Experimental estimation of the band gap in silicon and germanium from the temperature–voltage curve of diode thermometers

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Semiconductor diodes, in conjunction with a constant current source, are sometimes used as thermometers. It has been observed experimentally that, within a certain temperature range, the relation between temperature and voltage is almost linear. We show that this linearity is a direct consequence of the constancy of the current flowing through the diode, and that the parameters resulting from a least-squares fit to the experimental data can be used to determine the band gap energy of the semiconductor. We test the validity of our model by comparing our results to measurements on diodes made of germanium and silicon. If we take into account the simplifications used in our model, the results agree well with known values of the energy gaps.

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I. INTRODUCTION

The $p-n$ junction is an important element of most electronic devices, from a simple diode to a transistor to a sophisticated integrated circuit. Therefore, understanding of the $p-n$ junction is essential to the study of more complicated semiconductor devices and thus has attracted a considerable interest in undergraduate laboratories.[1–3] The study of the band gap structure of semiconductors is also important because it is directly related to its electrical properties. Several methods for the experimental determination of the band gap in silicon and germanium have been discussed.[1,4,5]

We propose another method, which is based on the application of diodes as thermometers, for the determination of the band gap. A short discussion of diode thermometers is given in Ref. 6. When a diode is used as a temperature sensor, the forward voltage drop across the $p-n$ junction is usually measured as a function of temperature. The relation between voltage and temperature is found to be almost linear within a certain temperature range characteristic of the semiconductor. Two questions that may arise are the following.

(1) Why is the temperature–voltage relation (we will take the voltage as the independent variable, because we use the diode as a thermometer) almost linear knowing that a diode is a highly nonlinear device?

(2) Can we extract any relevant information from the experimentally determined parameters a and b in the linear relation

$$T=aV+b? \tag{1}$$

We will show that the linearity of Eq. (1) is a consequence of the constant current source usually employed in diode thermometers, and that we can obtain the band gap energy E_g from a knowledge of a and b.

II. THEORY

The current–voltage characteristic of a $p-n$ junction can be described by the ideal diode equation

$$I=I_0[\exp(\mathrm{eV}/kT)-1], \tag{2}$$

where I is the current through the diode, I_0 is the maximum current for a large reverse bias voltage (formally $V\rightarrow-\infty$), e is the electron charge, V is the voltage across the diode, k is Boltzmann's constant, and T is the absolute temperature. Equation (2) is discussed extensively in the literature.[1,3,7] The following analysis is based on Eq. (2), and we assume that it describes accurately the $I-V$ characteristics of a $p-n$ junction.

We next introduce some simplifications. First, we will restrict ourselves to regions where we can neglect the number 1 in Eq. (2). If we take a forward bias of 0.1 V at room temperature, we obtain $\mathrm{eV}/kT=0.1/0.025=4$, and

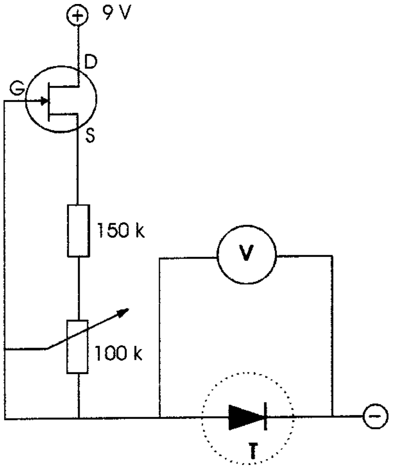

Fig. 1. The saturated drain current of the FET 2SK30A-GR is used as the constant-current source. The gate-source bias voltage was adjusted by a 100-kΩ potentiometer so that a constant current of 10 μA was obtained. The diode is at a variable temperature T.

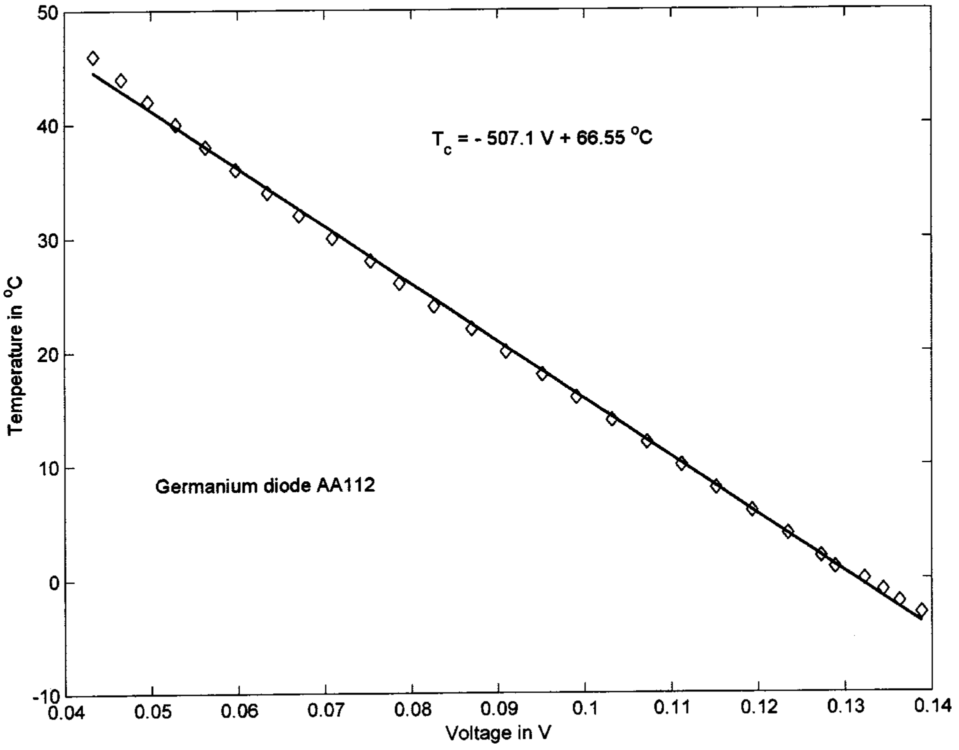

Fig. 2. Temperature vs voltage plot for the germanium diode AA112. The $T-V$ curve becomes nonlinear for temperatures above 40 °C. A constant current of 10 μA flows through the diode.

$\exp(eV/kT)\approx 55$. So, this approximation can be generally justified in the regions of interest. We will consider this point in Sec. V.

Second, we need an expression for the reverse current I_0, which depends strongly on temperature, but not on V. It can be shown[8] that I_0 is proportional to the Boltzmann factor $\exp(-E_g/kT)$ and to $T^{3+\gamma/2}$, where γ is a constant. We may thus write

$$I_0 = AT^{3+\gamma/2}\exp(-E_g/kT). \tag{3}$$

We can neglect the $T^{3+\gamma/2}$ dependence of I_0 in comparison to the exponential dependence on T and can treat $B = AT^{3+\gamma/2}$ as almost constant. Therefore,

$$I_0 = B\exp(-E_g/kT). \tag{4}$$

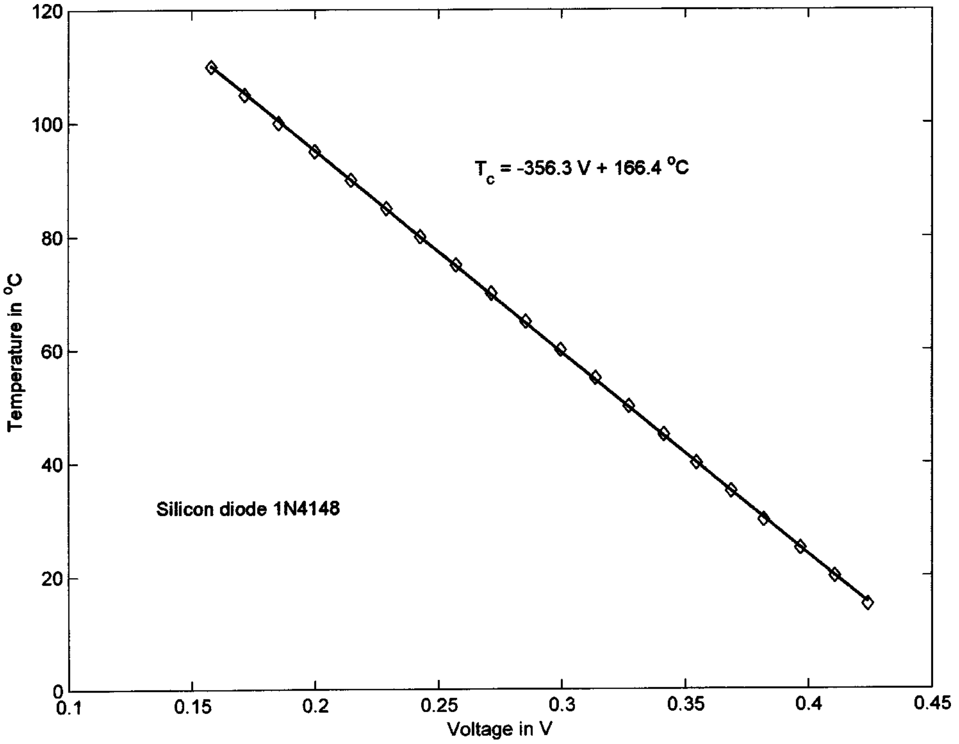

Fig. 3. Temperature vs voltage plot for the silicon diode 1N4148. A constant current of 10 μA flows through the diode.

Table I. Linear regression coefficients *a* and *b* obtained from the measured $T-V$ data of germanium and silicon, and the band gap energy calculated from them. For comparison, the last two columns contain the gap energies at 300 and 0 K from Ref. 8 (Fig. 8, p. 24).

	a (K/V)	*b* (K)	E_g (eV)	E_g (300 K) (eV)	E_g (0 K) (eV)
Germanium	−507.1	339.7	0.67	0.66	0.741
Silicon	−356.3	439.5	1.23	1.12	1.16

If we combine Eqs. (2) and (4), and neglect 1 in Eq. (2), we obtain

$$I=B\exp(-E_g/kT+eV/kT). \tag{5}$$

Equation (5) is given in the book of Sze (Ref. 8, p. 102). Experimentally we maintain the current *I* as a constant, and we may thus write

$$C=\mathrm{eV}/kT-E_g/kT, \tag{6}$$

where $C=\ln(I/B)$. If we rewrite Eq. (6), we have

$$T=(e/kC)V-E_g/kC, \tag{7}$$

which is the linear relation we were seeking, provided that E_g is constant. (In fact, E_g depends slightly on temperature, and we will consider this point later.) If we compare Eqs. (7) and (1), we see that $a=e/kC$ and $b=-E_g/kC$. We divide *b* by *a* and obtain

$$E_g=-(b/a)e. \tag{8}$$

Equation (8) relates the band gap energy E_g to the experimentally determined values of the parameters *a* and *b* in Eq. (1). This relation is the calibration curve that describes the operational characteristics of the diode thermometer in the linear region. Equations (7) and (8) answer the two questions raised in Sec. I.

In the following, we will discuss the measurement of the $T-V$ characteristics of a silicon and a germanium diode, do a least-squares fit of the linear region of the $T-V$ curve to obtain *a* and *b*, and use Eq. (8) to determine the energy gap of both materials.

III. EXPERIMENTAL PROCEDURE

Because the impedance of the diode varies considerably with temperature, we need a circuit that keeps the current constant. Figure 1 shows the electric circuit used to measure the $T-V$ calibration curve of the diode. The constant-current source was taken from Haruyana and McDonald[9] because of its simplicity, low cost, and good performance. It consists of a field effect transistor 2SK30A-GR which provides the constant current source. The current can be adjusted using the 100-kΩ variable resistor. We adjusted the circuit current to 10 μA. The whole circuit is powered by a 9-V battery.

The potential difference across the diodes was measured by a digital multimeter (Voltcraft M-4650 CR). A mercury thermometer[10] was used to measure the temperature of the diode. The diode was placed in direct contact with the mercury reservoir of the thermometer, and both were wrapped together in a thin PVC foil (as is used for food protection) for electrical insulation and waterproof purposes. A thin aluminum foil was then wrapped around the lower part of the thermometer and both were fixed mechanically using thin copper wires tied to a small aluminum block ($6\times19\times35\ \mathrm{mm}^3$) to increase its thermal response time. The whole system could be heated by an electrical heater or brought into contact with an ice–salt mixture to reach temperatures slightly less than 0 °C.

Voltage readings were taken at various temperatures while the aluminum block was slowly heated. During the cooling process, the voltage was recorded at the same temperatures. The two voltage readings at every temperature were averaged to reduce systematic errors. The experimental procedure was chosen mainly for its simplicity, rather than for its precision. The results of this procedure are given in Sec. IV.

IV. RESULTS

Figure 2 shows the measured temperature versus voltage data for the germanium diode AA112, and Fig. 3 shows similar data for the silicon diode 1N4181. The straight line is calculated from a least-squares fit of the experimental data and the corresponding equations are given inside the plots.

For the calculation of the band energy E_g, we need to replace the temperature T_c in the experimental T_c-V plot by the absolute temperature *T*. We obtain for germanium $T=T_c+273.15=-507.1V+339.70$. Therefore, according to Eq. (1), $a=-507.1$ K/V and $b=339.7$ K, giving $b/a=-0.670$ V, and from Eq. (8), $E_g=0.67$ eV. An analogous calculation holds for silicon. Table I summarizes the results for germanium and silicon.

V. DISCUSSION

In the derivation of Eq. (7) we assumed that $\exp(\mathrm{eV}/kT)\gg1$, so that we could neglect 1 in Eq. (1). As can be seen from Fig. 2, the $T-V$ data become nonlinear for temperatures higher than 40 °C (the experimental data for $T_c>40$ °C have been omitted in Fig. 2), and voltages less than 0.05 V. At this point, $eV/kT=1.85$, and $\exp(eV/kT)=6.4$, so that we cannot neglect 1 anymore. For this reason the linear relationship of Eq. (7) is violated for higher temperatures.

For silicon the $T-V$ curve is linear over the entire measured range. At the highest temperature, $T_c=110$ °C and $V=0.15$ V, so that $eV/kT=4.546$, and $\exp(eV/kT)=94.3$, which is still large compared to 1. Thus, no deviation from the linear $T-V$ characteristic is expected.

We assumed in our derivation of Eq. (7) that E_g does not depend on temperature, which is not strictly true. The temperature dependence of the band gap can be described by a universal function

$$E_g(T)=E_g(0)-\alpha T^2/(T+\beta), \tag{9}$$

where α and β are constants given in Ref. 8 (Fig. 8, p. 24). Let us calculate the variations of the band gap energy in the temperature ranges of our investigation. If T_L and T_H denote the lower and higher temperature limits, respectively, of the investigated temperature range, we obtain from Eq. (9)

$$\Delta E_g = E_g(T_H) - E_g(T_L) = \alpha[T_L^2/(T_L+\beta) - T_H^2/(T_H+\beta)]. \tag{10}$$

For germanium, $T_L = 270$ K, $T_H = 313$ K, $\alpha = 4.56 \times 10^{-4}$ eV/K, and $\beta = 210$ K. If we substitute these values into Eq. (10), we get $\Delta E_g(\mathrm{Ge}) = -0.016$ eV. In the same manner, for silicon, $T_L = 283$ K, $T_H = 383$ K, $\alpha = 7.02 \times 10^{-4}$ eV/K, and $\beta = 1108$ K, so that $\Delta E_g(\mathrm{Si}) = -0.029$ eV. Thus, to a first approximation, E_g can be considered to be constant in the temperature ranges investigated.

From a didactical point of view it is interesting to see how an experimental constraint (constant current) opens a new interpretation about an experimental fact (linear $T-V$ characteristic of a diode), namely the determination of the band gap as a secondary effect. If the $I-V$ characteristic of a $p-n$ junction obeys the ideal diode equation, and $\exp(eV/kT) \gg 1$, then the $T-V$ curve of the diode thermometer is expected to be linear, and the energy gap can be obtained from Eq. (8).

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[10] INCOTERM, −10 to 150 °C range, 1 °C scale.