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Phase diagram of non-local chiral quark models under compact star conditions

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Abstract. We study the properties of isospin asymmetric quark matter under compact stars constraints using a relativistic quark model with non local interactions in the mean field approximation. We consider a Gaussian regulator, and medium coupling ratio. We present the corresponding phase diagrams and discuss, in particular, the competition between chiral symmetry restoration and the various forms of two flavor color superconductivity.

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1. Introduction

In the last years much effort was focused on the construction and understanding of the QCD phase diagram[1]. On one hand, at high temperatures a deconfined weakly interacting quark matter phase takes place, the so-called Quark Gluon Plasma phase. The signatures of that phase are being investigated in relativistic heavy ion collisions. On the other hand, at low temperatures, in the region of low baryon chemical potential the chiral symmetry is broken due to the presence of a quark-antiquark condensate. However, when increasing the chemical potential, the quark-antiquark channel is expected to vanish, giving rise to a diquark condensate. This region is of great interest for the physics of the interior of neutron stars. For two flavor isospin symmetric quark matter the QCD phase diagram has been explored in the frame of different quark models. All of them agree in the sense that the two flavor color superconductivity phase (2SC) occurs at moderate chemical potentials. But when two non equal flavor chemical potentials are considered, and one imposes compact star conditions —i.e., electric charge and color neutrality conditions, to-

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gether with β equilibrium—, the situation is more complicated and different models lead to qualitatively different results[2,3]. The aim of this contribution is to report on a study of this problem using non-local chiral quark models[4–6]. This type of models have been previously used to investigate the phase diagram of isospin symmetric matter[7,8].

2. Formalism

The Euclidean action for the nonlocal chiral quark model in the case of two light flavors and anti-triplet diquark interactions reads

$$S_E = \int d^4x \, \left\{ \bar{\psi}(x) \left(-i\partial\!\!\!/ + m_c\right) \psi(x) - \frac{G}{2} j^f_M(x) j^f_M(x) - \frac{H}{2} \left[j^A_D(x)\right]^\dagger j^A_D(x) \right\} . \quad (1)$$

Here m_c is the current quark mass, which is assumed to be the same for both u and d quarks. Two alternative ways to introduce the non-locality have been considered in the literature[4]. One possibility[5] (that we call "Model I" hereon) is based in an instanton liquid picture of the QCD effective interactions. In this case the explicit forms of the non-local currents $j_{M,D}(x)$ appearing in Eq.(1) are

$$\begin{aligned} j^f_M(x) &= \int d^4y \; d^4z \; r(y-x) \; r(x-z) \; \bar{\psi}(y) \; \Lambda_f \; \psi(z) \, , \\ j^A_D(x) &= \int d^4y \; d^4z \; r(y-x) \; r(x-z) \; \bar{\psi}_C(y) \; i\gamma_5 \tau_2 \lambda_A \; \psi(z) \, , \end{aligned} \quad (2)$$

where $\Lambda_f = (\mathbb{1}, i\gamma_5\vec{\tau})$, $\vec{\tau}$ and $\lambda_{A=2,5,7}$ are Pauli and Gell-Mann matrices acting on flavor and color spaces, respectively, and we have used $\bar{\psi}_C(x) = \psi^{\,t}(x) \; \gamma_2\gamma_4$.

An alternative way[6] (that we call "Model II") is based on an effective one-gluon exchange picture. The corresponding form of the non-local currents $j_{M,D}(x)$ is in this case

$$\begin{aligned} j^f_M(x) &= \int d^4z \; g(z) \; \bar{\psi}(x+\frac{z}{2}) \; \Lambda_f \; \psi(x-\frac{z}{2}) \, , \\ j^A_D(x) &= \int d^4z \; g(z) \; \bar{\psi}(x+\frac{z}{2}) \; i\gamma_5 \tau_2 \lambda_A \; \psi(x-\frac{z}{2}) \end{aligned} \quad (3)$$

The functions $r(x-y)$ and $g(y)$ in Eqs. (2) and (3), respectively, are nonlocal regulators characterizing the interaction. The effective action in Eq. (1) might arise via Fierz rearrangement from some underlying more fundamental interactions.

The partition function $\mathcal{Z}$ for the model at temperature T and quark chemical potentials μ_{fc} is obtained in the usual way by going to momentum space and performing the replacements

$$\int \frac{d^4p}{(2\pi)^4} \quad \rightarrow \quad T \sum_{n=-\infty}^{\infty} \int \frac{d^3\vec{p}}{(2\pi)^3} \quad (4)$$

and $p_4 \to \omega_n - i\mu_{fc}$. Here p_4 is the fourth component of the (Euclidean) momentum of a quark carrying flavor f and color c, and ω_n are the Matsubara frequencies corresponding to fermionic modes, $\omega_n = (2n+1)\pi T$.

To proceed it is convenient to perform a standard bosonization of the theory. Thus, we introduce the bosonic fields σ, π_a and Δ_A, and integrate out the quark fields. In what follows we work within the mean field approximation (MFA), in which these bosonic fields are replaced by their vacuum expectation values $\bar{\pi}_a = 0$, $\bar{\sigma}$ and $\bar{\Delta}_A$. Moreover, we adopt the usual 2SC ansatz $\bar{\Delta}_5 = \bar{\Delta}_7 = 0$, $\bar{\Delta}_2 = \bar{\Delta}$. In this way, the mean field thermodynamical potential per unit volume reads

$$\Omega^{MFA} = -\frac{T}{V} \ln \mathcal{Z}^{MFA} = \frac{\bar{\sigma}^2}{2G} + \frac{|\bar{\Delta}|^2}{2H} - \frac{T}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^3\vec{p}}{(2\pi)^3} \ln \det \left[\frac{1}{T} \, S^{-1}(\bar{\sigma}, \bar{\Delta}) \right] . \quad (5)$$

The inverse propagator $S^{-1}(\bar{\sigma}, \bar{\Delta})$ is a 48×48 matrix in Dirac, flavor, color and Nambu-Gorkov spaces. Its determinant can be analytically evaluated leading to a rather simple expression in terms of the regulators. The corresponding explicit form will be given elsewhere[9]. For finite values of m_c, Ω^{MFA} turns out to be divergent. The regularization procedure used here amounts to define

$$\Omega^{MFA}_{(\mathrm{reg})} = \Omega^{MFA} \; - \; \Omega^{\mathrm{free}} \; + \; \Omega^{\mathrm{free}}_{(\mathrm{reg})} \; , \quad (6)$$

where Ω^{free} is obtained from Eq. (5) by setting $\bar{\Delta} = \bar{\sigma} = 0$, and $\Omega^{\mathrm{free}}_{(\mathrm{reg})}$ is the usual regularized expression for a free fermion gas.

The mean field values $\bar{\sigma}$ and $\bar{\Delta}$ are obtained from the coupled set of gap equations

$$\frac{d\Omega^{MFA}_{(\mathrm{reg})}}{d\bar{\Delta}} = 0 \qquad , \qquad \frac{d\Omega^{MFA}_{(\mathrm{reg})}}{d\bar{\sigma}} = 0 \quad . \quad (7)$$

So far we have introduced one chemical potential for each quark flavor and color. However, when the system is in chemical equilibrium not all of them are independent. For the 2SC ansatz, only one color-dependent chemical potential is needed to ensure color charge neutrality. Thus, the chemical potential for each different quark can be given in terms of only three independent quantities: the baryonic chemical potential $\mu_B = 3\mu$, the quark electric chemical potential μ_{Q_q} and one color chemical potential μ_8. The corresponding relations read

$$\begin{aligned} \mu_{ur} = \mu_{ug} &= \mu + \frac{2}{3}\mu_{Q_q} + \frac{1}{3}\mu_8 \qquad , \qquad \mu_{dr} = \mu_{dg} = \mu - \frac{1}{3}\mu_{Q_q} + \frac{1}{3}\mu_8 \\ \mu_{ub} &= \mu + \frac{2}{3}\mu_{Q_q} - \frac{2}{3}\mu_8 \qquad , \qquad \mu_{db} = \mu - \frac{1}{3}\mu_{Q_q} - \frac{2}{3}\mu_8 \end{aligned} \quad (8)$$

In the core of neutron stars, in addition to quark matter, we have electrons. Thus, within the MFA for the quark matter, and considering the electrons as a free Dirac gas, the full grand canonical potential is given by

$$\Omega = \Omega^{MFA}_{(\mathrm{reg})} + \Omega^e \; , \quad (9)$$

where Ω^e is the free energy of a free electron gas written in terms of the electron chemical potential μ_e. In addition, quark matter has to be in beta equilibrium with electrons. Thus, assuming that antineutrinos escape from the stellar core, we must have

$$\mu_{dc} - \mu_{uc} = \mu_e = -\mu_{Q_q} \ . \tag{10}$$

If we now require the system to be electric and color charge neutral, the number of independent chemical potentials reduces further. Namely, μ_e and μ_8 are fixed by the condition that the electric and color densities vanish:

$$\sum_{c=r,g,b} \left(\frac{2}{3}\,\rho_{uc} - \frac{1}{3}\,\rho_{dc} \right) - \rho_e = 0 \ , \qquad \frac{1}{\sqrt{3}} \sum_{f=u,d} \left(\rho_{fr} + \rho_{fg} - 2\rho_{fb} \right) = 0 \ , \tag{11}$$

where $\rho_e = -\partial\Omega/\partial\mu_e$ and $\rho_{fc} = -\partial\Omega/\partial\mu_{fc}$. Consequently, in the physical situation we are interested in, for each value of T and μ we should find the values of $\bar{\Delta}$, $\bar{\sigma}$, μ_e and μ_8 that solve Eqs. (7), supplemented by Eqs. (8) and (11).

3. Results

In this section we present some numerical results for the phase diagram using some specific regulator. As it has been shown in previous analyses[8], in general the results do not show a strong qualitative dependence on the shape of the regulator. Thus, we will consider here only the simple Gaussian regulator which written in momentum space reads

$$g(p^2) = \left[r(p^2) \right]^2 = \exp(-p^2/\Lambda^2) \ , \tag{12}$$

where Λ plays the rôle of an ultraviolet cut-off.

For definiteness we choose here, for both Model I and Model II, input parameters G, m_c and Λ which allow to reproduce the empirical values for the pion mass m_π and decay constant f_π, and lead to phenomenologically reasonable values for the chiral condensates and the dynamical quark masses at vanishing T and μ_{fc}. The parameters considered here for Model I are $G = 30$ GeV^{-2}, $m_c = 7.7$ MeV and $\Lambda = 760$ MeV . For Model II we use $G = 28.8$ GeV^{-2}, $m_c = 5.2$ MeV and $\Lambda = 817$ MeV.

The resulting phase diagrams for the standard value of the coupling ratio $H/G = 0.75$ are shown in Fig. 1. In both cases, for sufficiently low values of T, when increasing the chemical potential we reach a first order transition line (full line) that separates the chiral symmetry broken (CSB) phase (phase I) from a two flavor color superconducting (2SC) region (phase II). For intermediate values of the chemical potential, the effective pairing interaction is somewhat weaker in Model II (as compared to Model I), and this leads to the existence of a small region of mixed phase[10] (region V) at very low temperature in that model. We also observe that although for both models there exists a region of gapless two flavor color superconducting (g2SC) phase[11], such region is confined to a very narrow band (region IV) along the second order phase transition line (dashed line) that separates the color superconducting phases from the normal quark matter (NQM) phase.

Two critical points are shown in Fig. 1. The "end point" (EP) corresponds to the place where the first order phase transition line disappears. The "triple point" (3P) is the point at which the first order and second order transition lines meet. At this point the CSB, NQM and g2SC phases coexist. Comparing both panels of Fig. 1 we see that the positions of these critical points are quite dependent on the model considered. In fact, although the basic features of the phase diagrams remain unchanged, they are also quite sensitive to the input parameters used in each model[9].

Finally, it is interesting to compare the present results with those in Ref. [8], where isospin symmetric quark matter was considered. We see that although —as expected— the electric and color neutrality conditions that characterize the compact stars interior tend to reduce the size of color superconducting regions, they do not lead to their complete disappearance.

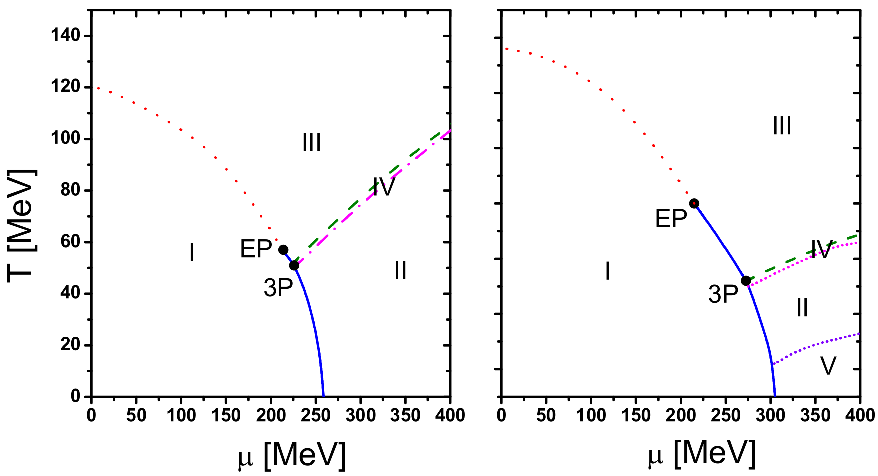

Fig. 1. Phase diagrams in the $T-\mu$ plane. Left and right panels correspond to Model I and Model II respectively. In both cases region I corresponds to the chiral symmetry broken phase, region II to the 2SC phase, region III to the normal quark matter (NQM) phase, region IV to the g2SC phase and region V to the mixed 2SC$-$NQM phase.

4. Conclusions

In this work we have studied the properties of isospin asymmetric quark matter under compact stars constraints using a relativistic quark model with non-local interactions in the mean field approximation. We have found that for both types of non-local models considered the corresponding phase diagrams display either 2SC or mixed 2SC−NQM phases in the region of low temperature and medium chemical potential relevant for compact star applications.

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