

(INDEPENDENT RESEARCH)

KEY ASSUMPTION (ISOTROPIC SYSTEMS)

$$4\pi P(t) = \int_0^{\infty} F(t') E(t-t') dt'$$

IMPORTANT POINTS:

1. LINEAR RELATIONSHIP
BETWEEN POLARIZATION
& ELECTRIC FIELD

2. CAUSALITY

$P @ t$ DEPENDS ON
VALUES OF E

@ EARLIER TIMES

3. 4π CONVENIENCE
(UNITS)

4. LOCAL RELATIONSHIP

$P(\vec{r})$ DEPENDS

ON $\vec{E}(\vec{r})$

~~$4\pi P(t) = \int_0^{\infty} F(t') E(t-t') dt'$~~

IF QUANTITIES DEPEND ON
TIME LIKE $e^{-i\omega t}$

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$$\Rightarrow 4\pi P = E \int_0^{\infty} F(t') e^{i\omega t'} dt'$$

OR, SINCE $D = E + 4\pi P$

$$E = 1 + \int_0^{\infty} F(t') e^{i\omega t'} dt' \quad (*)$$

TAKE $E(\omega) = E_1(\omega) + iE_2(\omega)$

$$\Rightarrow E(-\omega) = E^*(\omega) \quad \text{FIELDS ARE REAL}$$

$$\left\{ \begin{array}{l} E_1(-\omega) = E_1(\omega) \quad \text{REAL PART IS EVEN} \\ E_2(-\omega) = -E_2(\omega) \quad \text{IMAG. PART IS ODD} \end{array} \right.$$

FROM (*)

$$E_1(\omega) = 1 + \int_0^{\infty} F(t') \cos \omega t' dt'$$

$$E_2(\omega) = \int_0^{\infty} F(t') \sin \omega t' dt'$$

LET US NOW IMPOSE THE CONDITION

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$$F(t') = 0 \quad t' < 0$$

(THIS DOESN'T CHANGE ANYTHING)
EXCEPT THAT WE CAN CHANGE
THE LIMITS OF INTEGRATION

THEN, IF WE INVERT THE FOURIER TRANSFORM,
WE GET THAT

$$F(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [\epsilon(\omega) - 1] e^{-i\omega t} d\omega$$

$$\equiv \frac{1}{\pi} \int_0^{\infty} [(\epsilon_1 - 1) \cos \omega t + \epsilon_2 \sin \omega t] d\omega$$

↓
EVEN
↓
ODD

IF, FOR $t > 0$

$$\int_0^{\infty} (\epsilon_1 - 1) \cos \omega t d\omega \equiv \int_0^{\infty} \epsilon_2 \sin \omega t d\omega$$

⇒ $F \equiv 0 \quad t < 0$

FROM THERE, ONE CAN SHOW (HOMEWORK)

$$\left\{ \begin{aligned} \epsilon_1 &= 1 + \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\epsilon_2(\omega')}{\omega' - \omega} d\omega' \\ \epsilon_2 &= -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\epsilon_1(\omega') - 1}{\omega' - \omega} d\omega' \end{aligned} \right.$$

POLE @ $\omega = 0$
 $\frac{4\pi}{\omega}$ KK
~~POLE @ $\omega = 0$~~

OTHER EQUIVALENT EXPRESSIONS

$$\left\{ \begin{aligned} \epsilon_1 &= 1 + \frac{2}{\pi} \int_0^{\infty} d\omega' \epsilon_2(\omega') \left[\frac{\omega'}{\omega'^2 - \omega^2} \right] \\ \epsilon_2 &= \frac{2}{\pi} \int_0^{\infty} d\omega' \epsilon_1(\omega') \left[\frac{\omega}{\omega^2 - \omega'^2} \right] \end{aligned} \right.$$

ALSO EXPRESSIONS RELATING n TO K

MORE ELEGANT DERIVATION

SINCE

~~$\epsilon(t) = 1 + \int_0^t F(t') e^{i\omega t'} dt'$~~

$$\epsilon = 1 + \int_0^{\infty} F(t') e^{i\omega t'} dt'$$

PROVIDED $F(t') \rightarrow 0$ FOR $t' \rightarrow \infty$

ϵ IS ANALYTICAL IN UPPER HALF OF COMPLEX PLANE THEN, USING CAUCHY'S THEOREM (COMPLEX VARIABLES)

$$\epsilon(\omega) = -\frac{i}{2\pi} \oint_C \frac{\epsilon(\omega') - 1}{\omega' - \omega} d\omega'$$

SEE JACKSON P. 332

SOME GEN. PROPERTIES OF THE PERMITTIVITY (DIELECTRIC CONSTANT)

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START WITH

$$\epsilon_1 = 1 + \frac{2}{\pi} \int_0^{\infty} \epsilon_2(\omega') \frac{\omega'}{\omega'^2 - \omega^2} d\omega'$$

FOR $\hbar\omega \gg$ energy of interband transitions
EXPLAIN \uparrow

$$\epsilon_1(\omega) \approx 1 - \frac{2}{\pi\omega^2} \int_0^{\infty} \epsilon_2(\omega') \omega' d\omega'$$

$$\equiv 1 - \frac{8}{\omega^2} \int_0^{\infty} \sigma_1(\omega') d\omega'$$

$$\Rightarrow \epsilon_1(\omega) \equiv 1 - \frac{\omega_p^2}{\omega^2}$$

VALID FOR ALL SOLIDS \uparrow

$$\omega_p^2 = 8 \int_0^{\infty} \sigma_1(\omega') d\omega'$$

WE WILL LATER SEE THAT

$$\omega_p^2 = \frac{4\pi N e^2}{m}$$

$$\omega_p \approx 20 \text{ eV}$$

DRUDE MODEL (METALS)

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ASSUMPTION : DELTA PULSE OF ELECTRIC FIELD LEADS TO CURRENT DENSITY OF THE FORM

$$J = J_0 e^{-\gamma t}$$

SINCE $J(t) = \frac{\partial P}{\partial t} = \frac{1}{4\pi} \frac{\partial}{\partial t} \int_0^{\infty} F(t') E(t-t') dt'$

$$= \frac{1}{4\pi} \frac{\partial}{\partial t} \left[\int_0^{\infty} F(t') e^{-i\Omega(t-t')} E_{\Omega} d\Omega dt' \right]$$

INDEPENDENT OF Ω , IF $E \propto \delta(t)$

$$E(t) = 2\pi E_{\Omega} \delta(t)$$

$$= -\frac{i\Omega}{4\pi} E_{\Omega} \int_{-\infty}^{\infty} e^{-i\Omega t} d\Omega \int_0^{\infty} F(t') e^{i\Omega t'} dt'$$

$[E(\Omega) - 1]$

→ $J(t) = E_{\Omega} \int_{-\infty}^{\infty} e^{-i\Omega t} \sigma(\Omega) d\Omega$

$\times e^{i\Omega' t} dt$ + INTEGRATE USING THAT $\int_{-\infty}^{\infty} e^{-i\alpha\beta} d\beta = 2\pi\delta$

$$\sigma(\Omega') = \frac{1}{2\pi E_{\Omega}} \int_0^{\infty} J(t) e^{i\Omega' t} dt$$



$$\sigma(\omega) \propto \frac{1}{\delta - i\omega}$$

THE PROPORTIONALITY CONSTANT CAN BE ESTIMATED USING THAT

$$\sigma(\omega=0) \equiv \frac{Ne^2}{m}$$

$$\delta \equiv \frac{1}{\tau}$$

RELAXATION TIME

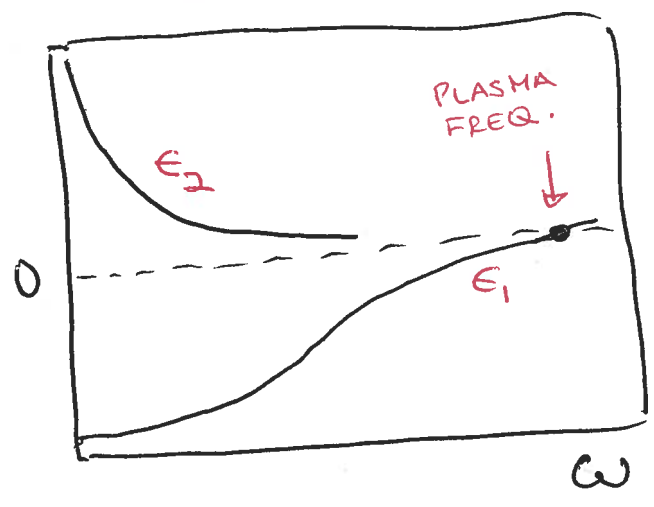
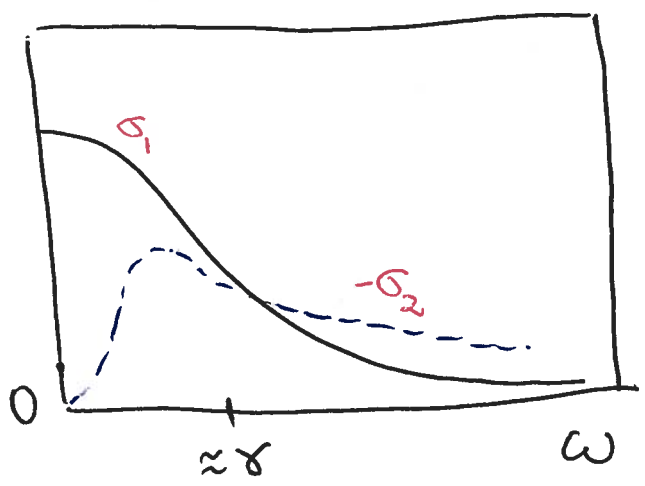
WE THEN HAVE

$$\epsilon_1(\omega) = 1 + \frac{4\pi}{\omega} \sigma_2(\omega) = 1 - \frac{4\pi Ne^2}{m} \frac{1}{\omega^2 + \delta^2}$$

CORRECT BEHAVIOR @ $\omega \rightarrow \infty$

$$\epsilon_2(\omega) = \frac{4\pi Ne^2}{m} \frac{\delta}{\omega} [\omega^2 + \delta^2]$$

DIVERGES FOR $\omega \rightarrow 0$ VERY IMPORTANT



LORENTZ MODEL

(CLASSICAL
OSCILLATORS

OR ASSEMBLY OF
INDEPENDENT
NEUTRAL ATOMS)

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... PHONONS, EXCITONS

LATER QM



$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = \frac{-e\bar{E}_0}{m} e^{-i\omega t}$$

$$\Rightarrow x = \frac{-e\bar{E}_0}{m} e^{-i\omega t} \frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

THE ELECTRIC-DIPOLE MOMENT PER UNIT
OF VOLUME, i.e., THE POLARIZATION IS:

$$P_x = -Nex$$

SINCE $\epsilon(\omega) = 1 + 4\pi P_x / E_x$

WE GET

$$\epsilon(\omega) = 1 + \frac{4\pi Ne^2}{m} (\omega_0^2 - \omega^2 - i\omega\gamma)^{-1}$$

LORENTZIAN

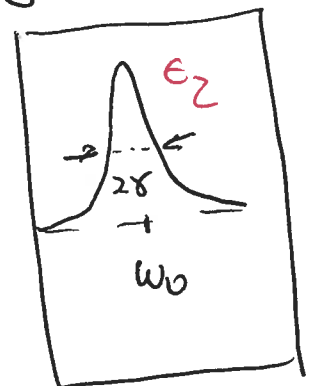
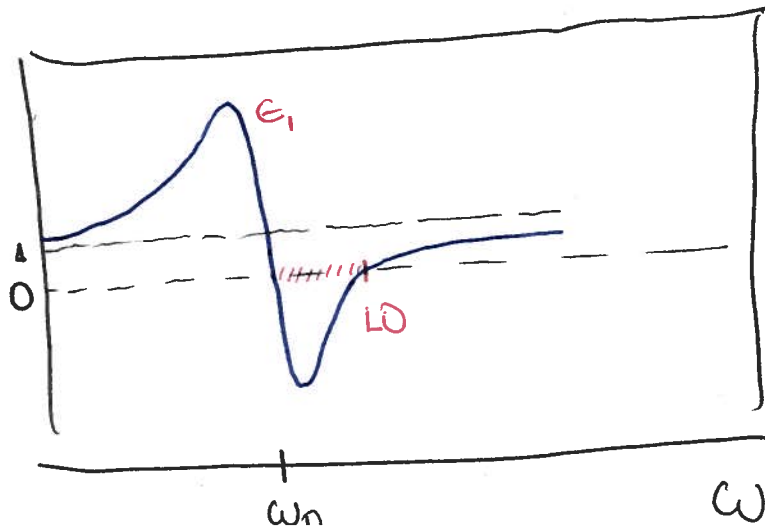
$$\epsilon_1(\omega) = \frac{1 + \omega_p^2 (\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + \omega^2 \gamma^2}$$

$$\epsilon_2(\omega) = \frac{\omega_p^2 \omega \gamma}{(\omega_0^2 - \omega^2)^2 + \omega^2 \gamma^2} \approx \frac{\omega_p^2 (\gamma/4\omega_0)}{(\omega - \omega_0)^2 + \gamma^2/4}$$

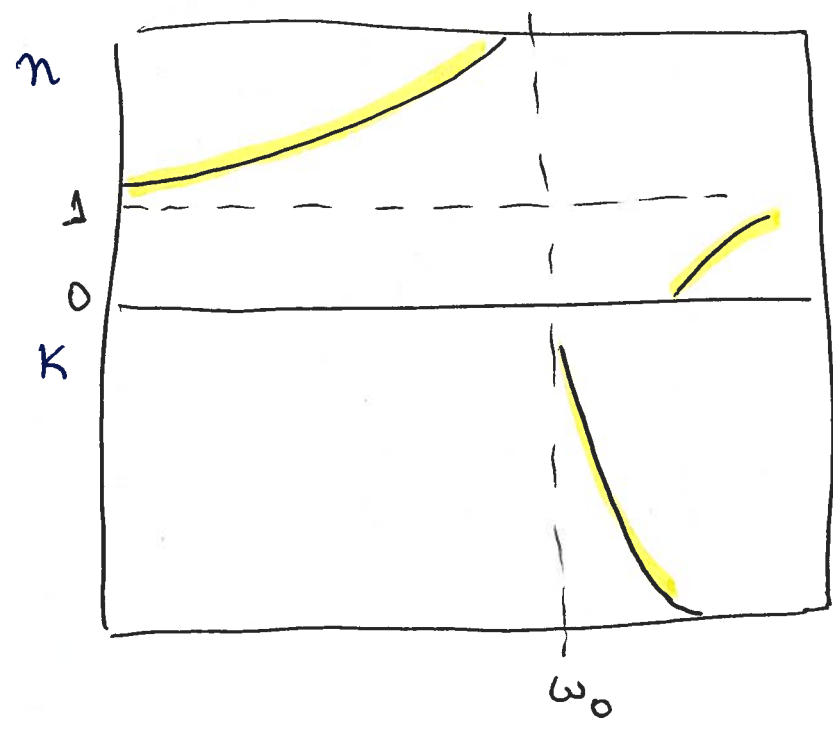
VERY DIFFERENT FROM METALS !
AT LOW FREQS.

$$\omega \rightarrow 0 \quad \left\{ \begin{array}{l} \epsilon_1 \rightarrow 1 + \frac{\omega_p^2}{\omega_0^2} > 1 \\ \epsilon_2 \rightarrow 0 \end{array} \right.$$

$$\omega \rightarrow \infty \quad \left\{ \begin{array}{l} \epsilon_1 \rightarrow 1 - \frac{\omega_p^2}{\omega^2} \\ \epsilon_2 \rightarrow \frac{\omega_p^2 \gamma}{\omega^3} \end{array} \right. \quad \text{IDENTICAL TO METALS}$$



HOW ABOUT THE REFRACTIVE INDEX?



REFLECTIVITY ?

