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Chiral Nuclear Effective Field Theory

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Abstract. New results regarding the renormalization of pion exchange and power counting in chiral nuclear effective field theory are discussed.

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1. Introduction

It has been known for about 30 years that QCD is the correct theory of the strong interactions that underline nuclear physics. Yet, progress in deriving the consequences of QCD to nuclear structure have been slow, in no small measure due to a mismatch of scales. QCD has an intrinsic mass scale, $M_{QCD} \simeq 1$ GeV, set by most hadronic masses (such as the nucleon mass m_N), above which it is formulated in terms of weakly coupled quarks and gluons. On the other hand, the typical nuclear binding energy per nucleon is $B/A \simeq 10$ MeV, and the relevant degrees of freedom in nuclei seem to be nucleons, pions, and perhaps delta isobars with momenta $Q \sim \sqrt{m_N B/A} \simeq 100$ MeV. The theory of strongly coupled quarks and gluons is incredibly complicated, and many of those who have thought about the problem admire it, I suspect, with the same awe we experience at the *cataratas* of Iguazú/Iguaçú.

What is needed in nuclear physics is a different formulation of QCD —a different starting point that nevertheless is equivalent to the standard formulation. Fortunately there exists a general framework —called Effective Field Theory (EFT)— to deal with problems such as this, which have two (or more) separate scales. As we go down in energy, fewer momentum modes, and so, in a sense, fewer degrees of freedom are accessible. In order to reproduce the underlying-theory results for low-energy observables, we need to renormalize the strength of interactions. EFT is a framework to construct the effective interactions systematically, at the same time maintaining desirable general principles such as causality and cluster decomposition.

Following Weinberg's original proposal [1], we have been developing EFTs for

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nuclear systems [2]. The goal is to understand the regularities of traditional nuclear physics from a QCD standpoint. In the process, it has been realized that this problem is much richer from an EFT perspective than originally thought. As a consequence, we have learned much about the structure of EFTs, such as the role of three-body forces and the discovery of limit cycles [2].

Here I want to focus on the newest of such discoveries, which concerns the renormalization of pion exchange. Most of the progress in nuclear EFTs has taken place at the lowest energies, where pion exchange can be treated as a short-range interaction. The range of nuclear phenomena is limited and the connection with QCD tenuous. Explicit pion exchange brings in serious difficulties, however, because it produces a singular potential that couples long- and short-range physics. Only now are we appreciating how much short-range physics it entails: short-range interactions that Weinberg implicitly assumed to be small are actually enhanced by the pion physics. We hope that eventually this insight will translate into even more accurate nuclear potentials for nuclear structure.

I will start by reviewing the basic ideas of EFT in Sect. 2 and of its chiral nuclear version in Sect. 3. In Sect. 4 I discuss the surprising renormalization of pion exchange and some of its consequences for power counting, while an outlook is offered in Sect. 5.

2. Effective Field Theories

EFT starts with the observation that the low-energy effective interactions consist of the sum of *all* possible interaction terms in a Lagrangian that involves only the fields representing low-energy degrees of freedom. Because of the uncertainty principle, each of these interaction terms can be taken as a local combination of derivatives of the fields. If the "integrating out" of the high-energy degrees of freedom is done appropriately, the effective Lagrangian will have the same symmetries as the underlying theory. The details of the underlying dynamics, on the other hand, are contained in the interaction strengths. The latter depend also on the details of how the low- and high-energy degrees of freedom are separated. This separation requires the introduction of a cutoff parameter Λ with dimensions of momentum. Both the interaction strengths and the quantum effects represented by loops depend on Λ. However, the cutoff procedure is arbitrary, so by construction observables are independent of Λ ("renormalization-group invariance").

If we denote the scales of the underlying theory and of the EFT by $M_{\rm hi}$ and $M_{\rm lo}$, respectively, the T matrix for any low-energy process acquires the schematic form

$$T(Q \sim M_{\rm lo}) = \mathcal{N} \sum_{\nu=\nu_{\rm min}}^{\infty} c_\nu(M_{\rm hi}, \Lambda) \left(\frac{Q}{M_{\rm hi}}\right)^{\nu} \mathcal{F}_\nu \left(\frac{Q}{M_{\rm lo}}; \frac{\Lambda}{M_{\rm lo}}\right), \tag{1}$$

where $\mathcal{N}$ is a common normalization factor, ν is a counting index starting at some value $\nu_{\rm min}$, the c_νs are parameters, and the $\mathcal{F}_\nu$s are calculable functions. We must have

$$\frac{\partial T(Q \sim M_{\rm lo})}{\partial \Lambda} = 0. \tag{2}$$

In order to maintain predictive power in the EFT it is necessary to truncate the sum in Eq. (1) in such a way that the resulting cutoff dependence can be decreased systematically with increasing order. We call such ordering "power counting". There are essentially two ways of doing this. One is to carry out the integration of high-energy degrees of freedom explicitly and infer the power counting from the sizes of the calculated terms. Another, which we use when we do not know or cannot solve the underlying theory, is to guess the sizes of the effective interactions. The simplest assumption is that the renormalized parameters are driven by short-range physics and are "natural", in the sense that they are in order of magnitude given by $M_{\rm hi}$ to a power determined by dimensional analysis. Whatever guess we make is confirmed *a posteriori*, by checking renormalization-group invariance and convergence of the truncation after the data is fitted order by order.

This framework has been applied in various contexts. It has been best explored in particle physics, where the $\mathcal{F}_\nu$'s could be obtained in perturbation theory. Much of the existing intuition about EFTs comes from these situations, in which naive dimensional analysis has been found to be a reliable tool for power counting. The greatest challenge to nuclear EFT is that, while we want to have expansions of the type (1), we also have to generate bound states. So, at least $\mathcal{F}_{\nu_{\rm min}}$ has to include an infinite number of (perturbative) Feynman diagrams. This interplay of perturbative and non-perturbative physics has been the source of most of the fun we have had. Renormalization and power counting are much less trivial than in a purely perturbative context.

The challenges brought up by non-perturbative renormalization led to the development of a simple nuclear EFT, where issues can be addressed at least partially by analytical means. The typical momentum of nucleons in the deuteron is $\aleph_1 \sim \sqrt{m_N B_d} \simeq 45$ MeV, which means that the deuteron is an object about three times larger than the bulk of the pion cloud around each nucleon, whose range is set by the inverse of the pion mass, m_π. For the ${}^1\mathrm{S}_0$ virtual bound state, the corresponding scale is even smaller, $\aleph_0 \sim \sqrt{m_N B_d'} \simeq 8$ MeV. With this resolution, all mesons (even the pion!) that can be exchanged among nucleons propagate for relative short times and distances. To address physics at this scale, one can then consider an EFT where the the meson cloud is represented by a multipole expansion: the Lagrangian contains only nucleon fields with "contact" interactions. In this "pionless" EFT, $M_{\rm hi} \sim m_\pi$ and $M_{\rm lo} \sim \aleph$ (with $\aleph$ some average of the $\aleph_i$s). An important subtlety in this case is that naive dimensional analysis fails in order to accomodate the fine-tuning that places bound states at the anomalously-small scale $\aleph$. Nevertheless, this EFT is now pretty well understood [2], including an explicit demonstration that its renormalization can be quite different from the renormalization of the related perturbative series. Perhaps the most important remaining issue is how far in A we can go within this EFT. (For "halo" nuclei, an EFT with additional explicit fields representing inert nuclear cores can also be formulated [2].)

Here I want to go back to the origins of nuclear EFT: the "pionful" theory, where $M_{\rm hi} \sim M_{QCD}$ and $M_{\rm lo} \sim m_\pi$. It is likely that for typical large nuclei we will have to employ this EFT.

3. Pionful EFT

In the pionful EFT, pions are explicit degrees of freedom, and (approximate) chiral symmetry plays a crucial role. The techniques to build the most general Lagrangian involving nucleons, pions, and delta isobars are by now standard —see, for example, Refs. [3, 2]. In the sector of $A = 0, 1$ nucleons this EFT reduces to well-known chiral perturbation theory (ChPT) [4, 3], where power counting based on naive dimensional analysis works well. Amplitudes can be written in the form (1) with $\nu = 2 - A + 2L + \sum_i V_i(d_i + f_i/2 - 2)$, where L is the number of loops and V_i the number of vertices with d_i derivatives or powers of m_π and f_i fermion fields. However, in the $A \geq 2$ sector power counting is much more subtle.

Weinberg [1] recognized that there is a breakdown of ChPT power counting in the propagation of two or more nucleons, caused by an infrared enhancement. If $A \leq 1$, all energy denominators are $\mathcal{O}(Q)$, but $A \geq 2$ diagrams with purely nucleonic intermediate states have small energy denominators of $\mathcal{O}(Q^2/m_N)$. Weinberg suggested that the calculation of a generic nuclear amplitude should consist of two steps. In the first step, one defines the nuclear potential as the sum of "irreducible" sub-diagrams that do not contain purely nucleonic intermediate states, and truncates the sum according to a simple extension of the standard ChPT power counting. In a second step, the potential is iterated to all orders, which can be done by using the Lippmann-Schwinger (LS) or Schrödinger equations.

The potential includes pion exchanges and contact interactions, which represent the contributions of more massive degrees of freedom. Assuming that contact interactions obey naive dimensional analysis, only a finite number of pion exchanges and contact interactions contribute to the potential at any given order. For example, in leading order Weinberg's power counting says [1] that the nuclear potential is a sum of two-nucleon ($2N$) potentials of the form

$$V = V_{1\pi}(\vec{q}\,) + V_c, \tag{3}$$

where

$$V_{1\pi}(\vec{q}\,) = -\left(\frac{g_A}{2f_\pi}\right)^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{(\vec{\sigma}_1 \cdot \vec{q}\,)\ (\vec{\sigma}_2 \cdot \vec{q}\,)}{\vec{q}^{\,2} + m_\pi^2} \tag{4}$$

is the one-pion-exchange (OPE) potential written in terms of the pion-nucleon coupling $g_A/2f_\pi$, the Pauli spin and isospin matrices $\vec{\sigma}_i$ and $\boldsymbol{\tau}_i$ of nucleon i, and the transferred momentum $\vec{q}$; and

$$V_c = \frac{1}{4\pi}\left(c_s\ P_s + c_t\ P_t\right), \tag{5}$$

is a contact interaction with projectors onto spin-triplet and spin-singlet S-wave states, P_t and P_s, and two strength parameters c_s and c_t that can be determined from $2N$ scattering data. It is possible to write

$$c_s = C_0 + m_\pi^2 D_2 + \ldots, \tag{6}$$

where the parameters C_0 and D_2 are independent of the quark masses. According to Weinberg's power counting, only C_0 appears in leading order. In subleading orders, other components of the nuclear potential are present [5,6], such as contact interactions with derivatives or powers of m_π^2 (*e.g.* D_2 in Eq. (6)), two- (TPE) and more-pion exchanges, and three- ($3N$) and more-nucleon interactions.

The resulting $2N$ [5] and $3N$ [6] potentials provide a quantitative description of few-nucleon systems [7]. In addition, this approach matches well with the Nijmegen energy-dependent partial-wave analysis (PWA) [8] of $2N$ scattering data, where the Schrödinger equation is solved with a long-range potential that consists of OPE and TPE (and the electromagnetic interaction), and a boundary condition with as many short-range parameters as needed for an optimal description of the observables. The pion mass and OPE parameters [9] and even TPE parameters [10] could be determined from $2N$ scattering data, in good agreement with values obtained from pion-nucleon scattering [11].

Yet, Weinberg's power counting has been criticized. A consistent power counting should provide sufficient counterterms at each order to absorb any cutoff dependence in the limit of large cutoffs. Because the solution of the LS equation is numerical in character, an explicit check of cutoff independence is challenging. This led Kaplan *et al.* [12] to examine a few of the diagrams contributing to the $2N$ T matrix coming from the iteration of Eqs. (4) and (5). They identified in two-loop diagrams ultraviolet divergences proportional to m_π^2 and $\vec{q}^{\,2}$ that are present in leading order but cannot be absorbed by the available counterterms. They concluded that pion exchange should not be fully iterated, but instead be treated in finite order in perturbation theory. Quantitative calculations at higher order showed, however, that this idea fails in some partial waves at $Q \simeq 100$ MeV [13].

It seems inevitable, then, that at such momenta pions have to be iterated. We are led to study the non-perturbative renormalization of OPE, which, based on experience with the pionless EFT, is not necessarily the same as that of the corresponding perturbative series.

4. Renormalization of Pion Exchange and Power Counting

OPE is a singular potential, and this has profound implications to the renormalization of the LS equation. The Fourier transform of Eq. (4) has $1/r^n$ singularities at small distances r: in spin-singlet channels $n = 1$, while in spin-triplet channels the tensor force has $n = 3$.

To understand the basic issue in the renormalization of singular potentials [14], consider two particles of reduced mass μ interacting through an uncoupled $-\lambda r_0^{n-2}/2\mu r^n$ potential, where $\lambda = \mathcal{O}(1)$ is dimensionless and r_0 sets the scale of curvature. We account for short-range physics by replacing the potential below a distance $R = 1/\Lambda$ by a square well of depth V_0, which represents a regularization of Eq. (3). Renormalization-group invariance requires that a $V_0(R)$ be found which keeps low-energy data invariant under a change in R.

Let us consider the S wave. If $n < 2$, the wavefunction for $r > R$ has regular and irregular components. However, if $n \geq 2$ and $\lambda > 0$, the zero-energy wavefunction for $R < r \ll r_0$ is

$$r\psi(r;0) \simeq \left(\frac{r}{r_0}\right)^{n/4} \cos\left(\frac{\sqrt{\lambda}}{n/2-1}\left(\frac{r}{r_0}\right)^{1-n/2} + \phi_n\right), \tag{7}$$

where the phase ϕ_n determines low-energy observables but is *not* fixed by the long-range potential. This solution, which cannot be obtained in perturbation theory, shows that one needs one piece of short-range physics in order to make the problem well defined. This is provided by $V_0(R)$ through matching at $r = R$. One can then show [14] that low-energy observables come out essentially R independent.

The extension of these results to $2N$ scattering is straightforward [15–17]. The perturbative problem [12] of the ultraviolet divergence in the $^1\mathrm{S}_0$ channel proportional to m_π^2 persists in this context [16]. In spin-singlet channels, OPE goes as $1 - m_\pi^2/q^2 + \ldots$ at high momentum. When iterated, the first term by itself introduces cutoff dependence in the S wave only, which can be removed by the chiral-symmetric counterterm C_0 in Eq. (6). The interference between the iteration of C_0 and the second term in OPE generates further cutoff dependence in the $^1\mathrm{S}_0$ wave, which in turn can be removed [12, 16] by the chiral-breaking counterterm $D_2 m_\pi^2$. This counterterm is enhanced with respect to naive dimensional analysis, and should be promoted to leading order if pion-mass effects are kept at this order, as it seems most efficient.

In spin-triplet channels, the situation is complicated by the tensor operator, which retains angular dependence even asymptotically. However, divergences associated with momenta, present in the $^3\mathrm{S}_1$-$^3\mathrm{D}_1$ coupled channel, *can* be absorbed into a single chiral-symmetric, momentum-independent counterterm, as prescribed by Weinberg's power counting [15–17].

Thus, with a simple amendment, Weinberg's power counting is consistent in a *non*-perturbative calculation of the S waves [16–18]. Since, neglecting angular-momentum factors, OPE is $\mathcal{O}(1/f_\pi^2)$, a crude estimate for the momentum of bound states is $Q \sim 4\pi f_\pi^2/m_N$. The deuteron and $^1\mathrm{S}_0$ virtual state have, for the observed value of the quark masses, binding momenta somewhat smaller than this estimate, indicating an amount of fine-tuning. However, if one varies the pion mass the momentum scales for the bound states acquire more natural values [16, 19].

OPE contributes, however, also in higher partial waves. Weinberg's power counting does not predict leading-order counterterms in these partial waves. It has been checked that higher spin-singlet [20, 17] and repulsive spin-triplet [17] waves are indeed cutoff independent.

Now, the same is *not* true when the OPE tensor force is attractive [17]. Examples are given in Fig. 1. In those channels, as Λ is increased, OPE becomes more

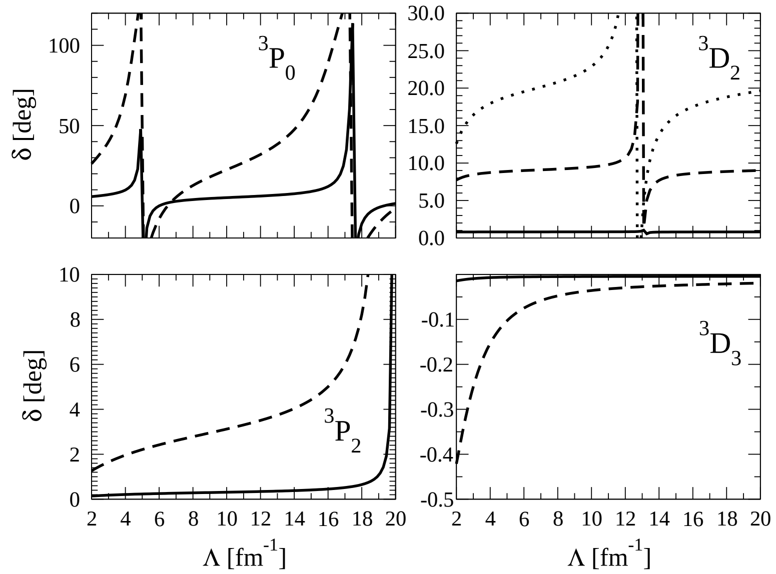

Fig. 1. Cutoff dependence of OPE phase shifts in attractive triplet channels at laboratory energies of 10 MeV (solid line), 50 MeV (dashed line), and 100 MeV (dotted line) [17].

and more attractive, creating bound states, which at certain values of the cutoff cross threshold and make the low-energy phase shifts go haywire. The short-range behavior of the wavefunction in these channels is similar to Eq. (7), but there is no counterterm to prevent the phase from depending on the arbitrary cutoff. In existing calculations, the renormalization issue has been sidestepped by choosing rather low cutoffs and by varying the cutoffs only in a very limited range [7]. The decrease in cutoff dependence over small cutoff ranges with increasing order has apparently been interpreted as consistent with the error expected from the truncation of the expansion.

Instead, the cutoff dependence observed over a large cutoff range [17] shows that not all necessary counterterms have been accounted for. The cutoff dependence is stronger in the lower waves. If we include [17] contact interactions —which in Weinberg's power counting are higher order—

$$\sum_{i=1}^{2} V_i + V_d = \sum_{i=1}^{2} \frac{c_i}{4} P_i(p'p) + c_d P_d(p'^2 p^2), \tag{8}$$

where $P_i(p'p)$ and $P_d(p'^2p^2)$ are projectors in the $^3\mathrm{P}_0$ ($i = 1$), $^3\mathrm{P}_2$-$^3\mathrm{F}_2$ ($i = 2$), and $^3\mathrm{D}_2$ (d) channels, the parameters c_i and c_d can be fitted so as to remove the cutoff dependence. The parameters then exhibit a limit-cycle-like behavior. One example is shown in Fig. 2. The resulting phase shifts are in good agreement with

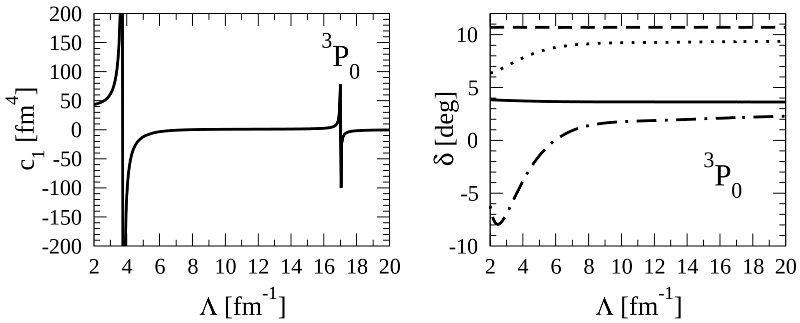

Fig. 2. Fit result [17] for the counterterm c_1 as a function of the cutoff, and the resulting cutoff dependence of the $^3\mathrm{P}_0$ phase shift at laboratory energies of 10 MeV (solid line), 50 MeV (dashed line), 100 MeV (dotted line), and 190 MeV (dash-dotted line).

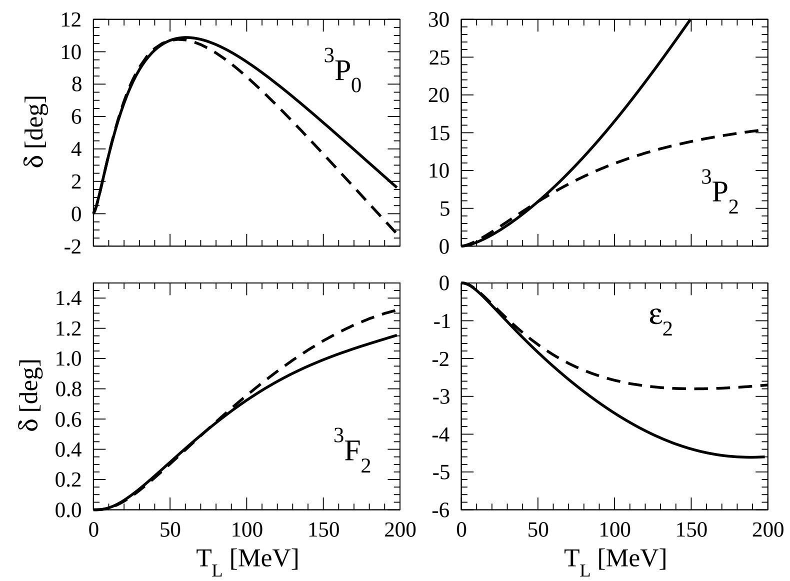

Fig. 3. Comparison [17] of properly renormalized, attractive triplet phase shifts (as function of the laboratory energy) for $\Lambda = 20\,\mathrm{fm}^{-1}$ (solid line) to the Nijmegen PWA (dashed line).

the Nijmegen PWA [8] at low energies, as illustrated in Fig. 3.

The conclusion is clear: a model-independent leading-order result can only be obtained if counterterms are promoted in all waves where the attractive OPE tensor potential is treated non-perturbatively. Implicit in Weinberg's power counting was the assumption that loops in the iteration of the potential do not bring significant new cutoff dependence. The parameters of contact interactions with derivatives or powers of m_π would thus be suppressed by powers of a large mass scale, M_{QCD}, and the effects of derivatives would scale as Q/M_{QCD}. However, we now see that Weinberg's implicit assumption is not correct. The short-range parameters needed to renormalize iterated OPE are enhanced in the infrared and driven by pion parameters, effects of derivatives scaling as Q/f_π (if we use $m_N \sim 4\pi f_\pi$).

Fortunately OPE does not need to be iterated in all waves, because of a kinematic suppression due to the centrifugal barrier. The appropriate counterterms will make OPE well defined, by selecting the correct phase in solutions such as (7), in the region $r \sim 1/f_\pi$. Therefore, the kinematic suppression can be estimated as for a regular potential. In the case of a central potential, it can be shown [21] that for $l \gg Qd$, where d is the range of the interaction, the l-wave phase shift is given by $\tan\delta_l \sim (Qd/(l+1/2))^{2l+1} \ll 1$ (barring fine-tuning). The ratio of the T matrix, and thus the potential, between $l+1$ and l is $\mathcal{O}(Q/lm_\pi)^2$, for large l. For $Q \sim m_\pi$, we are led to a suppression of $\mathcal{O}(1/l!^2)$. In the case of the tensor force, we expect $\mathcal{O}(1/l'!l!)$ for large l, l'. A more sophisticated argument for the momentum where OPE becomes non-perturbative in various waves is given in Ref. [22].

The above qualitative argument suggests that the effects of the corresponding higher-derivative counterterms are suppressed by a large (for large l) scale lf_π. Obviously, there might be other dimensionless factors missing here, but the fact that factors of l suppress OPE and its required counterterms in high-l waves must hold. For sufficiently large l, the suppression factor in counterterms becomes dominated by M_{QCD} (rather than lf_π), representing omitted QCD degrees of freedom, and the size of the counterterms is that assumed in Weinberg's power counting. On the other hand, for a finite number of low partial waves we find that perturbation theory is not sufficient for $Q \sim m_\pi$. Resummation is necessary and the cutoff dependence can be absorbed by one counterterm per partial wave. The favorable agreement of the leading-order calculation of Ref. [17] with data indicates that no additional inconsistencies are introduced.

The success of existing fits [7] based on Weinberg's power counting can be understood from the plateaus in the cutoff dependence of the counterterms (*c.f.* Fig. 2). Variation of the cutoff within a limited range on these plateaus will generate a band of values for observables. The error in a fit based on Weinberg's counting is likely dominated by the lowest partial wave without the required counterterm. As one goes to higher orders in Weinberg's counting, one acquires more counterterms, pushing the error to higher waves. The l suppression then ensures that the bands for the observables shrink, as observed [7], but, as we have seen, that does not imply Weinberg's counting is correct.

The corrected leading-order $2N$ interaction described above provides a model-independent $2N$ T matrix. We have verified [17] that the resulting triton binding energy is also cutoff independent. This is in contrast with the pionless EFT, where a $3N$ force is required [2] for renormalization already at leading order, and suggests that Weinberg's argument [1] for the smallness of few-nucleon forces in the pionful EFT holds.

5. Conclusion and Outlook

We have conjectured [17] that this mixture of perturbative treatment of higher partial waves, resummation of lower partial waves, and promotion of a finite number of counterterms is the most consistent approach to ChPT for nuclear systems. As we have seen, the leading-order nuclear potential consists of OPE plus the contact interactions required by its renormalization, and provides the $\mathcal{F}_{\nu_{\min}}$ in nuclear amplitudes of the type (1). In subleading orders, power counting naturally suggests a perturbative treatment of the subleading interactions that lead to the $\mathcal{F}_{\nu>\nu_{\min}}$s. The next-to-leading-order interactions consist in principle of TPE and counterterms with two more derivatives than leading order. Subsequent orders are constructed by the inclusion of successive powers of Q/M_{QCD}. The most-effective organizational scheme for subleading interactions probably relies on taking into account an explicit delta-isobar field [23]. The correctness of our modified power counting needs, of course, to be checked in future studies of higher orders [24].

After so many years and surprises, we hope that we are finally zeroing in the correct organizational scheme for the pionful EFT. This should lead to a model-independent nuclear potential with correct chiral-symmetry constraints, and better convergence and accuracy than found so far. It would remain to mine the rich soil of nuclear structure with this tool.

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