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HEAVY ION PHYSICS

QCD running coupling with diquarks

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Abstract. The running coupling constant is evaluated at order g^2 within an extended model of Quantum Chromodynamics with color antitriplet scalar diquarks. Asymptotic freedom is valid also when the matter fields are originated by diquarks composed by strongly correlated pair of quarks. Diquarks slightly enhances α_{QCD} at scales about 1 GeV, in qualitative agreement with data.

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1. Introduction

Correlated quarks in color anti-triplet states can play a role in low-energy QCD and hadron structure [1]. These pairs - diquarks - have an enhanced stability due to the color exchange-interaction between the quarks. The explicit diquark degree of freedom has been largely explored to model the nucleon structure (see e.g. [2])) and more recently appeared in the interpretation of light scalar mesons as an antidiquark-diquark nonet [3]. Although, it has been recognized that color antitriplet pairs of quarks in a symmetrical combination of flavors and spin 1 states may also be important to hadron structure, we study a color gauge invariant effective Lagrangian containing quarks, gluons and a scalar diquark, considered as an elementary field. Within a chiral effective Lagrangian, Hong and collaborators [4] have already introduced a color antitriplet diquark coupled to the gauge field. Our model is a direct extension of QCD with quarks and diquarks as the matter fields.

Our aim in this work is to calculate, within the extended renormalizable QCD model with diquarks, the running coupling constant. The strong correlation of quarks in pairs forming diquarks should be consistent with asymptotic freedom. For that purpose we evaluate the β-function up to second order in the coupling constant and by solving the appropriate RG-equation we obtain the contribution of diquarks to the QCD running coupling constant.

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2. Extended QCD with diquarks

The Lagrangian of the color antitriplet scalar diquark and quarks coupled to the gauge field is given by:

$$\mathcal{L} = -\frac{1}{4}G^{a,\mu\nu}G^{a}_{\mu\nu} + \bar{\Psi}\left(\imath \not{D}_q - m_q\right)\Psi - (D^{\mu}_{d}\phi)^{*}(D_{d,\mu}\phi) - m_d^2\phi^{*}\phi, \tag{1}$$

where $\phi(x)$ is the diquark field with mass m_d, $G^a_{\mu\nu}$ is the gauge field tensor and Ψ the quark field. The covariant derivative for diquark and quark fields are written as:

$$\left[(D_d)_\mu\phi\right]_i = \partial_\mu\phi_i - igA^a_\mu(T^a_{ij})^{*}\phi_j \ \ and \ \ \left[(D_q)_\mu\psi\right]_i = \partial_\mu\psi_i + igA^a_\mu T^a_{ij}\psi_j \ . \tag{2}$$

Note that the covariant derivative of the diquark field is constructed accordingly to the conjugate representation of the color group, as the diquark belongs to the color anti-triplet representation.

The generators of the color group are given by the matrices T^a (a=1 to 8) which obey comutation relations: $[T^a, T^b] = if^{abc}T^c$, with the f^{abc} being the structure constants of the group. The gauge field tensor is written as

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{abc}A^b_\mu A^c_\nu, \tag{3}$$

in terms of the gluon field A^a_μ.

For infinitesimal gauge group transformations the fields transform as:

$$\begin{aligned} A^{a\prime}_\mu &= A^a_\mu + f^{abc}\Lambda^b A^c_\mu + \frac{1}{g}\partial_\mu\Lambda_a \ , \\ \psi^\prime_i &= \psi_i - i\Lambda^a T^a_{ij}\psi_j \ , \quad \phi^\prime_i = \phi_i + i\Lambda^a (T^a_{ij})^{*}\phi_j \ , \end{aligned} \tag{4}$$

where Λ_a are infinitesimal functions. The model of Eq. (1) with the covariant derivatives from Eq. (2) is gauge invariant and renormalizable. Based on that, we calculate the scale dependence of the coupling constant evaluating the renormalization constants up to $O(g^2)$. Our results are shown in the next sections.

3. Field and coupling constant renormalization

The renormalization constants of the fields and vertices in the bare Lagrangian,

$$\begin{aligned} \mathcal{L}_B = &-\frac{1}{4}G^{\mu\nu}_B G_{B\mu\nu} + \bar{\Psi}_B\left(\imath\not{\partial} - g_B \not{A}^a_B T^a - m_B\right)\Psi_B - \\ &\left(\partial^\mu\phi_B - igA^{\mu\ a}_B T^{a*}\phi_B\right)^{*}\left(\partial_\mu\phi_B - igA^a_{B\mu}T^{a*}\phi_B\right) - m^2_{dB}\phi^{*}_B\phi_B \ , \end{aligned} \tag{5}$$

are obtained using dimensional regularization. The bare Lagrangian can be written in terms of the renormalized fields, masses and coupling constant as:

$$\mathcal{L}_B = Z_2\bar{\Psi}\imath\not{\partial}\Psi + Z_1 g\mu^{\frac{\epsilon}{2}}\bar{\Psi}\not{A}^a T^a\Psi - (m+C)\bar{\Psi}\Psi + Z_5 g^2\mu^{\epsilon} f^{abc}A^b{}_\mu A^c{}_\nu f^{ade}A^{d\mu}A^{e\nu}$$

$$-\frac{Z_3}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)\left(\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a}\right) - Z_4 g\mu^{\frac{\epsilon}{2}}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right) f^{abc} A^{b\mu} A^{c\nu} + ...$$
$$+Z^d_2 \partial^\mu \phi^* \partial_\mu \phi - (m_d + D)\phi^*\phi + iZ^d_1 g\mu^{\frac{\epsilon}{2}} A^{\mu a}\left(T^{a*}\phi\right)^* \partial_\mu \phi + ...$$
$$+Z^d_3 g^2 \mu^\epsilon A^{\mu a} A^b_\mu \left(T^{a*}\phi\right)^* T^{b^*}\phi, \quad (6)$$

where $\epsilon = d - 4$ (d number of dimensions), g and the masses are the physical ones. Comparing the bare Lagrangian given by Eqs. (5) and (6) and taking into account the renormalization of the fields, i.e., $\Psi = \sqrt{Z_2}\Psi_B$, $A^a_{B\mu} = \sqrt{Z_3}A^a_\mu$, $\phi_B = \sqrt{Z^d_2}\phi_B$, one arrives at several identities, and among them:

$$g_B = g\mu^{\frac{\epsilon}{2}} \frac{Z_1}{Z_2\sqrt{Z_3}} = g\mu^{\frac{\epsilon}{2}} \frac{Z^d_1}{Z^d_2\sqrt{Z_3}} = ... \quad (7)$$

which are indeed valid as a consequence of Ward identities. In particular the last term in the above equality comes from the gluon-diquark-diquark vertex. The running coupling constant is derived from renormalization group invariance expressed by $\frac{dg_B}{d\mu} = 0$.

4. Running coupling constant from the quark-gluon vertex

The diquarks contributes to the standard calculation of the quark running coupling constant through the renormalization of the gluon field, which fluctuates in a pair of diquark-antidiquark or when diquark bubble emerges in the gluon propagation. The Feynman rules for the interaction of the gluon field with diquarks from the effective Lagrangian, Eq. (1), are shown in fig. 1.

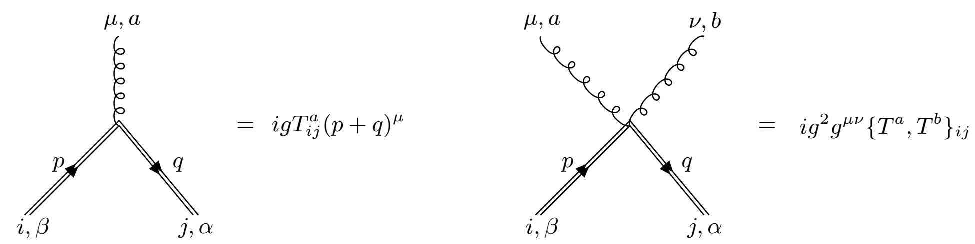

Fig. 1. Feynman rules for diquark-gluon coupling.

The corrections to the gluon propagator in order g^2 due to the vaccum polarization coming from diquark fluctuations are shown in fig. 2. A straightforward calculation with dimensional regularization gives the diquark contribution to the gluon polarization tensor as:

$$\Pi^{\mu\nu}_{ab}(d + d2) = \frac{g^2}{48\pi^2\varepsilon} n^d_F \delta_{ab}[(p^\mu p^\nu - p^2 g^{\mu\nu})] \ , \quad (8)$$

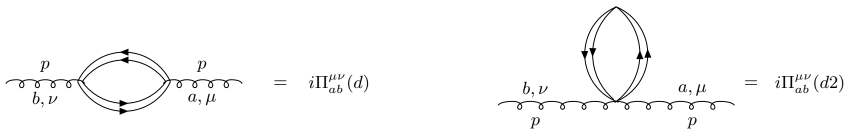

Fig. 2. $O(g^2)$ diquark contributions to vacuum polarization effects in the gluon propagation. Left diagram: vaccum fluctuation in diquark pairs. Right diagram: diquark-bubble.

and together with the standard results for QCD with quarks [5], we obtain a new value for the gluon renormalization constant:

$$Z_3 = 1 + \frac{g^2}{8\pi^2\varepsilon}(5 - \frac{2n_F^q}{3} - \frac{n_F^d}{6}) \ . \tag{9}$$

Taking into account the values of Z_1 and Z_2 [5] we can evaluate α_{QCD} for a model with quarks and diquarks, or only diquarks as matter field. The result is

$$\alpha_{QCD}^{q+d}(Q^2) = \frac{4\pi}{(11 - \frac{2n_F^q}{3} - \frac{n_F^d}{6})\log\frac{Q^2}{\Lambda^2}} \ , \tag{10}$$

and when only diquarks are considered, i.e., $n_F^q = 0$, we get

$$\alpha_{QCD}^{d}(Q^2) = \frac{4\pi}{(11 - \frac{n_F^d}{6})\log\frac{Q^2}{\Lambda^2}} \ . \tag{11}$$

We notice that even when strong correlations between quark pairs are included through an independent diquark field, for $n_F^q = 15$ from six quark flavors (u, d, s, c, b, t), the running coupling constant is still dominated by gluon self-interactions. Therefore, the asymptotic freedom behavior is maintained thanks to the non-abelian nature of the gauge field.

5. Running coupling constant from the diquark-gluon vertex

In this section we present results for the renormalization constants Z_1^d and Z_2^d of the diquark-gluon vertex and diquark field, respectively. Using Z_1^d and Z_2^d and Z_3 from Eq. (9), we verify the validity of the identity (7).

We begin by showing the correction in $O(g^2)$ for the diquark self-energy as illustrated in fig 3. We present only the terms in the diquark self-energy which contribute to the diquark field and mass renormalization. Our result is:

$$-i\Sigma_{ab}(d1 + d2) = \frac{-ig^2 C_2(F)}{8\pi^2\varepsilon}\delta_{ab}[2p^2 + m_d^2] + 0(\varepsilon) \ , \tag{12}$$

where $C_2(F) = 4/3$ is the second Casimir constant for the $SU(3)$ group. The coefficient of the p^2 term in the r.h.s. of Eq. (12) contributes to the renormalization

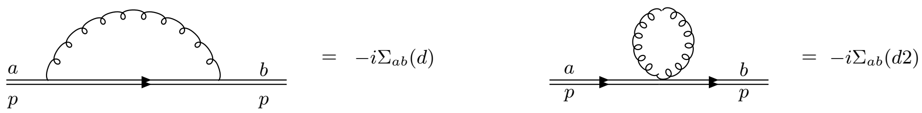

Fig. 3. $O(g^2)$ corrections to the diquark self-energy.

of the diquark field, which is given by:

$$Z_2^d = 1 + \frac{g^2 C_2(F)}{4\pi^2 \varepsilon} \ , \tag{13}$$

while the constant term in (12) is cancelled by a mass counterterm.

Next, we are going to show the results in $O(g^2)$ of the corrections to the diquark-gluon vertex. The Feynman diagrams corresponding to these corrections are shown in figs. 4 and 5.

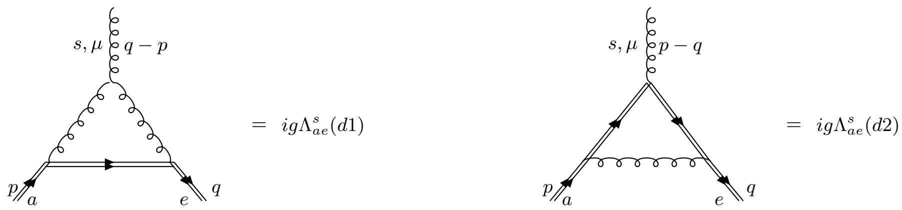

Fig. 4. $O(g^2)$ corrections to the diquark-gluon vertex, named $d1$ and $d2$ given by right and left diagrams, respectively.

Evaluating all vertex corrections from $d1$ to $d4$ one easily gets:

$$ig\Lambda^s_{ae}(d1 + d2 + d3 + d4) = \frac{ig^3}{8\pi^2 \varepsilon} T^s_{ae} (p+q)^\mu [-2C_2(F) + C_2(G)] \ , \tag{14}$$

with $C_2(G) = N_c$ being number of colors. The renormalization of the diquark-gluon

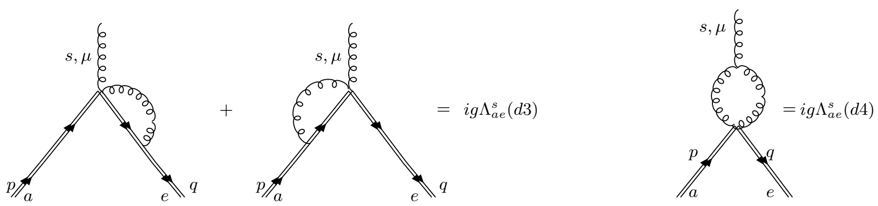

Fig. 5. $O(g^2)$ corrections to the diquark-gluon vertex, named $d3$ and $d4$ given by right and left diagrams, respectively.

vertex is obtained from Eq. (14) as:

$$Z_1^d = 1 - \frac{g^2}{24\pi^2\varepsilon} \ . \tag{15}$$

Finally, introducing the renormalization constants Z_3, Z_2^d and Z_1^d from Eqs. (9), (13) and (15) in Eq.(7), we obtain the same β-function as already derived from renormalization of the quark-gluon coupling constant. Consequently, we obtain the same result for the running coupling constant shown in Eq.(10).

6. Conclusion

The running coupling constant is evaluated in $O(g^2)$ within Quantum Chromodynamics model extended to include strongly correlated quark pairs in color antitriplet states with $J^\Pi = 0^+$. The gluon self-interactions dominates the asymptotic freedom property even when matter fields are formed only by strongly correlated quark pairs. For u, d, s, c, b and t flavors, strongly correlated diquarks slightly increases α_{QCD}. Our calculation goes beyond the perturbative expansion by including nonperturbative effects, i.e., the dynamical correlation of quark pairs, which enhances α_{QCD} in qualitative agreement with the data [6]. At scales below ~ 1 GeV where nonperturbative effects are expected to dominate, possibly our results extends the range of validity of Eq. (11) giving a glimpse to the onset of confinement and a theoretical basis to formulate diquark interactions for application in exotic hadron phenomenology.

Acknowledgments

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