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Semiclassical theory of fusion and breakup reactions

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Abstract. We discuss the use of the semiclassical approximation of Alder and Winther in the description of fusion and breakup reactions with weakly bound nuclei. A comparison with realistic CDCC calculations shows that the method leads to accurate predictions for the breakup cross section. We show how the method can be extended to fusion reactions and apply it to a schematic two-channel model, in which the breakup states are approximated by a single effective channel. The complete fusion cross section so evaluated compares very well with predictions of fully quantum mechanical coupled-channels calculations.

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1. Introduction

The effects of channel coupling on the fusion of weakly bound nuclei on heavy targets has attracted great interest over the last decade [1]. Recently, several measurements of fusion cross sections in reactions with stable and radioactive weakly bound nuclei have been performed [2].

The first calculations of the complete fusion cross section for weakly bound projectiles lead to different conclusions. Some calculations predicted a suppression of this cross section [3] while others predicted enhancement [4]. However, these calculations were based on very schematic models, which did not take into account all the relevant properties of the breakup channel. Recently, realistic coupled-channels calculations of breakup [5] and fusion cross sections were performed [6]. These calculations were based on the Continuum Discretized Coupled-Channel (CDCC) method. Although the CDCC method is the natural way to describe coupled-

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channels problems involving the continuum, calculations based on this method are very complicated and requires considerable computer power. In this paper, we review some recent works based on the semiclassical theory of Alder and Winther (AW) [7]. The paper is organized as follows. In section 2, we present the semiclassical approach to the coupled-channels problem. In section 3 we discuss its application to study the breakup of weakly bound projectiles. In section 4, we show how the method can be used to calculate the complete fusion cross section in collisions of weakly bound projectiles. In section 5 we present the calculation of upper bounds to the incomplete fusion cross section. Finally, in section 6, we give our conclusions.

2. The semiclassical coupled-channels equations

Let us consider the collision described by the Hamiltonian

$$H = H_0(\mathbf{r}) + h(\xi) + V(\xi, \mathbf{r}), \tag{1}$$

where $\mathbf{r}$ is the projectile-target separation vector and ξ stands for the set of relevant intrinsic coordinates of the projectile and the target. The AW method consists of treating the relative motion by classical mechanics whereas the intrinsic motion is handled as a time-dependent quantum mechanics problem. Solving the classical equations of motion with the Hamiltonian H_0, one obtains the classical trajectory $r_b(t)$ for each impact parameter b and total energy E. The intrinsic wave funtion is then the solution of the Schrödinger equation with the time-dependent Hamiltonian

$$\mathcal{H}(\xi, t) = h(\xi) + V(\xi, r_b(t)) \equiv h(\xi) + V(\xi, t). \tag{2}$$

That is,

$$\mathcal{H}(\xi, t)\ \psi(\xi, t) = i\hbar\ \frac{\partial \psi(\xi, t)}{\partial t}. \tag{3}$$

Expanding $\psi(\xi, t)$ in terms of the eigenfuntions of h (truncated at $\alpha = N$) and inserting the expansion in eq.(3), one obtains the set of coupled differential equations on the time-variable,

$$i\hbar \dot{a}_\alpha(t) = \sum_\beta V_{\alpha,\beta}(t)\ e^{i(\varepsilon_\alpha - \varepsilon_\beta)t/\hbar}\ a_\beta(t); \qquad \alpha, \beta = 0, 1, ..., N. \tag{4}$$

Above, $V_{\alpha,\beta}(t) = \int d\xi\ \varphi^*_\alpha(\xi)\ V(\xi, t)\ \varphi_\beta(\xi)$, where φ^*_α (φ_β) is the eigenstate of h with eigenvalue ε_α (ε_β). The above equations should then be solved with the initial conditions

$$a_\alpha(t \to -\infty) = \delta(\alpha, 1), \tag{5}$$

which means that before the collision the projetile-target system was in its ground state.

The cross sections are then given in terms of the final values of the Alder-Winther amplitudes. The integrated cross section for channel-α is given by the integral over impact parameter

$$\sigma_\alpha = 2\pi \int P_\alpha(b)\ b\ db; \qquad \text{with} \quad P_\alpha(b) = |a_\alpha(t \to \infty)|^2 . \tag{6}$$

Angular distributions in channel-α can be expressed in terms of the elastic cross section and the final population of this channel,

$$\frac{d\sigma_\alpha(\theta)}{d\Omega} = \frac{d\sigma_{el}(\theta)}{d\Omega}\ P_\alpha(b_\theta)\ . \tag{7}$$

Above, b_θ is the impact parameter associated with the scattering angle θ through the classical trajectory. Usually, the elastic cross section is approximated by its classical value. In this case, it should be multiplied by a factor $A_{abs}(b_\theta)$, which accounts for the absorption along the classical trajectory.

3. Breakup reactions

The procedure described in the previous section is particularly useful in problems involving a large number of coupled channels, where fully quantum mechanical calculations become very complicated. This is the situation where the elastic channel is strongly coupled to breakup states. In this case, one of the collision partners, usually the projectile, breaks into one or more fragments moving in the continuum. The channel label is then continuous and the problem becomes very complicated. This difficulty is usually handled by the CDCC method [8], where the continuum is discretized in a set of bins of variable size. Recently it has been used in the realistic calculations of Nunes and Thompson [5] to study ^{8}B breakup in the ^{8}B + ^{58}Ni collision, at E_{lab} = 26 MeV. The resulting angular distribution is represented by a solid line in figure 1. The authors have pointed out that continuum-continuum couplings play a very important role in the calculations.

Marta *et al.* [9] studied the same problem with the Alder-Winther method. They discretized the continuum using the same bins and angular momentum states as in ref. [5]. Their results correspond to the dashed line in figure 1. The agreement with the CDCC calculations is very good. This suggests that the semiclassical method may be an important tool to study nuclear reactions induced by weakly bound projectiles.

4. Extension to fusion reactions

An extension of the AW theory to fusion reactions has been proposed in ref. [10]. Before discussing this work, we recall the evaluation of the fusion cross section in a quantum mechanical coupled-channels calculation. For the time being, we assume that all channels are bound and have spin zero. Assuming that the channel-coupling interaction is real, the total fusion cross section is a sum of separate contributions

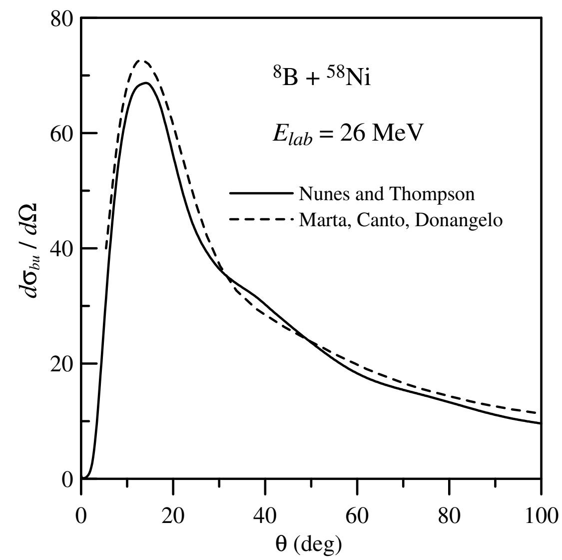

Fig. 1. ^{8}B breakup angular distributions obtained with the semiclassical method (dashed line) and with CDCC calculations (solid line).

from each channel. Carrying out partial-wave expansions one gets

$$\sigma_{TF} = \sum_{\alpha} \left[\frac{\pi}{k^2} \sum_{l} (2l+1) \; P_l^F(\alpha) \right], \tag{8}$$

with

$$P_l^F(\alpha) = \frac{4k}{E} \int dr \; |u_{\alpha l}(k_\alpha, r)|^2 \; W_\alpha^F(r). \tag{9}$$

Above, $u_{\alpha l}(k_\alpha, r)$ represents the radial wave function for the l^{th}-partial-wave in channel α and W_α^F is the absolute value of the imaginary part of the optical potential associated to fusion in this channel.

The basic idea of ref. [10] is to approximate the fusion probabilities as

$$P_l^F(\alpha) \simeq \bar{P}_l^{(\alpha)} \; T_l^{(\alpha)}(E_\alpha). \tag{10}$$

Above, $T_l^{(\alpha)}(E_\alpha)$ is the probability that a particle with energy $E_\alpha = E - \varepsilon_\alpha$ and reduced mass $\mu_\alpha = m_0 A_P A_T / (A_P + A_T)$ tunnels through the potential barrier in channel-α, and $\bar{P}_l^{(\alpha)}$ is the probability that the system is in that channel at the point of closest approach on the classical trajectory. This method was used to evaluate complete (CF) and incomplete fusion (ICF) cross sections in reactions induced by weakly bound projectiles. For simplicity, the ground state was assumed to be the only bound state of the projectile, with breakup processes producing two fragments, F_1 and F_2. In this way, labels $\alpha = 0$ and $\alpha \neq 0$ correspond, respectively, to the GS and the breakup states represented by two unbound fragments. Neglecting any sequential contribution, CF can only arise from the elastic channel. Thus the sum over channels in eq.(8) is reduced to a single term with

$$\bar{P}_l^{(0)} \equiv P_l^{surv} = |a_0(t_{c.a.})|^2 \,, \tag{11}$$

where the amplitude a_0 is evaluated along a trajectory with impact parameter $b = l/k$. The factor P_l^{surv} is usually called survival (to breakup) probability. The CF cross section can then be written

$$\sigma_{CF} = \frac{\pi}{k^2} \sum_l (2l+1)\ \ P_l^{surv}\ T_l^{(0)}(E). \tag{12}$$

The accuracy of this procedure was checked in a preliminary two-channel calculation for the scattering of ^{6}He projectiles on a ^{238}U target, at near barrier energies [10]. The weakly bound ^{6}He nucleus dissociates into ^{4}He and two neutrons, with threshold energy $B = 0.975$ MeV. The elastic channel is strongly coupled to the breakup channel and the influence of this coupling on the fusion cross section is very important. In this model, the breakup channel is represented by a single effective state [11]. For simplicity, the effective channel was treated as a bound state but it was assumed to contribute only to incomplete fusion. In this way, the CF cross section was given by eq.(12). The threshold energy was neglected and the same potential barrier was used for both channels. The optical potential was given by Woods-Saxon parametrizations with $V_0 = -60$ MeV, $r_{0r} = 1.25$ F, $a_r = 0.65$ F, $W_0 = -50$ MeV, $r_{0i} = 1.0$ F and $a_i = 0.1$ F. The form factor had the radial dependence of the electric dipole coupling with an arbitrary strength chosen in such a way that the coupling modifies the cross section of the one dimension penetration barrier appreciably. The CF cross section was shown to be in very good agreement with the results of a full coupled-channels calculation at above barrier energies. However, the agreement was very poor at sub-barrier energies. The semiclassical calculation underestimated the CF cross section drastically [12].

To improve the semiclassical model at sub-barrier energies, one can resort to the analytical continuation method, which consists of introducing the imaginary part of the time variable to obtain a classical trajectory in the sub-barrier region [13]. This procedure is illustrated in figure 2, where the time scale is chosen such that $t = 0$ at the external turning point, r_e. Along the incident branch of the trajectory (A), the time develops on the real axis as the system approaches the barrier. At $r = r_e$, the trajectory splits into two parts: the reflected branch (B) and the classically forbidden transmission branch (C). On the former, which is not

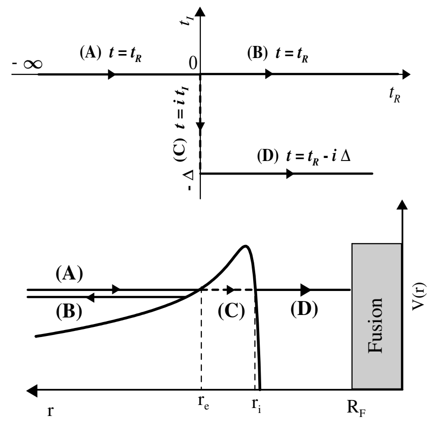

Fig. 2. Analytical continuation of the time variable. The upper panel shows the branches of the classical trajectory and the lower panel the evolution on the complex time plane.

relevant to the fusion process, the time remains real. Along the branch (C), the real part of t remains equal to zero while its imaginary part develops on the negative part of the imaginary axis, until this trajectory reaches the exit point r_i, at $t = -i\Delta$. This trajectory is then continued into the internal classically allowed region (D), towards the strong absorption radius, R_F, where fusion occurs. Over this branch, the real part of t grows whereas its imaginary part keeps the value $t_I = -\Delta$. The fusion probability is then evaluated in terms of the elastic Alder-Winther amplitude calculated along the trajectory $A - C - D$. The survival probabilities become

$$P_l^{surv} = \left|a_0(t_F)\right|^2 , \tag{13}$$

where t_F is the complex value of the time variable at which the system reaches the strong absorption radius.

In order to account for the excitation energy in the breakup channel and simulate the irreversible nature of the breakup process [14], the complex value $E = Q - i\Gamma/2$ was assigned to the energy of the effective channel. One uses the fact that an exponentially decaying state with mean life $\tau = \hbar/\Gamma$, can be obtained through the inclusion of a constant imaginary potential equal to $-i\Gamma/2$ in the system Hamiltonian. This procedure requires some care. Solving the AW equations does not present difficulties since the population of the resonant state is vanishingly small as $t \to -\infty$. The numerical solution of the coupled-channel equations, however, requires attention. To handle this situation one should switch-off the $-i\Gamma/2$ imaginary potential at some distance much larger than the range of the potentials, and then match the radial wave functions with their asymptotic forms [12].

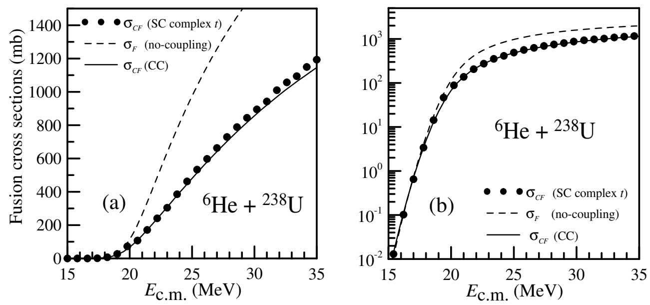

Fig. 3. Quantum mechanical (full line) and semiclassical (full circles) CF cross sections for $B = 0.975$ MeV and $\Gamma = 2$ MeV. At sub-barrier energies, the contribution from the elastic channel alone is already as large as the cross section in the no-coupling limit (dashed line). This is a consequence of channel coupling.

Let us now consider the CF cross sections obtained with the above discussed procedures. The results of the improved semiclassical calculation (solid circles) are shown in figure 3, in comparison with results of the CC method (solid line) and in the no-coupling limit. In order to exhibit the details above and below the barrier, the cross sections are represented on a linear (a) and on a logarithmic (b) scale. Comparing the semiclassical estimate for σ_{CF} with the CC values, we conclude that the improved semiclassical model leads to accurate results, above and below the Coulomb barrier.

5. Incomplete fusion

In ref. [11], the ICF cross section was associated with the contribution from the effective breakup channel to eq.(8). However, this procedure may be wrong, since the potential barrier for the fragments may be very different from the one in the entrance channel. In this section we present a simple model to take into account this effect. For simplicity, we consider collisions of light weakly bound projectiles with a heavy target, where $E_{c.m.} \simeq E_{lab} = E$, and assume that the share of the incident energy corresponding to each fragment is proportional to its mass. Neglecting the relative momentum of the fragments, the total energy in the laboratory frame after breakup is $E' = E - B$, where B is the breakup threshold. The energy of the i^{th} fragment ($i = 1, 2$) is given by

$$E_i = \frac{A_i}{A_P}\, E', \tag{14}$$

where A_i and A_P are respectively the mass numbers of the i^{th} fragment and of the projectile. The angular momentum of the i^{th} fragment, l_i, scales with respect to the projectile-target angular momentum, l, in a similar way. That is,

$$l_i = \frac{A_i}{A_P}\; l' = \frac{A_i}{A_P}\; \sqrt{\frac{E'}{E}}\quad l. \tag{15}$$

The ICF cross sections for the fragments F_1 and F_2 are then approximated by the expressions

$$\sigma_{ICF_1} = \frac{\pi}{k^2}\sum_l (2l+1)\;\; \bar{P}_l^{bup}\;\; T_{l_1}^{(F_1)}(E_1)\;\; \left[1 - T_{l_2}^{(F_2)}(E_2)\right], \tag{16}$$

$$\sigma_{ICF_2} = \frac{\pi}{k^2}\sum_l (2l+1)\;\; \bar{P}_l^{bup}\;\; T_{l_2}^{(F_2)}(E_2)\;\; \left[1 - T_{l_1}^{(F_1)}(E_1)\right], \tag{17}$$

with the total ICF cross section given by

$$\sigma_{ICF} = \sigma_{ICF_1} + \sigma_{ICF_2}. \tag{18}$$

Above, $T_{l_i}^{(F_i)}(E_i)$ is the probability that a fragment F_i with energy E_i and angular momentum l_i tunnels through the barrier associated with its interaction with the target. In each case, $\left[1 - T_{l_j}^{(F_j)}(E_j)\right]$ is the non-tunneling probability for the other fragment. The factor

$$\bar{P}_l^{bup} = |a_1(t_{c.a.})|^2 \tag{19}$$

is the population of the effective breakup channel when the system reaches closest approach, along the classical trajetory with impact parameter $b = l/k$.

A similar procedure leads to the sequential contribution to complete fusion, corresponding to the situation where both fragments tunnels through their interaction barriers with the target. One obtains

$$\sigma_{SCF} = \frac{\pi}{k^2}\sum_l (2l+1)\;\; \bar{P}_l^{bup}\;\; T_{l_1}^{(F_1)}(E_1)\;\; T_{l_2}^{(F_2)}(E_2). \tag{20}$$

It is clear that the schematic two-channel model provides a very crude description of the breakup process. For quantitative calculations of the ICF cross section one should resort to more realistic continuum discretization procedures, as the one used in the CDCC method. However, this simple model may be used to make predictions of upper bounds to the ICF cross section. For this purpose, one assumes that the populations of the breakup channel has their largest possible values. One sets : $\bar{P}_l^{bup} = 1$ for all partial-waves. To illustrate the application of this procedure, we show two examples studied in ref. [15]: (a) ^{7}Li + ^{209}Bi, with the breakup process ^{7}Li $\rightarrow ^3$H + ^{4}He, and (b) ^{6}He $+^{238}$ U, where the breakup process is ^{6}He $\rightarrow$ 2n $+^4$ He. The reduced cross sections at energies slightly above the barrier are shown in figures 4 (a) and (b), respectively. For comparison, the fusion cross sections of the one-dimensional barrier penetration model (BPM) are also shown. In the case of ^{7}Li projectiles, the ICF cross section may be very important. This is consistent

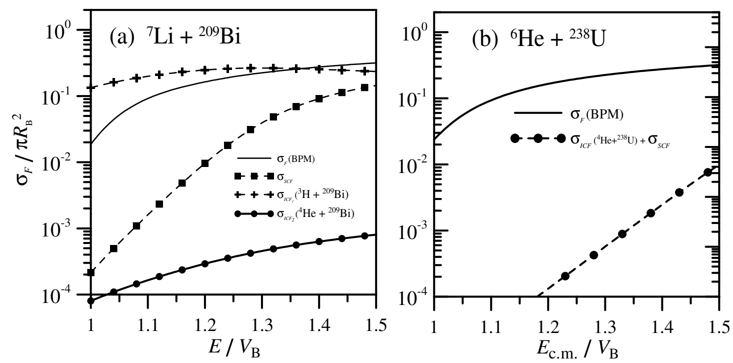

Fig. 4. Upper bounds for incomplete fusion for two systems. For details, see the text.

with the experimental results of Dasgupta *et al.* [2], which indicate that ICF may be responsible for a reduction of $\simeq 30\%$ in the CF cross section at above-barrier energies. The cross section for the fusion of ^{3}H is much larger than that for ^{4}He. This follows from the lower mass and Coulomb barrier for the former fragment. We now consider ICF in the $^6\mathrm{He} + ^{238}\mathrm{U}$ collision. The dashed line represents the upper bound for the fusion of the ^{4}He fragment with ^{238}U. From the experimental point of view, this corresponds to complete fusion, since the whole projectile's charge is captured by the target. Formally, however, it is incomplete fusion. Since the ^{4}He fragment carries roughly $2/3$ of the incident energy whereas its barrier is slightly higher than that for the projectile, it has a very low fusion cross section. The conclusion is that the fusion of ^{4}He with the target at near-barrier energies is negligible. In this way, the formal and the operational definitions for CF are approximately equivalent.

6. Conclusions

The application of the semiclassical method of Alder and Winther to fusion and breakup of weakly bound nuclei has been discussed. The ^{8}B breakup cross section in collisions with a ^{58}Ni target was calculated using a continuum discretized basis for the breakup channel. The results were very close to those of a fully quantum mechanical calculations using the same states.

Complete fusion cross sections were evaluated in a schematic two-channel model, in which the breakup states were mocked up by a single effective channel. In order to describe the fusion cross section at sub-barrier energies it was necessary to carry out the analytical continuation of the time to complex values. A comparison with results of a fully quantum mechanical calculation indicated that the method leads to accurate results, above and below the barrier. The extension of the model to incomplete fusion was discussed and upper bounds for this cross section have been calculated.

The semiclassical method discussed in this paper has the advantages of employing the correct barriers for incomplete fusion and to allow the description of sequential complete fusion. A more quantitative calculation of the fusion cross section using a realistic discretization of the continuum is under development.

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