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# Quantum Entanglement in Quasispin Models

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**Abstract.** We discuss the concept of quantum entanglement of mixed states, and its behavior in quasispin systems at finite temperature. We examine in particular the limit temperatures for different kinds of entanglement and their relation with the mean field critical temperature.

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Entanglement is one of the most fundamental features of quantum mechanics, representing the potential of composite quantum systems to exhibit correlations with no classical analogue. Recognized already by E. Schrödinger, interest on entanglement has grown enormously in the last few years since its potential for developing radically new forms of information transmission and processing (quantum teleportation, cryptography, computation, etc. [1]) was unveiled, being now considered a *resource* [1]. Nevertheless, many aspects of entanglement remain to be investigated, particularly in mixed states of many-body systems. We will discuss here some features of the *thermal* entanglement in quasispin models familiar to nuclear physics, where progress has been recently made (see [2, 3] and references therein).

A *mixed state* of a bipartite quantum system $A+B$, represented by a density matrix $\rho$, is said to be *separable* or *classically correlated* when it can be written as a statistical superposition of product densities, i.e., $\rho=\sum_\alpha q_\alpha\rho^A_\alpha\otimes\rho^B_\alpha$, $q_\alpha>0$ [4]. Otherwise, $\rho$ is *entangled* or *inseparable*. Separable states satisfy Bell inequalities [1] and other classical properties which can be violated by entangled states. While pure states ($\rho^2=\rho$) are separable just for tensor products $\rho=\rho_A\otimes\rho_B$, this is not the case for mixed states, where the determination of separability is in general an NP hard problem, except for simple systems. Nonetheless, it is known that any mixed state is separable if it is sufficiently close to the completely random state. This implies in particular that any thermal state $\rho(T)\propto\exp[-H/kT]$ of a finite system, where $T$ denotes the temperature and $H$ its Hamiltonian, becomes separable above a *finite* limit temperature for entanglement $T_L$ [2,3]. In other words, there is always a temperature $T_L$ above which the correlations between the system components are of classical character. The value of $T_L$ for bipartite entanglement can be estimated

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from the vanishing of the *negativity* $N[\rho] = (\mathrm{Tr}\,|\rho^{\mathrm{tp}}| - 1)/2$ [5], where tp denotes partial transposition, which is an entanglement measure that is identically zero in any separable state, but positive in any distillable entangled state.

As illustration, we consider a quasispin system described by a Hamiltonian $H = \varepsilon \sum_p s_z^p - \sum_{i,p\neq q} v_i^{pq} s_i^p s_i^q$, where $s_i^p$, $i = x, y, z$, denote spin 1/2 operators at "site" $p$, $p = 1, \ldots, n$. Hamiltonians of this type describe different quantum computing devices [2] and have also been widely employed in nuclear physics for different studies [6], where they arise through the mapping $s_z^p = \sum_{\nu=\pm} \nu c_{p\nu}^\dagger c_{p\nu}/2$, $s_\pm^p = c_{p\pm}^\dagger c_{p\mp}$, with $c^\dagger, c$, fermion operators. Fig. 1 depicts the limit temperatures for non-zero negativities between *global* bipartitions $\{m-(n-m)\}$ of the above system ($m = 1, \ldots, n/2$) in the fully connected Lipkin case $v_x^{pq} = -v_y^{pq} = \frac{v}{n-1}$, $v_z^{pq} = 0$ [2], together with the limit temperature for *pairwise* entanglement, derived from the *reduced* two-spin density $\rho_2 = \mathrm{Tr}_{n-2}\,\rho$, which is *lower* than the previous ones. We depict in addition the critical temperature $T_c$ for the deformed Hartree-Fock solution, existing just for $|v| > 1$, which is also lower. Entanglement may then exist well *beyond* the limits for symmetry-breaking mean field solutions, being here present also for $|v| < 1$. It can be shown as well that $T_L$ may differ significantly from $T_c$ even in larger systems [2], and that the behavior of entanglement for $T < T_L$ is not necessarily monotonous [3]. Entanglement provides thus a new promising perspective for the analysis and use of quantum correlations in many-body systems.

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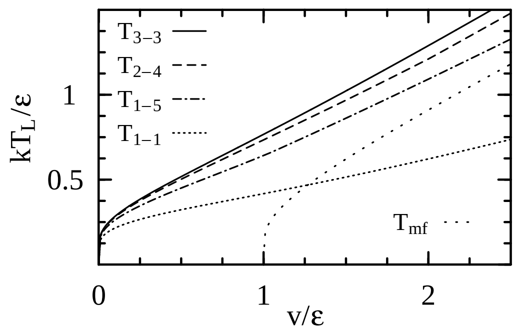


**Fig. 1.** Limit temperatures $T_{m-(n-m)}$ for non-zero global negativities in the $n = 6$ Lipkin model, for different bipartitions. The limit temperature for pairwise entanglement $T_{1-1}$ and the mean field critical temperature $T_{\mathrm{mf}}$ are also depicted.

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