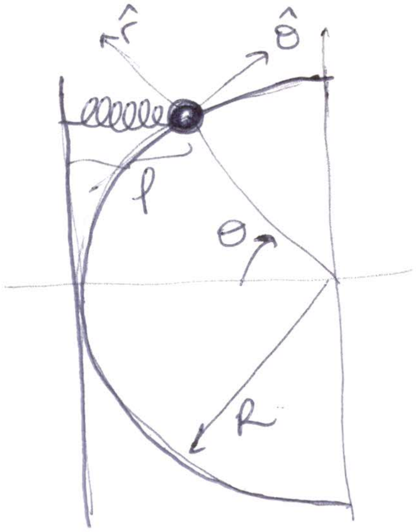


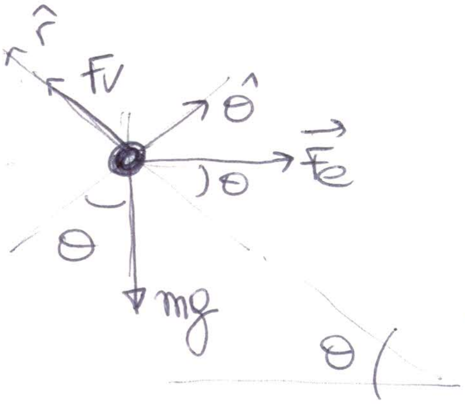


$$|\vec{Fe}| = k|l - l_0|$$

$$l_0 = R$$

$$l = R - R\cos\theta$$

$$|\vec{Fe}| = kR\cos\theta$$

y como $l_0 = R$ el resorte está siempre comprimido

a)

$$\begin{cases} \hat{r}: & Fv - \overbrace{(kR\cos\theta)}^{Fe}\cos\theta - mg\,\text{sen}\,\theta = -mR\dot{\theta}^2 \\ \hat{\theta}: & \underbrace{(kR\cos\theta)}_{Fe}\,\text{sen}\,\theta - mg\cos\theta = mR\ddot{\theta} \end{cases}$$

$$\longrightarrow \boxed{\ddot{\theta} + g/R\cos\theta - \frac{k}{m}\cos\theta\,\text{sen}\,\theta = 0}$$ Ec. mto.

b) $\ddot{\theta} = \dfrac{d\dot{\theta}}{d\theta}\cdot\dot{\theta} \rightarrow$

$$\rightarrow \int_{\pi/2}^{\theta} kR\cos\theta\,\text{sen}\,\theta\,d\theta - mg\int_{\pi/2}^{\theta}\cos\theta\,d\theta = mR\int_{0}^{\dot{\theta}}\dot{\theta}\,d\dot{\theta}$$

$$\frac{kR}{2}\text{sen}^2\theta\,\Big|_{\pi/2}^{\theta} - mg\,\text{sen}\,\theta\,\Big|_{\pi/2}^{\theta} = \frac{mR\dot{\theta}^2}{2}$$

$$\therefore \frac{1}{2}mR\dot{\theta}^2 = \frac{kR}{2}(\text{sen}^2\theta - 1) - 2mg(\text{sen}\,\theta - 1)$$

$$\boxed{Fv = kR\cos^2\theta + mg\,\text{sen}\,\theta + kR(1 - \text{sen}^2\theta) + 2mg(\text{sen}\,\theta - 1)}$$

c) Equilibrio: $\left(\ddot{\theta}(\theta_{eq})=0\right) \rightarrow$

$$\rightarrow \quad kR\cos\theta_{eq}\,\mathrm{sen}\,\theta_{eq} - mg\cos\theta_{eq} = 0$$

$$\underbrace{\cos\theta_{eq}}_{0}\,\underbrace{\left(kR\,\mathrm{sen}\,\theta_{eq} - mg\right)}_{0} = 0$$

$\cos\theta_{eq} = 0 \rightarrow \boxed{\theta_{eq} = \pi/2}$

o $\boxed{\mathrm{sen}\,\tilde{\theta}_{eq} = mg/kR} \rightarrow \exists$ si $0 < \frac{mg}{kR} < 1$

$0 < mg < kR$

Estabilidad

$$F_\theta(\theta) = kR\cos\theta\,\mathrm{sen}\,\theta - mg\cos\theta$$

$$\frac{dF_\theta}{d\theta}(\theta) = kR\left(-\mathrm{sen}^2\theta + \underbrace{\cos^2\theta}_{1-\mathrm{sen}^2\theta}\right) + mg\,\mathrm{sen}\,\theta$$

$$\boxed{\frac{dF_\theta}{d\theta}(\theta) = kR\left(1 - 2\,\mathrm{sen}^2\theta\right) + mg\,\mathrm{sen}\,\theta}$$

$\boxed{\theta_{eq} = \pi/2}$ $\quad \frac{dF_\theta}{d\theta}(\theta_{eq}=\pi/2) = -kR + mg < 0$ (EST) si $kR > mg$

$> 0$ (IN) si $mg > kR$

$\boxed{\theta_{eq} = \tilde{\theta}_{eq}}$ $(\mathrm{sen}\,\tilde{\theta}_{eq} = mg/kR)$ $\rightarrow \quad \frac{dF}{d\theta}(\tilde{\theta}_{eq}) = kR\left(1 - 2\frac{(mg)^2}{(kR)^2}\right) + mg\left(\frac{mg}{kR}\right)$

$$= kR - \frac{(mg)^2}{kR} = kR\left[1 - \left(\frac{mg}{kR}\right)^2\right] > 0 \quad \text{(IN)}$$

si $mg < kR$

| | | | |
|---|---|---|---|
| $mg < kR \rightarrow \exists$ 2 $\theta_{eq}$ | $\theta_{eq} = \pi/2$ | EST | |
| | $\theta_{eq} = \arcsen(mg/kR)$ | IN | |
| $mg > kR \rightarrow \exists$ 1 $\theta_{eq}$ | $\theta_{eq} = \pi/2$ | IN | |