Acta Phys. Hung. A 0/0 (2005) 000–000

HEAVY ION PHYSICS

Boson mapping treatment of atoms-photon systems.

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Received 9 December 2005

Abstract. A system which consists of A identical atoms, each of them having three atomic levels, and one optical photon is analyzed in terms of an algebraic approach. The symmetries exhibit by the model are used to construct an exact boson mapping of the Hamiltonian. The image-Hamiltonian reduces to a form which contains one boson and two atomic levels.

Keywords: two-photon Dicke model, boson mapping, effective atomic excitations.
PACS: 03.65.Fd, 42.50.-p, 42.50.Hz

1. Introduction

The study of models which contain the interactions between photons and atoms, like the Jaynes-Cummings model [1], is of interest for several branches of physics [2, 3]. Generally speaking these models may be linear [4–8] or quadratic [9–14] in the photon variables.

Klimov et al. have shown in [15] that, a collection of A identical three levels atoms interacting with a quantized radiation field in a perfect cavity, can be described, by an effective interaction consisting of a collection of two-level atoms with a quantized radiation field, at the two-photon level.

In the present work, we aim at the interpretation of the sequential rotations introduced in [15], in terms of: (i) a bi-linear expansion of the atomic sector of the Hamiltonian, and (ii) a subsequent boson mapping. The resulting, mapped, Hamiltonian describes the effective interaction of two level atomic pairs with the external photon.

1219-7580/04/ $ 20.00

2. Formalism

The system consists of A identical three-level atoms in interaction with a radiation field [15]. The atoms and the photon are placed in a perfect cavity. The creation (annihilation) operator for the $i-th$ atomic level ($i=0,1,2$), is denoted by $b_i^\dagger(b_i)$. The operators $b_i^\dagger$ and b_i obey boson commutation relations.

The Hamiltonian of the system reads

$$H = \omega a^\dagger a + \sum_i E_i S^{ii} + g_1\ (a\ S_+^{01} + a^\dagger\ S_-^{01}) + g_2\ (a\ S_+^{12} + a^\dagger\ S_-^{12}), \tag{1}$$

with

$$S_+^{ij} = b_j^\dagger b_i, \quad S_z^{ij} = \frac{1}{2}(S^{jj} - S^{ii}). \tag{2}$$

In the expression of H of Eq. (1), ω is the energy of the photon, $a^\dagger(a)$ is the one photon-creation (-annihilation) operator, E_i is the energy of the $i-th$-atomic level, and g_1 and g_2 are coupling constants describing the absorption (emission) of a photon in the presence of an upward (downward) atomic excitation between levels 0 and 1 (term proportional to g_1), and between levels 1 and 2 (term proportional to g_2). The two-photon resonance condition is satisfied by $E_2 - E_0 = 2\omega$.

The operators ${S_+^{ij}, S_-^{ij}, S_z^{ij}}_{i\neq j}$ generate the $(A+1)(A+2)/2$-dimensional symmetric representation of the $su(3)$ algebra. By making use of group techniques, the above Hamiltonian can be solved exactly. Notice that the operator $L = a^\dagger a + 2S_z^{02}$ commutes with H.

We shall introduce the following boson mapping [16]

$$\begin{array}{rcl} S^{00} & = & \beta_0^\dagger\beta_0 \ = \ \hat{n}_0, \quad S^{22} = \beta_2^\dagger\beta_2 \ = \ \hat{n}_2, \quad S^{11} = A - \hat{n}_0 - \hat{n}_2, \\ S_+^{01} & = & \sqrt{A - \hat{n}_0 - \hat{n}_2}\ \beta_0, \quad S_+^{21} = \sqrt{A - \hat{n}_0 - \hat{n}_2}\ \beta_2, \quad S_+^{02} = \beta_2^\dagger\beta_0,. \end{array} \tag{3}$$

The boson image of the Hamiltonian is

$$\begin{array}{rcl} H_B & = & -\frac{2\Delta}{3}A + \omega(a^\dagger a + \beta_2^\dagger\beta_2 - \beta_0^\dagger\beta_0) + \Delta(\beta_0^\dagger\beta_0 + \beta_2^\dagger\beta_2) + \\ & & (g_1 a^\dagger \beta_0^\dagger \sqrt{A - \hat{n}_0 - \hat{n}_2} + g_2 a^\dagger \sqrt{A - \hat{n}_0 - \hat{n}_2}\beta_2 + h.c.) \end{array} \tag{4}$$

Notice that, due to mapping procedure of Eq. (3), the atomic term has been replaced by a constant and that the bosonic Hamiltonian contains the interactions between the photon and the atomic levels with $i = 0$ and 2. Following [16], we have taken $E_0 + E_1 + E_2 = 0$, that is $E_0 = \Delta/3 - \omega$. The boson image of the symmetry operator reads $N_b = a^\dagger a + \hat{n}_2 - \hat{n}_0$.

3. Results and Discussion.

In the following we shall show some results for the time evolution of the atomic inversion, $S_z(t)$. The radiation field was assumed to be initially in a coherent state with $< a^\dagger a >= 100$, and the atoms in their ground state. Figure 1 show the results corresponding to the exact and boson image Hamiltonian, for the parameters $g_1 = 1$, $g_2 = 2$, $\Delta = 1000$, $A = 3$. Both lines coincide within the resolution of the figure.

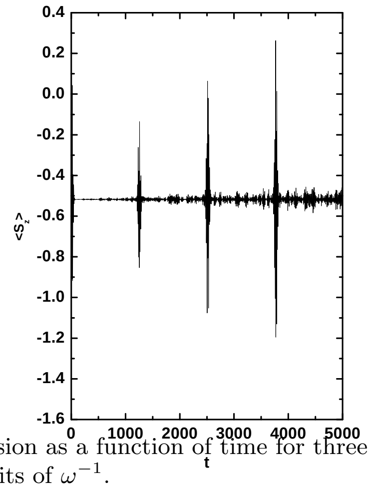

Fig. 1. Atomic inversion as a function of time for three atoms. Since $\hbar = 1$, the time, t, is given in units of ω^{-1}.

4. Conclusions.

In this work, the proposed Hamiltonian, which allows for the activation of three atomic levels, is reduced by the boson mapping to a two-atomic levels situation, with linear interactions in the photon sector. The solutions of the transformed Hamiltonian (4) coincide with the exact ones, and those of [15]. This is evident, particularly, for the appearance and disappearance of the modulation of the revivals. These findings may signal the existence of a certain class of equivalence between boson mapping techniques and algebraic reductions, like the ones implemented in [15]. To conclude, in view of the agreement between the results of the different approximations considered, it becomes evident that the boson mapping is a suitable method for the treatment of atoms-photon interactions.

Acknowledgment

This work has been partially supported by the CONICET (Argentina).

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