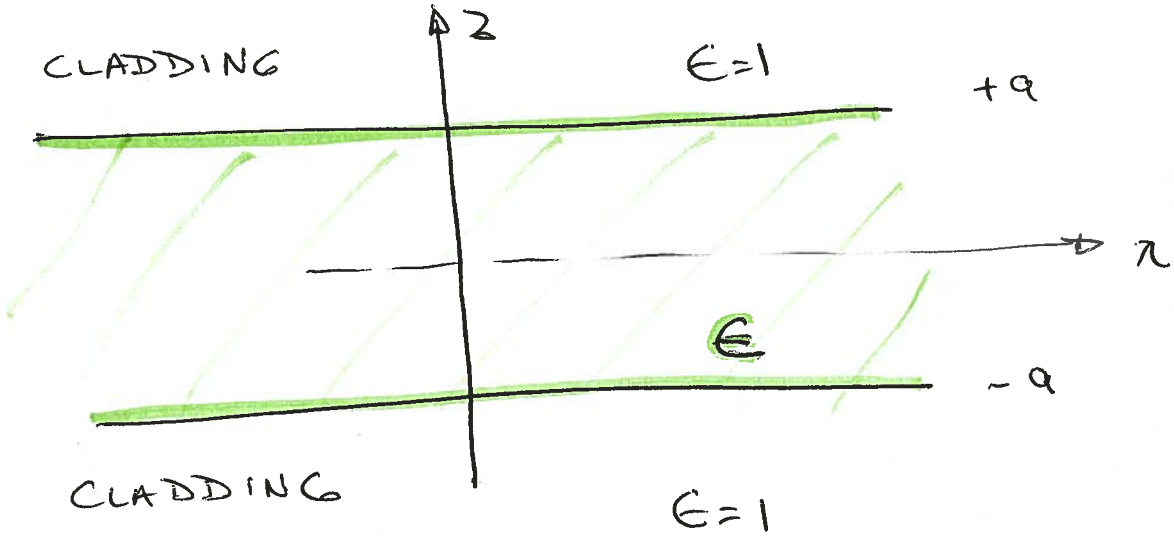
CONSIDER NOW A SLAB



FIELDS DECAY EXPONENTIALLY IN CLADDING LAYERS

TM MODES

$z > a$

$$\left.\begin{aligned} H_y &= A \\ E_x &= iA\alpha c/\omega \\ E_z &= -Aqc/\omega \end{aligned}\right\} \times e^{iqx-\alpha z}$$

$$\boxed{q^2-\alpha^2=k_0^2}$$

$z < a$

$$\left.\begin{aligned} H_y &= B \\ E_x &= -iB\alpha c/\omega \\ E_z &= -Bqc/\omega \end{aligned}\right\} \times e^{iqx+\alpha z}$$

$|z| \le a$

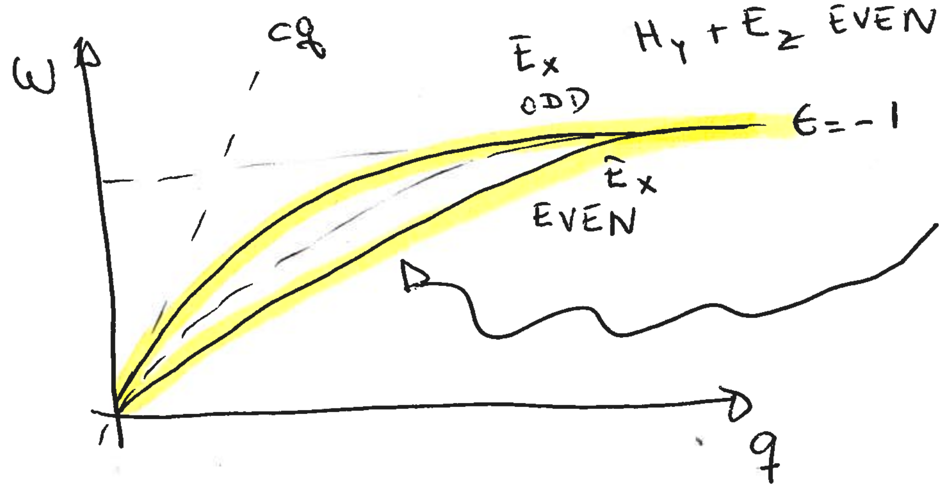
$$\boxed{q^2 - \beta^2 = \epsilon k_0^2}$$

$$\begin{cases} H_y = C e^{iqx} e^{\beta z} + D e^{iqx} e^{-\beta z} \\ E_x = -i \dfrac{C\beta}{\omega\epsilon} e^{iqx} e^{\beta z} + i \dfrac{D\beta c}{\omega\epsilon} e^{iqx} e^{-\beta z} \\ E_z = -C \dfrac{q}{\omega\epsilon} e^{iqx} e^{\beta z} - D \dfrac{qc}{\omega\epsilon} e^{iqx} e^{-\beta z} \end{cases}$$

AFTER APPLYING BOUNDARY CONDITIONS, THE SOLUTIONS ARE:

$$\tanh\left[\sqrt{q^2 - \epsilon k_0^2}\, a\right] = \begin{cases} -\epsilon \sqrt{\dfrac{q^2 - k_0^2}{q^2 - \epsilon k_0^2}} \\ -\dfrac{1}{\epsilon} \sqrt{\dfrac{q^2 - \epsilon k_0^2}{q^2 - k_0^2}} \end{cases}$$

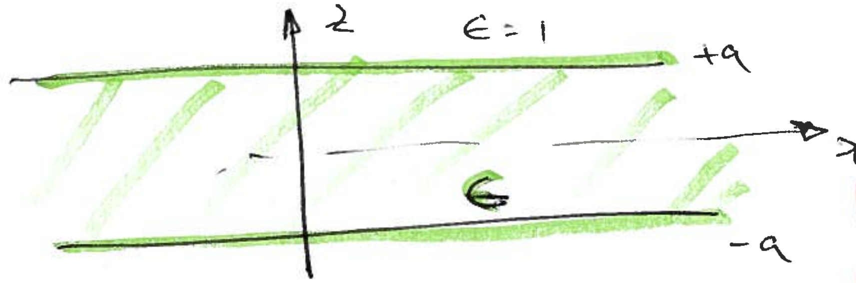
FOR A DRUDE METAL,



NOT SFCE. BUT **SLAB** MODES

@ LARGE q, LOCALIZED @ SFCES

• ELECTROSTATIC APPROXIMATION



$$\boxed{q \gg k_0}$$

$$\vec{E} = -\vec{\nabla}\phi \Rightarrow \Delta\phi = 0$$

BOUNDARY CONDITIONS: ϕ + D_z ARE CONTINUOUS

$$\begin{cases} z > a & \phi = A e^{-qz} e^{iqx} \\ |z| < a & \phi = \left(B e^{-qz} + C e^{qz}\right) e^{iqx} \\ z < a & \phi = D e^{qz} e^{iqx} \end{cases}$$

$$\begin{cases} A e^{-qa} = B e^{-qa} + C e^{qa} \\ -\cancel{q} A e^{-qa} = \left[-\cancel{q} B e^{-qa} + \cancel{q} C e^{qa}\right]\epsilon \\ D e^{-qa} = B e^{qa} + C e^{-qa} \\ \cancel{q} D e^{-qa} = \left[-\cancel{q} B e^{qa} + \cancel{q} C e^{-qa}\right]\epsilon \end{cases}$$

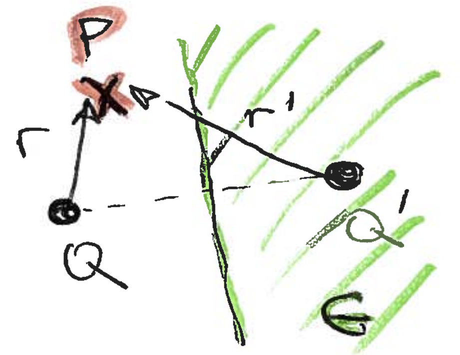
$$\Rightarrow e^{-4qa} = \left(\frac{\epsilon+1}{\epsilon-1}\right)^2 \qquad \boxed{\tanh qa = \begin{cases} -\epsilon \\ -\dfrac{1}{\epsilon} \end{cases}}$$

FOR $a \to \infty$, THE ELECTROSTATIC APPROX. GIVES $\boxed{\epsilon = -1}$

CONSIDER THE PROBLEM OF A CHARGE Q @ AN ARBITRARY DISTANCE FROM A PLANE BOUNDARY

USING METHOD OF IMAGES:

$$\phi_p = \frac{Q}{r} + \frac{Q}{r'}\left[\frac{1-\epsilon}{1+\epsilon}\right] \xrightarrow[\epsilon=-1]{} \infty$$



NOTE: PLASMONS ARE NOT THE ONLY MODES THAT GIVE EVANESCENT BEHAVIOR IN CLADDING LAYERS

DIELECTRICS GIVE WAVEGUIDE MODES (OPTICAL FIBER MODES), FOR WHICH THE DISPERSION IS:

$$\tan\left(k_0 a\sqrt{2\Delta} - p\frac{\pi}{2}\right) = f\sqrt{\frac{1}{\xi^2} - 1}$$

p: INTEGER

$$\Delta = \frac{\epsilon - 1}{2\epsilon} \qquad \boxed{\epsilon > 1} \qquad \xi = \left[\frac{1 - \beta^2/k_0^2}{2\Delta}\right]^{1/2}$$

$$f = \begin{cases} 1 & \text{TE MODES} \\ \dfrac{1}{1-2\Delta} & \text{TM MODES} \end{cases}$$

- TOTAL INTERNAL REFLECTION

PLASMONS IN SPHERICAL PARTICLES

MIE SOLVED THE PROBLEM OF SCATTERING OF A PLANE WAVE BY A SPHERE OF RADIUS a IN 1908.

THERE ARE TWO TYPES OF MODES:

$$\begin{cases} H_r = 0 & \text{(TM-MODES OF E-WAVES)} \\ E_r = 0 & \text{(TE-MODES OF H-WAVES)} \end{cases}$$

DEFINE: $x = \omega a / c$

SPHERICAL BESSEL FUNCTIONS

$$\Rightarrow f(x,\epsilon) = \frac{\left[x\, h_\ell^{(1)}(x)\right]'}{\left[\sqrt{\epsilon}\, x\, j_\ell(\sqrt{\epsilon}\, x)\right]'} \times \frac{j_\ell(\sqrt{\epsilon}\, x)}{h_\ell^{(1)}(x)}$$

THEN, $f = 1/\epsilon$ (TM) OR $f = 1$ (TE) GIVE THE NATURAL FREQUENCIES.

- AS FOR SLABS, THERE ARE PLASMON-LIKE SOLUTIONS ($\epsilon < 0$) AS WELL AS CAVITY-LIKE MODES FOR DIELECTRICS.

CONSIDER NOW THE ELECTROSTATIC LIMIT : $x \ll 1$ $\quad |\sqrt{\epsilon}|\, x \ll 1$

USING THAT

$$j_s(\rho) \approx \frac{\rho^s}{(2s+1)!!} \qquad h_s^{(1)}(\rho) \approx -i\,\frac{(2s-1)!!}{\rho^{s+1}}$$

WE GET $\quad f \approx \frac{-s}{s+1} \Rightarrow \boxed{\epsilon = -\frac{(s+1)}{s}}$

BUT, FOR $s=0$ $\quad \tanh|\sqrt{\epsilon}|x = \frac{1}{|\sqrt{\epsilon}|x}$

SAME RESULT CAN BE OBTAINED BY SOLVING POISSON'S EQUATION:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}\right] = 0$$

SOLUTIONS ARE OF THE FORM

$$\boxed{\Phi = Y_{lm}(\theta,\phi)\,R(r)}$$

WHERE $\quad \frac{1}{r^2}\frac{d}{dr}\left(r^2\dot{R}\right) = \frac{l(l+1)}{r^2}R$

FOR $\quad R \propto r^t \Rightarrow t(t+1) = l(l+1)$

THUS
$$\begin{cases} R = A r^{l} & (r<a) \\ R = \dfrac{B}{r^{l+1}} & (r>a) \end{cases}$$

USING BOUNDARY CONDITIONS FOR Φ & D_r, WE GOT $\boxed{\epsilon = -\dfrac{(l+1)}{l}}$

- THE MODE $l=0$ CORRESPONDS TO $\epsilon = -\infty$. THE FIELD VANISHES INSIDE THE SPHERE - HOWEVER, ONE CANNOT APPLY THE ELECTROSTATIC APPROXIMATION
- THE MODE $l=1$ BEHAVES AS AN ELECTRIC DIPOLE. IT CORRESPONDS TO $\epsilon=-2$. THIS IS THE ONLY MODE THAT COUPLES TO A UNIFORM ELECTRIC FIELD:

$$\begin{cases} r<a & \Phi = \tilde{A}\, r \underbrace{\cos\theta}_{Y_{10}} \\ r>a & \Phi = \dfrac{\tilde{B}}{r^2}\cos\theta - \xi \underbrace{r\cos\theta}_{z} \end{cases}$$

EXTERNAL FIELD ALONG $\hat{z}$

$$\tilde{A} a = \tilde{B}/a^2 - \xi a \quad + \quad \tilde{A} = -2\tilde{B}/a^3 - \xi$$

$$\tilde{B} = \xi a^3 \frac{\epsilon - 1}{\epsilon + 2} \qquad \tilde{A} = -\frac{3}{\epsilon+2}\xi$$

THEY DIVERGE FOR $\epsilon \to -2$

ALSO, NOTE THAT THE TERM $B\cos\theta/r^2$ CAN BE WRITTEN AS

$$\boxed{\frac{\tilde{B}\,\hat{e}_z \cdot \hat{r}}{r^3}}$$

THIS REPRESENTS THE POTENTIAL DUE TO AN ELECTRIC DIPOLE ORIENTED ALONG $\hat{z}$.

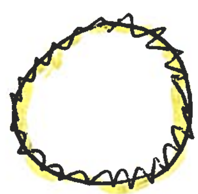
THE (INDUCED) DIPOLE IS GIVEN BY

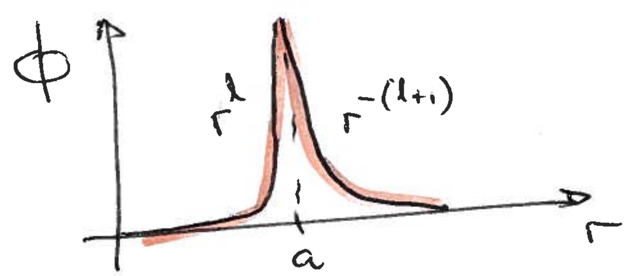
$$\vec{p} = V\alpha\vec{E} \quad \text{WHERE } V = \tfrac{4}{3}\pi a^3$$

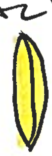
AND THE ELECTRIC POLARIZABILITY IS

$$\boxed{\alpha = \frac{3(\epsilon - 1)}{4\pi(\epsilon + 2)}}$$

- NOTE THAT FOR $l \to \infty$, $\epsilon \to -1$. THIS IS AGAIN THE SFCE. PLASMON WHICH WE STUDIED FOR THE SEMI ∞ SLAB. IT IS STRONGLY LOCALIZED ON THE SFCE. OF THE SPHERE





EXACT (ANALYTICAL) RESULTS CAN BE OBTAINED FOR PROLATE  AND OBLATE  SPHEROIDS, & ELLIPTICAL CYLINDERS.

IGNORING RETARDATION (i.e., IN THE ELECTROSTATIC LIMIT) MANY OTHER SHAPES CAN BE SOLVED.

→ Bohren & Huffman, Absorption & Scatt. of Light by Small Particles (Wiley, 1983 & 2004)