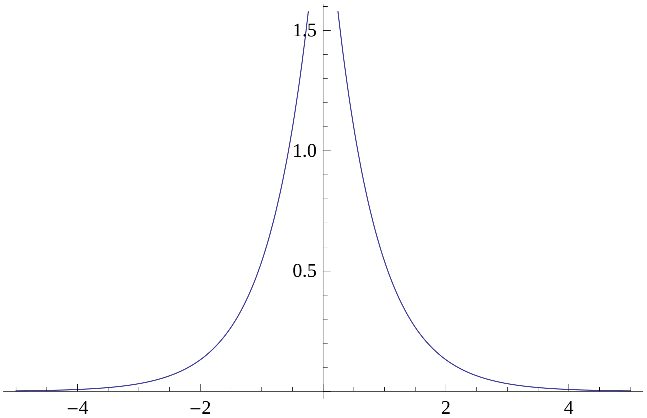


In[529]:=
```
Yk[k_] = FourierTransform[Yx[x], x, k]
```

Out[529]= $\frac{1}{2} e^{-\sqrt{2}\ \text{Abs}[k]} \sqrt{\pi}$

In[530]:= `Plot[ Yk[k] , {k, -5, 5}]`

Out[530]=



In[532]:= `Clear[x, k, σ, a, Gk];`

$$G[x\_] := \frac{1}{\sqrt{\sigma \sqrt{\pi}}} e^{-\frac{(x-a)^2}{2 \sigma^2}}$$

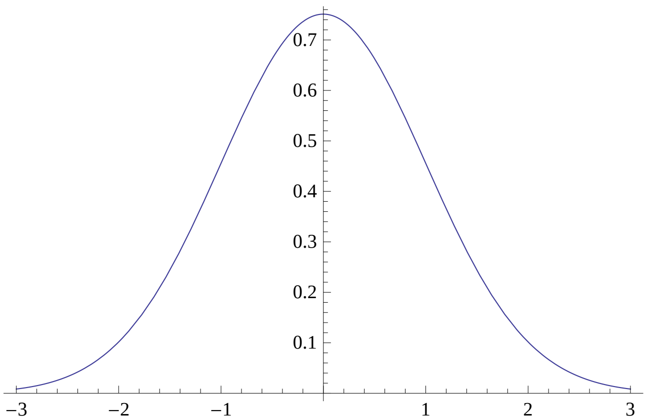
In[534]:= `Gk[k_] = FourierTransform[G[x], x, k]`

Out[534]= $\frac{e^{i a k - \frac{k^2 \sigma^2}{2}} \text{Abs}[\sigma]}{\pi^{1/4} \sqrt{\sigma}}$

In[535]:=
```
Clear[x, k, σ, a, Gk];
Gk[k_] := FourierTransform[G[x], x, k]
σ = 1 ;
a = 0 ;
```

In[539]:= `Plot[ G[x] , {x, -3, 3}]`

Out[539]=

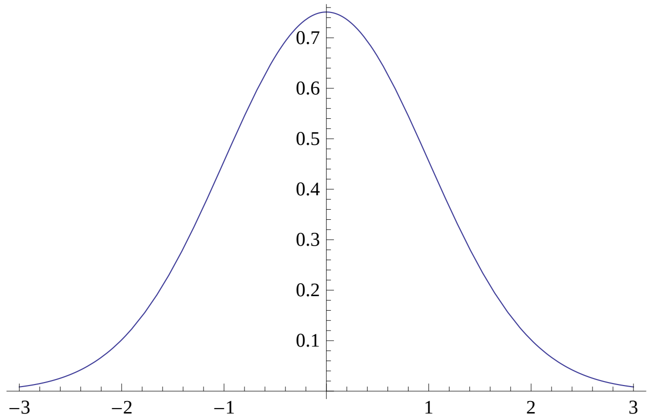


In[540]:=
`Gk[1.5]`

Out[540]= 0.243855

In[541]:=
```
Plot[ Re[Gk[k]] , {k, -3, 3}]
```

Out[541]=



Ejercicios

1. Comprobar que Gk^2 esta normalizado
3. Como varian los anchos y posiciones de la transformada?

Valores Medios

In[547]:=
```
ClearAll[σ, a]
```

In[548]:=
```
amed :=
  Assuming[{σ > 0, Element[σ, Reals]}, Integrate[G[x] * x * G[x], {x, -Infinity, Infinity}]]
```

In[549]:=
```
a2med :=
  Assuming[{σ > 0, Element[σ, Reals]}, Integrate[G[x] * x^2 * G[x], {x, -Infinity, Infinity}]]
```

In[550]:=
```
amed
```

Out[550]= a

In[551]:=
```
a2med
```

Out[551]= $a^2 + \frac{\sigma^2}{2}$

In[552]:=
```
σ x := a2med - (amed)^2
```

In[553]:=
```
σ x
```

Out[553]= $\frac{\sigma^2}{2}$

In[554]:=
```
P[x_] := -I h * G'[x]
```

In[555]:=
```
P[2]
```

Out[555]= $\frac{i\,(2 - a)\, e^{-\frac{(2-a)^2}{2\sigma^2}}\, h}{\pi^{1/4}\, \sigma^{5/2}}$

In[556]:=
```
pmed := Assuming[{σ > 0, Element[σ, Reals]}, Integrate[G[x] * P[x], {x, -Infinity, Infinity}]]
```

In[557]:=
```
pmed
```

Out[557]= 0

In[558]:=
```
P2[x_] := -I h * P'[x]
```

In[559]:=
```
p2med :=
 Assuming[{σ > 0, Element[σ, Reals]}, Integrate[G[x] * P2[x], {x, -Infinity, Infinity}]]
```

In[560]:=
```
p2med
```

Out[560]= $\frac{h^2}{2\,\sigma^2}$

In[561]:=
```
σ p := p2med - (pmed)^2
```

In[562]:=
```
σ p
```

Out[562]= $\frac{h^2}{2\,\sigma^2}$

In[563]:=
```
σ xσ p
```

Out[563]= $\frac{h^2}{4}$