

Lock-in detection with DataStudio

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The ScienceWorkshop data acquisition system with DATASTUDIO software is capable of performing lock-in detection of periodic signals. I discuss how DATASTUDIO can be used for teaching this detection technique and doing real experiments. Along with a demonstration of lock-in detection, a determination of the Verdet constant with an ac magnetic field is described. © 2006 American Association of Physics Teachers.

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I. INTRODUCTION

Lock-in detection is a powerful experimental technique for measuring weak periodic signals in the presence of signals of other frequencies or noise.¹⁻⁴ The method rests on the exact knowledge of the frequency of the expected signal. The signal is measured by a detector controlled by a reference voltage taken from the oscillator governing the process under study. The frequency of the expected signal therefore strictly coincides with that of the reference and the phase shift between them remains constant. The output signal of the detector is averaged over a sufficiently long time to suppress irregular pulses caused by signals of other frequencies or noise. The effective bandwidth of a lock-in detector is inversely proportional to the averaging time, which is easy to change. The detector is thus always tuned to the signal to be measured and has a readily adjustable bandwidth.

The operation of a lock-in detector may be explained as follows. An electronic switch controlled by a reference voltage periodically alters the polarity of a signal fed to an integrating RC circuit (see Fig. 1). The integration time constant is much longer than the period of the signal. When measuring a sinusoidal voltage $U_0 \sin(\omega t - \varphi)$, where φ is the phase shift between this voltage and the reference, the average output voltage of the detector equals

$$\begin{aligned} U_{\text{out}} &= \frac{U_0}{T} \left[\int_0^{T/2} \sin(\omega t - \varphi) dt - \int_{T/2}^T \sin(\omega t - \varphi) dt \right] \\ &= \frac{2U_0}{\pi} \cos \varphi, \end{aligned} \quad (1)$$

where $T = 2\pi/\omega$.

A dc voltage at the output of the detector appears only when the signal contains a component of the reference frequency. The output voltage is proportional to the amplitude of this component and to the cosine of the phase shift between it and the reference voltage. Therefore, lock-in detectors are often called phase-sensitive detectors. They incorporate an adjustable phase shifter to achieve maximum output voltages. When the signal contains no component of the reference frequency, the averaged output voltage remains zero. The reason is the absence of fixed phase relations between the reference and signals of other frequencies or noise.

Amplifiers employing the lock-in detection technique are called lock-in amplifiers. Due to the narrow effective bandwidth, they provide high noise immunity. With this technique, signals even much weaker than the noise become measurable. Some lock-in amplifiers incorporate two phase-sensitive detectors governed by references whose phases are

shifted by 90° . The use of two detectors makes it possible to measure the signal and its phase shift relative to the reference. Lock-in detectors are also efficient for measuring small phase changes. In this case the phase shift between the signal and the reference is set at 90° . Under such conditions the output voltage of the detector is zero, but the sensitivity to phase changes is maximum.

Temple⁵ has described a student lock-in amplifier and some experiments using it, including the determination of the velocity of sound in air based on phase measurements. Wolfson⁶ reported on an experiment on the operation of each section of a lock-in amplifier. Scofield⁷ considered the frequency components of the signal that are present at various stages of a lock-in amplifier. Moriarty *et al.*⁸ reported on the use of LABVIEW, a signal processing and waveform analysis tool in the undergraduate laboratory. Among other experiments, the authors considered the lock-in detection technique. Recently, Behringer and Brincat⁹ described a project involving a beam-chopping process, which is used for detecting weak optical signals.

II. DEMONSTRATION OF THE LOCK-IN OPERATIONS

For a demonstration of the lock-in detection technique with DATASTUDIO,¹⁰ we use a light-emitting diode (LED) fed through a variable resistor by a square wave voltage from the signal generator incorporated into the ScienceWorkshop 750 Interface (Fig. 2). This voltage of frequency 173 Hz also serves as the reference. A photodiode detects the pulses of the light emitted, which are in phase with the voltage applied to the LED. The output voltage of the photodiode is amplified by an ac amplifier and then measured by the voltage sensor (PASCO, CI-6503). The calculate tool of DATASTUDIO calculates the product of the reference voltage and the incoming signal, and its average value, and thus reproduces the operations of a phase-sensitive detector.

To demonstrate the selectivity of the lock-in detection technique, light from a fluorescent lamp is also directed onto the photodiode. The light causes an unlocked signal containing an ac component of doubled line frequency, that is, 100 or 120 Hz. The results presented here relate to measurements of the locked signal only (173 Hz) and to measurements when the unlocked signal added is about five times larger than the locked signal. DATASTUDIO displays the signals from the photodiode (see Fig. 3). When the unlocked signal is added, it is difficult to observe the initial locked signal. The results of the lock-in operations are also seen on the screen of a computer. The measurements confirm that the lock-in detector measures a locked signal even when it is much

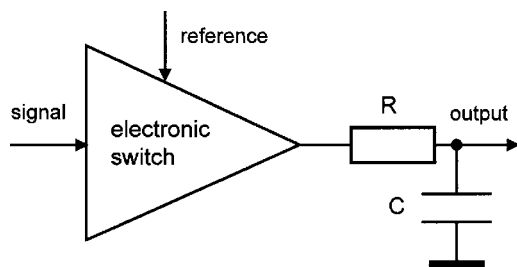


Fig. 1. Basic diagram of a lock-in detector. An electronic switch controlled by a reference voltage and an integrating RC circuit perform the necessary operations. The averaged output voltage is proportional to the ac input signal of a frequency, which coincides with that of the reference and to the cosine of the phase shift between them.

smaller than signals of different frequencies or noise. For the locked signal only, there is no need for long averaging. When the unlocked signal is added, the result becomes close to the correct value after averaging for one second (Fig. 4). This result is the main goal of the demonstration. Another demonstration shows the role of the phase shift between the reference and the signal (Fig. 5). If other data acquisition systems are used, it is important that they can perform multiplication and averaging operations.

Three experiments were modified to use DATASTUDIO as a lock-in detector. One experiment measures the velocity of sound from phase relationships between the loudspeaker driving voltage and the corresponding signal picked up by a microphone.⁵ The phase of the signal is clearly seen from the measurements. The second experiment determines the magnetic constant (the magnetic permeability of free space) μ_0 . An ac current from the signal generator feeds a pair of Helmholtz coils. The data acquisition system measures the emf generated in a small probe coil positioned between them as a function of the amplitude and frequency of the current feeding the Helmholtz coils. From the data, μ_0 can be determined using the relation for the magnetic field produced by the Helmholtz coils and Faraday's law for sinusoidal magnetic fields.

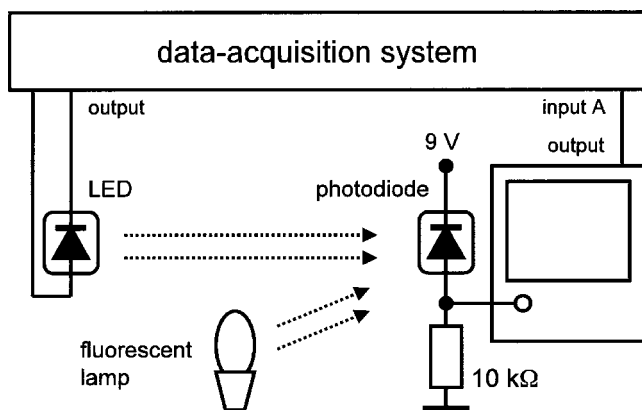


Fig. 2. Diagram of the setup demonstrating lock-in detection with DATASTUDIO. A light-emitting diode is the source of a locked signal, while the illumination by a fluorescent lamp provides an unlocked signal of double line frequency.

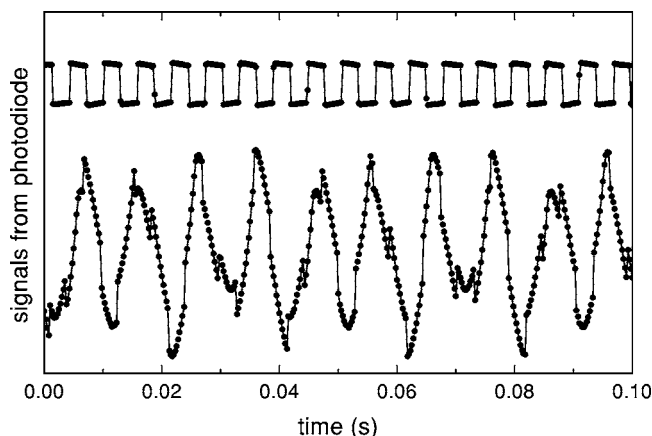


Fig. 3. Signals from the photodiode for a 173-Hz locked signal (top) and when the unlocked signal added is about five times larger than the locked signal (bottom).

III. DETERMINATION OF THE VERDET CONSTANT

The third experiment is the determination of the Verdet constant employing ac magnetic fields and the lock-in detection of small ac signals due to periodic Faraday rotation. In 1845 Michael Faraday discovered the rotation of the plane of polarization of light passing through a medium in a magnetic field aligned parallel to the direction of propagation. The Faraday effect was one of the earliest indications of a relation between electromagnetism and light. The theoretical treatment of the Faraday effect involves the quantum-mechanical theory of dispersion, including the effect of the magnetic field on the atomic or molecular energy levels.^{11,12} The Faraday rotation is difficult to observe in a teaching laboratory because high magnetic fields are necessary.^{13,14} The experiment described in the following shows how to observe this effect with moderate ac magnetic fields using DATASTUDIO to perform the lock-in detection operations.

The setup for the measurements is similar to that described earlier,¹⁵ but the lock-in detection operations are performed by DATASTUDIO. Using the ac technique, measurements are possible with magnetic fields much weaker than is necessary for measurements with dc fields. The theory of ac measurements of the Verdet constant is simple.¹⁵ When a polarized light beam passes through a sample, the angle of Faraday rotation θ is proportional to the thickness of the sample d and the strength of the magnetic field B ,

$$\theta = VBd, \quad (2)$$

where V is the Verdet constant of the material, which depends on the wavelength of light.

The light beam then traverses through the analyzer, which is set at the angle of $\pi/4$ with respect to the polarizer. According to the cosine-squared rule (Malus's law),¹⁶ the light intensity measured by the detector is given by

$$I = I_m \cos^2(\pi/4 - \theta), \quad (3)$$

where I_m is the maximum light intensity. The angle θ is very small and Eq. (3) can be simplified to

$$I = I_m(1 + 2\theta)/2. \quad (4)$$

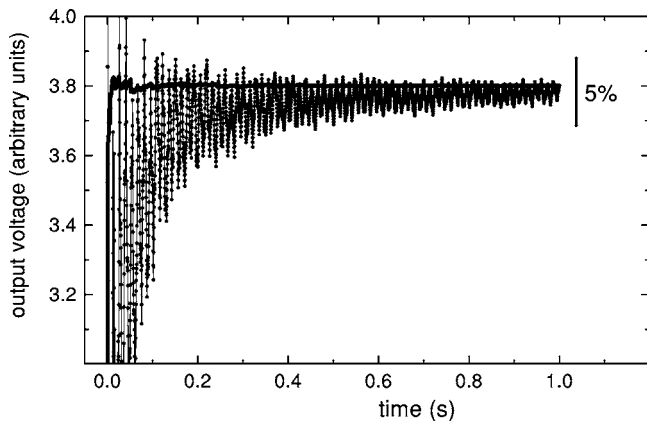


Fig. 4. The averaging process for signals shown in Fig. 3. For the locked signal alone, there is no need for a long averaging time (bold line). When the unlocked signal is added, the result becomes close to the correct value after one second averaging (oscillating curve).

Because the ac magnetic field is $B=B_0 \sin \omega t$, the rotation angle is $\theta=\theta_m \sin \omega t$. The light intensity thus contains a dc and an ac component,

$$I=I_0+2I_0\theta_m \sin \omega t=I_0+\Delta I \sin \omega t, \quad (5)$$

where ΔI is the amplitude of the ac component of the light intensity. The voltages on the load resistor of the photodiode corresponding to the dc and ac components of the light intensity are U_0 and U , respectively.

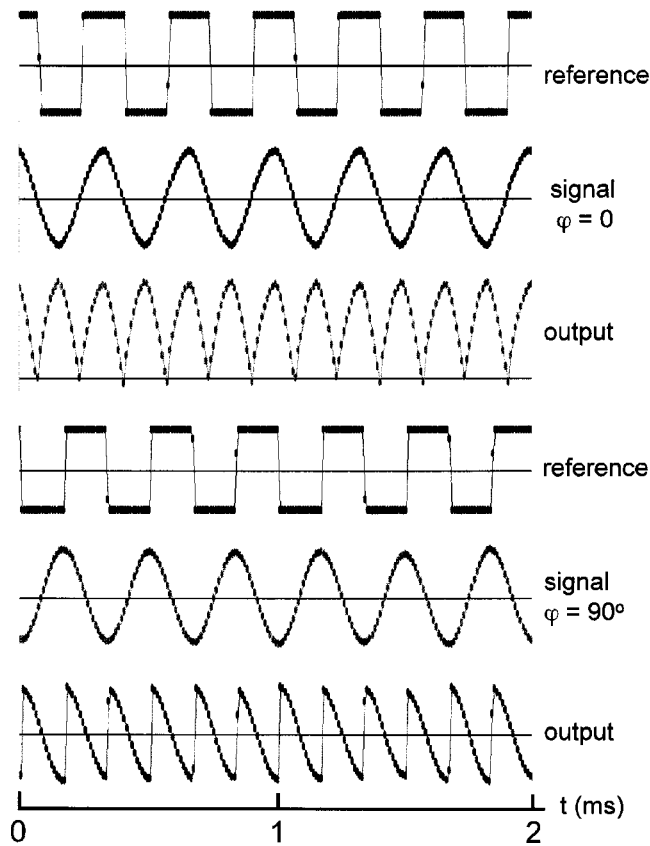


Fig. 5. Diagram showing how the detector senses the phase shift between the reference and the signal. For a 90° or 270° phase shift, the average output voltage becomes zero.

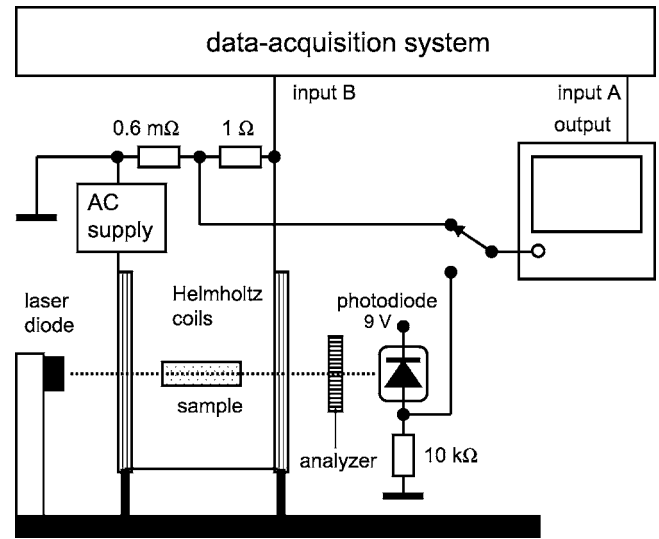


Fig. 6. Arrangement for the Faraday rotation experiment with ac magnetic fields.

The amplitude of the angle of rotation, θ_m , is related to U and U_0 as:

$$\theta_m=U/2U_0, \quad (6)$$

where U is the amplitude of the ac voltage across the load of the photodiode. The Verdet constant is determined from Eq. (2), using the amplitude of the magnetic field.

The laser beam from a laser diode (PASCO, OS-8525A) passes through a sample subjected to an ac magnetic field (see Fig. 6) produced by a pair of Helmholtz coils (PASCO, EM-6711) connected to an ac source. The source includes a variac and an isolating transformer (the details are not shown in Fig. 6). The axial magnetic field produced by the Helmholtz coils in the center of the system is given by

$$B=\frac{8\mu_0NI}{a\sqrt{125}}, \quad (7)$$

where a is the radius of the coil, and I and N are the current in the coils and number of turns in one coil, respectively. Equation (7) is also valid for an ac current. In our case, $N=200$, $a=10.5$ cm, and $B(\text{G})=17.1 I(\text{A})$. The ac current through the coils equals 2 A (rms).

The laser beam ($\lambda=0.67 \mu\text{m}$) is polarized, so that only an analyzer is needed. It is set at 45° with respect to the polarization of the laser beam. After the analyzer the light intensity is measured by a photodiode loaded by a 10-k Ω resistor. A multimeter (not shown in Fig. 6) measures its dc output voltage and an oscilloscope displays the ac component. The oscilloscope serves also as an amplifier giving a gain of $G=100$. One voltage sensor measures the amplified voltage (input A of the interface), and the second voltage sensor measures the voltage drop across a 1- Ω resistor connected in series with the Helmholtz coils (input B). The second voltage is measured with high sensitivity of the voltage sensor. When a voltage is measured with high sensitivity, DATASTUDIO restricts the acquired voltage by 0.1 V. Therefore a sinusoidal voltage with an amplitude much larger than 0.1 V is automatically converted into a square wave voltage of amplitude 0.1 V. This conversion is important for obtaining the desired form of the reference voltage. The sample rate is 4000 Hz,

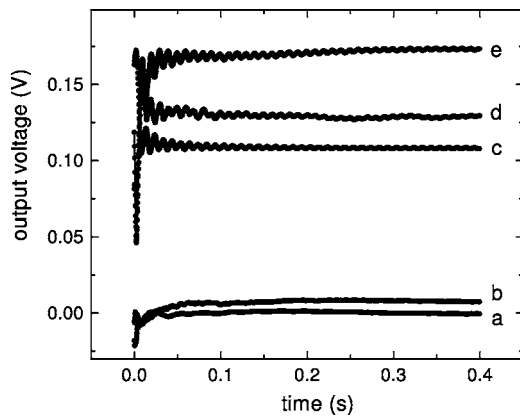


Fig. 7. Output voltages after lock-in operations when determining the Verdet constants: (a) without a sample, (b) empty quartz cuvette, (c) U_R , the voltage on the resistor R , (d) water, and (e) NaCl.

and each run lasts 0.4 s. Two samples were measured; water in a quartz cuvette ($d=64$ mm) and a NaCl crystal ($d=31$ mm). DATASTUDIO performs the lock-in detection operations and displays the results.

In the measurements the periodic Faraday rotation is directly compared to the ac magnetic field applied to the sample. In this way possible errors caused by the amplifier of the oscilloscope and by the data acquisition system itself are excluded. A resistor $R=0.6$ m Ω is put in series with the Helmholtz coils and the 1- Ω resistor. Under operating conditions, the voltage drop on this resistor is comparable to the ac voltages provided by the photodiode. This voltage drop, $U_R=IR$, is measured by the same measuring system including the oscilloscope and the ScienceWorkshop 750 Interface. DATASTUDIO displays the results of this measurement and the measurements of the ac voltage from the photodiode (Fig. 7).

It is easy to deduce the expression for the calculation. The Verdet constant equals

$$V = \theta_m/B_m d = KU/2U_R U_0 d, \quad (8)$$

where B_m is the amplitude of the ac magnetic field. The voltages U and U_R are the output voltages of the measuring system after the lock-in detection operations when measuring the Faraday rotation and the voltage drop across the resistor R , respectively. The coefficient K takes into account the value of this resistor and the relation between the current and the magnetic field produced by the Helmholtz coils. The rotation angle in Eq. (8) is given in radians; for comparison with available data it should be converted into minutes. After the necessary substitutions, the Verdet constant in our case equals

$$V = 0.0603U/U_R U_0 d. \quad (9)$$

Equation (9) for the Verdet constant contains the ratio of two voltages, U and U_R , determined by the lock-in detection operations, the dc voltage on the load of the photodiode, U_0 , and the thickness of the sample, d . The total gain of the measuring system is thus not needed. Only the resistor R needs to be precisely known. For this purpose we use a cali-

brated shunt (100 A, 60 mV, 0.5%). If the voltage U_0 is given in volts and the thickness of the sample d in meters, the Verdet constant in Eq. (9) is given in min $G^{-1} m^{-1}$. The contribution from the cuvette should be taken into account. Values of U_0 and d should be introduced for each sample. For the example shown, U_0 equals 1.05 V (water) and 1.10 V (NaCl).

Earlier, the Verdet constants of water and NaCl were determined over a range of wavelengths.^{17,18} The Verdet constants obtained in our experiment are in good agreement with these results.

IV. CONCLUSION

We have shown how DATASTUDIO software performs the lock-in detection operations. In some cases, DATASTUDIO can replace expensive lock-in amplifiers. However, this approach is not applicable for measuring high frequency signals because of the limited rate of data processing.

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