1:

$D(E)dE = \frac{1}{(2\pi)^2}\int dS_k \qquad \int dS_k = \int dl_E dK_\perp$

dl_E *is the line of constant energy, defined by* $E = \frac{\hbar^2}{2m}(k_x^2 - k_y^2)$

$dl_E = \sqrt{dk_x^2 + dk_y^2}$

$k_y = \sqrt{\frac{2mE}{\hbar^2} - dk_x^2}$

$dk_y = \frac{k_x dk_x}{\sqrt{\frac{2mE}{\hbar^2} - k_x^2}}$

$dk_y^2 = \frac{k_x^2 dk_x^2}{\frac{2mE}{\hbar^2} - k_x^2}$

So $dl_E = \sqrt{1 + \frac{k_x^2}{\frac{2mE}{\hbar^2} - k_x^2}} = \left(\frac{\frac{2mE}{\hbar^2}}{\frac{2mE}{\hbar^2} - k_x^2}\right)^{\frac{1}{2}}$

and dE = $\frac{\hbar^2}{m}\sqrt{\frac{2mE}{\hbar^2}}dK_\perp$

So D(E) = $\frac{1}{(2\pi)^2}\int \frac{\frac{2mE}{\hbar^2}}{\sqrt{\frac{2mE}{\hbar^2} - k_x^2}} \frac{dk_x dK_\perp}{\frac{\hbar^2}{m}\sqrt{\frac{2mE}{\hbar^2}}dK_\perp} = \frac{m}{(2\pi\hbar)^2}\int \frac{dk_x}{\sqrt{\frac{2mE}{\hbar^2} - k_x^2}} = \frac{m}{(2\pi\hbar)^2}\ln(k_x + \sqrt{\frac{2mE}{\hbar^2} - k_x^2})$

Look at the edge of the Brillion Zone: $E = E - E_0, \; k_x \gg E - E_0$

So $D(E) = \frac{m}{(2\pi\hbar)^2}\ln(k_x + \frac{2m(E-E_0)}{\hbar^2 k_x} - k_x)$

So $D(E) \approx \ln(E - E_0^*)$

2:

$\epsilon_2 \propto (E - E_0)^{1/2}$ at the van hove singularity means that

$$\epsilon_2 \propto \frac{1}{w^2}(E - E_0)^{1/2}$$

Then $\epsilon_1 = 1 + \frac{A}{\pi} P \int_{-\infty}^{\infty} \frac{\sqrt{w'-w_0}}{w'^2(w'-w)} dw'$

2 residues, at $w' = 0 \; and \; w' = w$

$w' = 0$:

$$\pi \mathbf{i} \lim_{w'\to 0}\left(\frac{\partial}{\partial w'} \frac{\sqrt{w'-w_0}}{(w'-w)}\right) = \pi \mathbf{i} \lim_{w'\to 0}\left(\frac{1}{2\sqrt{w'-w_0}(w'-w)} - \frac{\sqrt{w'-w_0}}{(w'-w)^2}\right)\backslash$$

$$= \pi \frac{(-w+2w_0)}{2w^2\sqrt{w_0}}$$

$w' = w$:

$$\pi \mathbf{i} \frac{\sqrt{w-w_0}}{w^2} = \pi \frac{\sqrt{w_0-w}}{w^2}$$

So $\epsilon_1 = 1 + A\,(w - 2w_0 + 2\sqrt{w_0}\sqrt{w_0 - w})/(2w^2\sqrt{w_0})$

3, a) Near $k=0$, k is approximately constant, so we can take it out:

$\sum_n |\langle u_0 | \vec{E}\cdot\hat{p} | u_n\rangle|^2 \approx \sum_n k^2 |\langle u_0 | \vec{\epsilon}\cdot\hat{p} | u_n\rangle|^2$

Let us also assume all but one band is sufficiently far away to be ignored, so that we ignore the series. Then

$$\frac{1}{m^*} = \frac{1}{m} + \frac{2}{m^2}\frac{|\langle u_v|\hat{p}|u_c\rangle|^2}{E_0 - E_n}$$

$|\langle u_v|\hat{p}|u_c\rangle|^2 = P$, so

$$\frac{1}{m^*} = \frac{1}{m} + \frac{2}{m^2}\frac{P^2}{E_g}$$

$$\frac{E_g}{4}\left(\frac{1}{m^*} - \frac{1}{m}\right) = \frac{P^2}{2m^2}$$

$$\frac{P^2}{2m} = \frac{E_g}{4}\left(\frac{m}{m^*} - 1\right)$$

b) These do not work in this model, as Si and Ge are indirect band gap materials. Therefore we cannot remove k from the sum.

3.

	Energy Gap (eV)	m*	Lattice Constant	P^2	P^2/2m
Si	1.107	1.100	5.43072	-2.571E+04	-0.025159091
Ge	0.670	0.550	5.65754	1.401E+05	0.137045455
Alpha - Sn	0.080	0.020	6.49120	1.002E+06	0.98
Te	0.330	0.190	4.45000	3.594E+05	0.351710526
III-V compounds					
AlSb	1.600	0.090	6.13550	4.133E+06	4.044444444
GaP	2.240	0.350	5.49050	1.063E+06	1.04
GaAs	1.350	0.068	5.65315	4.728E+06	4.625735294
GaSb	0.670	0.050	6.09540	3.253E+06	3.1825
InP	1.270	0.067	5.86875	4.519E+06	4.42130597
InAs	0.360	0.022	6.05838	4.089E+06	4.000909091
InSb	0.165	0.014	6.47877	2.969E+06	2.905178571
II-VI compounds					
ZnO	3.200	0.380	4.63000	1.334E+06	1.305263158
CdO	2.500	0.100	4.69530	5.749E+06	5.625
CdS	2.400	0.165	5.83200	3.103E+06	3.036363636
CdSe	1.740	0.130	6.05000	2.975E+06	2.911153846
HgSe	0.300	0.030	6.08400	2.478E+06	2.425
HgTe	0.150	0.017	6.46230	2.216E+06	2.168382353
Halite structure compounds					
PbS	0.370	0.160	5.93620	4.963E+05	0.485625
PbSe	0.260	0.300	6.12430	1.550E+05	0.151666667
PbTe	0.250	0.210	6.45400	2.403E+05	0.235119048
Others					
Bi2Te3	0.130	0.580	4.38000	2.405E+04	0.023534483
Mg2Si	0.770	0.460	6.33800	2.309E+05	0.225978261
Mg2Sn	0.210	0.370	6.76500	9.136E+04	0.089391892

Average: 2.016877623 eV