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HEAVY ION PHYSICS

Nuclear structure aspects of neutrinoless $\beta\beta$ decay: limits on the electron neutrino mass

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Abstract. We discuss some features of the nuclear structure elements participant in the calculation of the mass sector of the half-life of the neutrinoless double beta decay, and the consequences upon the adopted limits of the electron-neutrino mass.

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1. Introduction

The neutrino mediates nuclear single-β decay transitions and most rare nuclear processes, like double-β decay transitions. The present knowledge about the neutrino is based on the oscillation data, extracted from SNO and SK results, and from limits extracted from CHOOZ, Kamland and WMAP [**?**]. The details about the analysis of neutrino mixing angles and phases can be found elsewhere [**?**, **?**]. The oscillation data show that the neutrino is a massive particle. The neutrino oscillations between mass eigenstates clearly demonstrate the existence of physics beyond the Standard Model (SM) of electro-weak interactions. The possible extensions of the SM depend largely on the determination of the neutrino absolute mass scale[**?**, **?**].

In this determination enter: a)the nuclear physics, needed to describe the structure of the nuclear states which participate in double-beta decays, and b)the models of the neutrino, needed to specify the mass-mixing mechanism.

The interplay between model dependent inputs and experimental results is well illustrated in neutrino-nuclear physics, since to extract the information about the neutrino mass from the experimentally determined limits on the non-observability

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of neutrinoless double-beta decay process one needs also to calculate a set of nuclear matrix elements[?]. This is not an easy task, since the dependence of actual values of the involved nuclear matrix elements upon model parameters is crucial. In this talk we shall discuss the question of the nuclear structure effects on the matrix elements governing the mass sector of the neutrinoless double beta decay, and, particularly, the so-called g_{pp}-dependence [?, ?], following the point of view of [?], where it is argued that the effective parameters appearing in the nuclear matrix elements of the zero-neutrino mode may be fixed from the adjustment to the observed two-neutrino mode. Also, we shall comment on the possible universality (if any) of the matrix elements.

2. Nuclear Structure Sector

The two-neutrino double-beta decay mode ($2\nu\beta\beta$), a process which is allowed by the selection rules of the standard model of electro-weak interactions, consists of an uncorrelated pair of virtual, subsequent, single $\beta^(\mp)$ decays connecting a (A, N, Z) nucleus with a $(A, N \mp 2, Z \pm 2)$ one. It is not dependent on neutrino properties and its is severely limited by lepton-phase space limitations. It exists only because of the presence of pairing interactions in nuclei, i.e: the ground state energy of a double-odd mass nucleus with $(A, N \mp 1, Z \pm 1)$ is higher than the ground state energies of the initial (A, N, Z) and final $(A, N \mp 2, Z \pm 2)$ nuclei. The half-life of the $2\nu\beta\beta$ decay is given by the expression [?]

$$\left[t^{2\nu}_{1/2}(J_f)\right]^{-1} = \frac{(g_A G_F cos\theta_C)^4}{96 ln2 \pi^7 m_e^2} | \sum_m < J_f^+ || \sum_i \sigma(i)\tau^{\pm} || 1_m^+ >$$
$$< 1_m^+ || \sum_i \sigma(i)\tau^{\pm} || 0_{gs}^+ > |^2 \int F_0(Z_f, \epsilon_2) F_0(Z_f, \epsilon_2) D(K, L) \omega_1^2 \omega_2^2 p_1 p_2 \epsilon_1 \epsilon_2$$
$$\delta(\epsilon_1 + \epsilon_2 + \omega_1 + \omega_2 + E_f - M_i) d\omega_1 d\omega_2 d\epsilon_1 d\epsilon_2 \quad (1)$$

The final leptonic state has four leptons (two neutrinos and two electrons) and there is not a helicity requirement between the first $(e_1, \bar{\nu}_1)$ and second $(e_2, \bar{\nu}_2)$ vertices. The zero-neutrino mode ($0\nu\beta\beta$) mode is a process where the virtual decays are connected, i.e: the anti-neutrino emitted in the first vertex is absorbed as a neutrino in the second vertex. It is not allowed in the Standard Model, where neutrinos are massless, purely left-handed Dirac particles. If we focus our attention on the mass sector of the half-life we obtain [?]

$$\begin{aligned}\left[t^{2\nu}_{1/2}(J_f)\right]^{-1} &= C^{(0\nu)}_{mm} \frac{< m_\nu >^2}{{m_e}^2} \\ C^{(0\nu)}_{mm} &= G^{(0\nu)}_1 \left[({M_{GT}}^{(0\nu)})(1 - \chi_F) \right]^2 \end{aligned} \quad (2)$$

where $\chi_F = \frac{M_F^{(0\nu)}}{M_{GT}^{(0\nu)}}$ is the ratio between the matrix elements of the double Fermi and Gamow-Teller operators:

$$\begin{aligned} M_{GT}^{(0\nu)} &= \sum_a < 0_F^+||h_+(r_{mn}, E_a)\sigma_n.\sigma_n||0_I^+ > \\ M_F^{(0\nu)} &= \sum_a < 0_F^+||h_+(r_{mn}, E_a)||0_I^+ > \end{aligned} \quad (3)$$

and $< m_\nu >$ is the average electron-neutrino mass (its definition in terms of neutrino mass eigenvalues and mixing amplitudes is given below, see. Eq.(5))). To calculate these matrix elements one needs to compute the nuclear charge density $\rho =< J^\pi, n \mid (Y_k\sigma)_{\lambda,\mu} f_k(r)\tau^- \mid 0^+_{I(F)} >$ for the multipole operators resulting from the partial wave expansion of the neutrino potential $h_+(r_{mn}, E_a)$. The nuclear states participant in the decay path are calculated in the quasiparticle random phase approximation [?]. The parameters of the model are the coupling strenghts g_{ph}, which may be fixed by the energetics of giant resonances (although few multipoles are known) and g_{pp}, which is unknown, except for symmetry considerations. The model dependence may be minimized by adopting either one of the following procedures: a)g_{pp} fixed by single-beta decay data, or b)g_{pp} fixed by $2\nu\beta\beta$ decay data.

3. Neutrino Sector

The relation between the flavor and mass neutrinos eigenstates is given by the mixing-matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \otimes \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad (4)$$

The matrix elements of the mixing matrix U are determined from the oscillation data [?], while the average mass scale may be fixed from the limits on neutrino densities determined by WMAP [?]. Table (??) shows a compilation of the adopted values extracted from the experiments quoted in [?]. The quantities δm^2_{ij} are the differences between the squares of the mass eigenvalues ($\delta m^{@}_{ij} = m_j^2 - m_i^2$) extracted from solar and atmospheric neutrino oscillation data, θ_{solar} and θ_{atm} are the oscillation mixing angles extracted from solar and atmospheric data and Ω_ν is the neutrino density extracted from satellite data. Concerning the mass-eigenvalues, the following cases, known as *mass hierarchies* may be present: a) $m_1 \approx m_2 < m_3$ (Normal); b) $m_1 \approx m_2 > m_3$ (Inverse);c)$m_1 \approx m_2 \approx m_3$ (Degenerate). Table (??) shows the value of the average mass value $< m_\nu >$

$$< m_\nu >= \sum_{i=1}^{3} m_i U_{ei}^2 = m_1 U_{e1}^2 \pm m_2 U_{e2}^2 \quad , \quad (5)$$

relevant for the nuclear double-beta decay [?]. The fact that nearly vanishing average masses may be obtained is one of the main results of the present calculations.

Table 1. Current limits on oscillation parameters

$\delta m^2{}_{12} = \delta m^2{}_{\mathrm{solar}}$	$5 \times 10^{-5}\mathrm{eV}^2 \to 1.1 \times 10^{-4}\mathrm{eV}^2$
$\delta m^2{}_{23} = \delta m^2{}_{\mathrm{atm}}$	$10^{-3}\mathrm{eV}^2 \to 5 \times 10^{-3}\mathrm{eV}^2$
$\sin^2 2\theta_{\mathrm{solar}}$	≈ 0.86
$\sin^2 2\theta_{\mathrm{atm}}$	≈ 1.0
Ω_ν	< 0.71 eV

Table 2. Average electron-neutrino mass $< m_\nu >$. The results shown in the column labelled $U(a)$ have been obtained by using the matrix U extracted from the best fit of the oscillation data, the results labelled $U(b)$ have been obtained by using a maximal mixing between mass eigensates.

Hierarchy		$< m_\nu >$	U(a)	U(b)
Normal	$(m_1 = 0)$	$< m_\nu >_-$	-0.010	-0.012
		$< m_\nu >_+$	0.011	0.012
Inverse	$(m_3 = 0)$	$< m_\nu >_-$	0.105	0.087
		$< m_\nu >_+$	0.234	0.235
Degenerate	(*extreme*)	$< m_\nu >_-$	0.107	0.088
		$< m_\nu >_+$	0.237	0.237

It means that even after the massive character of the neutrino has been conclusively established by the oscillation data, the positive signals of neutrinoless double beta decay may be hampered by the cancellation of $< m_\nu >$, which is operative if CP is conserved. This is an important result, considering the planned sensitivity of future double-beta decay experiments [**?**].

4. Joint Nuclear Structure and Neutrino Data

The experimental limits on the half-life of the neutrinoless double beta decay are fixed by the analysis of events (emission of two-electrons with a sum energy equals to the Q-value of the decay) and it is expressed by the ratio

$$F_N = t_{1/2}^{(0\nu)} C_{mm}^{(0\nu)} = (< m_\nu > / m_e)^{-2} \quad , \tag{6}$$

Clearly, if one aims at the extraction of the value of the average electron-neutrino mass, $(< m_\nu > / m_e)^{-2}$, from the data, $t_{1/2}^{(0\nu)}$, the nuclear structure factor $C_{mm}^{(0\nu)}$ must be known accurately enough. The phase space factors used in the calculations are shown in Table (**??**), together with the matrix elements extracted from compilation of the published results and the ones of the present calculations [**?**,**?**,**?**]. To illustrate the degree of dependence of the nuclear matrix elements we shall focuss on the case of the decay of ^{76}Ge.

Table 3. Phase-space factors, $G_1^{(0\nu)}$, and calculated double Gamow-Teller nuclear matrix elements $M_{GT}^{0\nu)}$ (N.M.E.), of Eq. (2). The values shown in the third column have been extracted from the literature and the ones shown in the fourth column are the results of the present calculations. The values of the average electron-neutrino mass, Eq.(5), are shown in the last column, in units of eV.

System	$G_1^{(0\nu)} \times 10^{14}$	N.M.E.	N.M.E.(this work)	$< m_\nu >$
^{48}Ca	6.43	1.08-2.38		8.70-19.0
^{76}Ge	0.63	2.98-4.33	3.33	0.30-0.43
^{82}Se	2.73	2.53-3.98	3.44	4.73-7.44
^{96}Zr	5.70	2.74	3.55	19.1-24.7
^{100}Mo	11.30	0.77-4.67	2.97	1.38-8.42
^{116}Cd	4.68	1.09-3.46	3.75	2.37-8.18
^{128}Te	0.16	2.51-4.58		9.51-17.4
^{130}Te	4.14	2.10-3.59	3.49	1.87-3.20
^{136}Xe	4.37	1.61-1.90	4.64	0.79-2.29

5. Results

In view of the large spreading exhibited by the results (see Table (**??**)) it is desirable to estimate the uncertainty introduced by model assumptions. We have considered two aspects, namely: a)the suppression of the matrix elements governing the $2\nu\beta\beta$ mode, and, b)the relevance of this suppression for the $0\nu\beta\beta$ mode.

The suppression induced by the particle-particle channel of the two-body interaction is rather evident [**?**]. However, it is operative on one multipole ($J^\pi = 1^+$), only. Since in principle all multipoles are allowed, we may investigate the effect of this suppression relatively to other multipole contributions to the matrix element. The fact that matrix elements of the neutrinoless double beta decay mode may be dominated by few multipole contributions to the charge density is illustrated by the results shown in Figure (**??**). Even if the contribution of the $J^\pi = 1^+$ multipole is nearly suppressed it represents a minor effect on the total matrix element. Thus, it seems that the procedure of [**?**] may not necessarily lead to a phenomenological value of g_{pp} (strength of the $J^\pi = 1^+$ channel of the residual proton-neutron interaction. The calculations include multipole contributions of normal and abnormal parity, up to $10^\pm$. The most evident contribution is the one of the $J^\pi = 2^-$ multipole. The previous results, for the decay of ^{76}Ge, show a tendency which is not restricted to this case. We have found the same features in other cases [**?**, **?**].

6. Conclusions

We may summarize in the following: a)The matrix elements governing the mass sector of the $0\nu\beta\beta$ decay do not depend much on g_{pp}, rather, they are dominated by few multipole contributions, which are not so sensitive to the parametrization

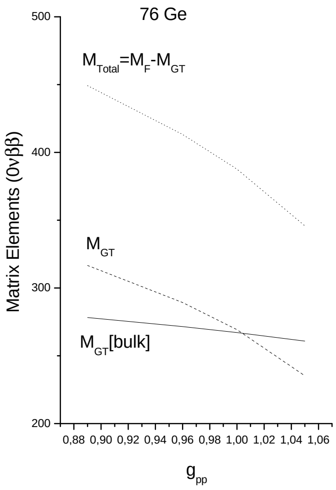

Fig. 1. Dependence of the total matrix element due to Gamow-Teller and Fermi transitions, on the parameter g_{pp}.

of the two-body interaction in the particle-particle channel. This feature may be of some importance in dealing with the experimental possibilities of measuring single beta decay transitions of these few multipolarities;

b)The matrix elements of the $2\nu\beta\beta$ and $0\nu\beta\beta$ decays are not related, therefore the adjustment of the nuclear parameters by the fit to the observed half-life of the $2\nu\beta\beta$ does not necessarily guarantees that the theoretical uncertainties in the calculation of the $0\nu\beta\beta$ are reduced.

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