The Magnetic Helicity of an Interplanetary Hot Flux Rope

S. Dasso*, C.H. Mandrini† and P. Démoulin**

*Instituto de Astronomía y Física del Espacio (IAFE), Buenos Aires, Argentina.
Instituto de Ciencias, Universidad de Gral. Sarmiento, Los Polvorines, Buenos Aires, Argentina.
On leave from Instituto de Física del Plasma (INFIP), Buenos Aires, Argentina.
† Instituto de Astronomía y Física del Espacio, Buenos Aires, Argentina.
**Observatoire de Paris, DASOP, France.

Abstract. In the last years, interest in the study of the relationship between the magnetic helicity of solar active regions and the one contained in the interplanetary structures has grown. This has lead us to compute the helicity content of an interplanetary hot tube observed by Wind on October 24-25, 1995, applying three different approaches in cylindrical geometry: a linear force-free field, a constant twist angle, and a non force-free model with constant current. We have fitted the set of free parameters for each of the three models, finding that the determined magnetic helicity values are very similar when using the same orientation for the flux tube. From our point of view, these results imply that, whatever be the model used, magnetic helicity is a well-determined quantity and, thus, it is worth using it to understand the link between solar and interplanetary phenomena.

INTRODUCTION

Observations of helical magnetic structures in the solar atmosphere and solar wind have attracted considerable attention in the last years, with the consequent interest in magnetic helicity studies, both in the solar and interplanetary contexts. Magnetic helicity is one of the few global quantities which is preserved even in resistive MHD on time scales shorter than the global diffusion time scale [1]. Therefore, it can be used to link phenomena under very different physical conditions.

Interplanetary flux ropes, of which magnetic clouds are a subset, present in general a helicoidal structure that is supposed to be the trace of the torsion of the corresponding plasmoids ejected from the solar surface. These interplanetary phenomena can be modeled in cylindrical geometry using three different approaches: a linear force-free field model [2], a uniform twist model (or Gold-Hoyle model, see e.g. [3]) or a non force-free model with constant current [4]. These models are physically different, being not evident at all which of them give the best representation of interplanetary flux ropes.

In this work, we first derive the analytical expressions of the magnetic helicity for the three models mentioned above. Then, we apply them to a hot tube observed by Wind on October 24-25, 1995. We fit the set of free parameters for each of the three models and present our results. We find that the values of the magnetic helicity computed for each of the models are very similar for the same orientation of the tube. This orientation is computed using a minimum variance analysis [5]. However, if the orientation of the flux rope is fitted together with the model dependent physical parameters, what we have done for the constant current model [4], we obtain a value that is lower by a factor of $\sim$ 4-5. Even though, the three models give quite close values when we compute the magnetic helicity per unit of volume, $\sim$ 0.3-0.4 nT^2 AU. We suggest that our results imply that magnetic helicity is a well-determined quantity, and can be exploited to study the link between solar and interplanetary phenomena.

MAGNETIC HELICITY OF FLUX ROPES

The magnetic helicity of a field $\vec{\mathbf{B}}$ within a volume V is defined by $H = \int_V \vec{\mathbf{A}} \cdot \vec{\mathbf{B}}\, dV$, where the vector potential $\vec{\mathbf{A}}$ satisfies $\vec{\mathbf{B}} = \vec{\nabla} \times \vec{\mathbf{A}}$. However, the helicity defined as above is physically meaningful only when the magnetic field is fully contained inside the volume V (i.e., at any point of the surface S surrounding V, the normal component $B_n = \vec{\mathbf{B}} \cdot \widehat{\mathbf{n}}$ vanishes). This is so because the vector potential is defined only up to a gauge transformation ($\vec{\mathbf{A}}' = \vec{\mathbf{A}} + \vec{\nabla}\Phi$), then H is gauge-invariant only when $B_n = 0$. For cases where $B_n \neq 0$ (as on both legs of the interplanetary flux tubes) it has been shown that a relative magnetic helicity (H_r) can be defined [6]. This relative helicity is obtained subtracting the helicity of a

CP679, *Solar Wind Ten: Proceedings of the Tenth International Solar Wind Conference,*
edited by M. Velli, R. Bruno, and F. Malara

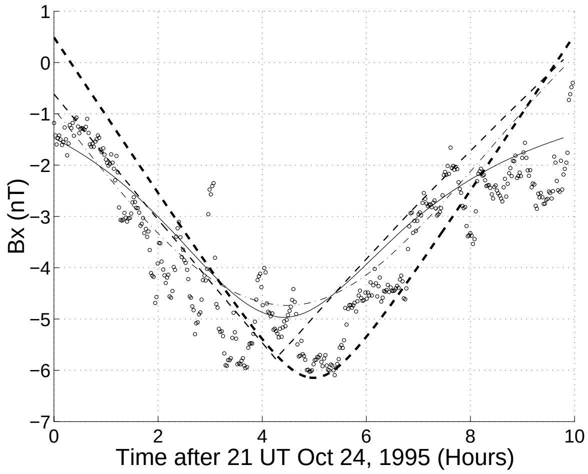

FIGURE 1. B_x component of the magnetic field (in GSE) for the flux rope observed on 24-25 October, 1995. Circles correspond to the observed field, solid line to Gold-Hoyle model, dash-dotted line to the linear force-free field model; while thin and thick dashed lines to the constant current model using a minimum variance method and a direct fit for the orientation of the tube, respectively.

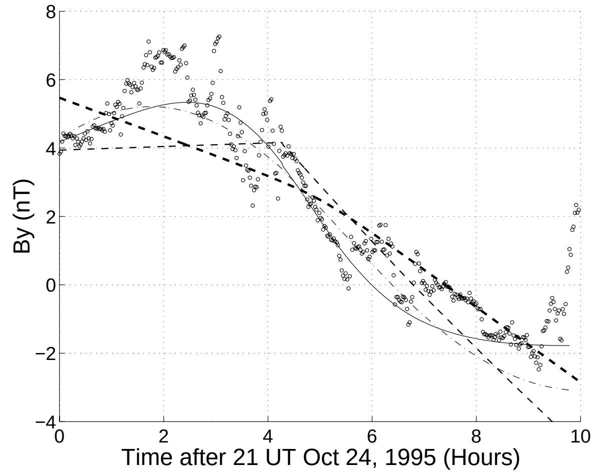

FIGURE 2. B_y component of the magnetic field (in GSE) for the flux rope observed by Wind on 24-25 October, 1995. The convention for the curves and the data is the same as in Fig. 1.

reference field $\vec{\mathbf{B}}_\mathbf{0}$ having the same distribution of B_n on S:

$$H_r = H - \int_V \vec{\mathbf{A}}_\mathbf{0} \cdot \vec{\mathbf{B}}_\mathbf{0}\, dV \quad . \tag{1}$$

H_r is gauge$-$invariant and it does not depend on the common extension of $\vec{\mathbf{B}}$ and $\vec{\mathbf{B}}_\mathbf{0}$ outside V, as was shown by [6] and [7]

The magnetic topology of an interplanetary flux rope can be modeled locally as a cylindrical structure with $\vec{\mathbf{B}}(\vec{\mathbf{r}}) = B_\phi(r)\hat{\phi} + B_z(r)\hat{\mathbf{z}}$. The reference field $\vec{\mathbf{B}}_0$ can be chosen as $\vec{\mathbf{B}}_0(r) = B_z(r)\hat{\mathbf{z}}$, and the vector potential as $\vec{\mathbf{A}}_0 = A_{0,\phi}(r)\hat{\phi} + A_{0,z}\hat{\mathbf{z}}$, with $A_{0,z}$ a constant value and $A_{0,\phi}(r)$ satisfying $rB_{0,z}(r) = \frac{\partial}{\partial r}(rA_{0,\phi}(r))$, in order to satisfy $\vec{\mathbf{B}}_0 = \vec{\nabla} \times \vec{\mathbf{A}}_0$.

Thus, the relative magnetic helicity per unit length (L) along the tube can be expressed independently of A_0 and B_0 as,

$$H_r/L = 4\pi \int_0^R rdr \;\; A_\phi B_\phi \tag{2}$$

where R is the radius of the magnetic tube.

MAGNETIC HELICITY OF THE LINEAR FORCE-FREE FIELD

The general static, axially symmetric magnetic field of a linear force-free configuration ($\vec{\nabla} \times \vec{\mathbf{B}} = \alpha \vec{\mathbf{B}}$) was obtained by [2]. However, it has been shown that only one harmonic of this solution is enough to describe the main tendency of 'in situ' measurements for interplanetary magnetic flux ropes, as magnetic clouds (e.g., [8, 9, 10]). Thus, the field is well modeled by

$$\vec{\mathbf{B}} = B_0 J_0(\alpha r)\hat{\mathbf{z}} + B_0 J_1(\alpha r)\hat{\phi} \tag{3}$$

where J_n is the Bessel function of the first kind of order n, B_0 is the strength of the field and α is a constant.

Computing H_r from Eq. (2) and taking $\vec{\mathbf{A}} = \vec{\mathbf{B}}/\alpha$, it is possible to obtain the relative helicity for this force-free field as follows,

$$\frac{H_r}{4\pi L} = \frac{B_0^2}{\alpha} \int_0^R dr\; rJ_1^2(\alpha r) \tag{4}$$

A numerical integration of this equation gives [11]:

$$\frac{H_r}{L} \sim 0.70 B_0^2 R^3 \tag{5}$$

MAGNETIC HELICITY OF A UNIFORMLY TWISTED FIELD

The non-linear force-free field having a uniform twist has been used to model interplanetary flux ropes (e.g., [12]). The components of $\vec{\mathbf{B}}$ for this configuration are [3],

$$\vec{\mathbf{B}} = \frac{B_0}{1+b^2r^2}\hat{\mathbf{z}} + \frac{B_0 br}{1+b^2r^2}\hat{\phi} \tag{6}$$

In this magnetic configuration the amount by which a given line is twisted when going from one end of the tube to the other (b) is independent of the radius of the tube r.

From (2), and considering:

$$\vec{\mathbf{A}} = -\frac{B_0}{2b}\ln(1+b^2r^2)\widehat{\mathbf{z}} + \frac{B_0}{2b^2r}\ln(1+b^2r^2)\widehat{\phi}, \qquad (7)$$

the relative helicity results

$$\frac{H_r}{L} = \frac{\pi B_0^2}{2b^3}[\ln(1+b^2R^2)]^2 \qquad (8)$$

MAGNETIC HELICITY OF A CONSTANT CURRENT FLUX ROPE

A non force-free model has been recently proposed by [13] and [4] to describe interplanetary structures. This model assumes a constant current density such as $\vec{\mathbf{J}}(\vec{\mathbf{r}}) = J_\phi\widehat{\phi} + J_z\widehat{\mathbf{z}}$, where J_ϕ and J_z are constants. Thus, the magnetic field of this configuration is obtained as $B_z(r) = \frac{4\pi}{c}J_\phi(R-r)$ and $B_\phi(r) = \frac{2\pi}{c}J_z r$, where R is the radius of the interplanetary tube and c is the speed of light. Being the trajectory of the spacecraft $r(t)$, it can be seen that $B_z(r(t))$ and $B_\phi(r(t))$ behave as the function modulo when the impact parameter (p) of the satellite is $p=0$.

From (2), and considering:

$$\vec{\mathbf{A}} = \frac{4\pi}{c}J_\phi r(R/2 - r/3)\widehat{\phi} - \frac{\pi}{c}J_z r^2\widehat{\mathbf{z}}, \qquad (9)$$

the relative helicity results,

$$\frac{H_r}{L} = \frac{28\pi^3}{15c^2}J_\phi J_z R^5 \qquad (10)$$

RELATIVE MAGNETIC HELICITY IN THE INTERPLANETARY HOT TUBE

We apply the analytical results derived in the previous section to the hot tube observed by Wind on October 24-25, 1995. The one minute cadence magnetic data have been downloaded from the public site: $http://cdaweb.gsfc.nasa.gov/cdaweb/istp-public/$.

Using a minimum variance analysis [5], [12] found a well-defined direction for the principal axis of the tube and $p \approx 0$ for this particular event. The variance coordinates have been used to obtain the physical parameters that best fit the observations of the flux tube for the three models discussed above. Furthermore, in order to test the validity of the minimum variance method, we have found the direction of the flux tube fitting directly its orientation (and consequently its radius) and physical parameters for the constant current model using GSE (Geocentric Solar Ecliptic) coordinates, as described in [13]. The fitting has been done in all cases using the Levenberg-Marquardt method [14]. Figures 1, 2, and 3 show the three components of the measured magnetic field, together with the curves obtained from every model.

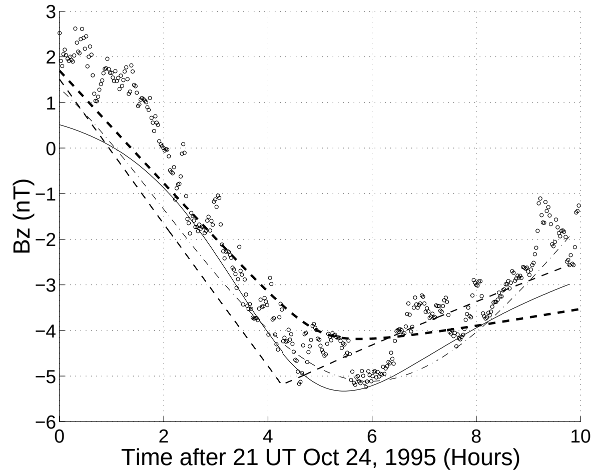

FIGURE 3. B_z component of the magnetic field (in GSE) for the flux rope observed by Wind on 24-25 October, 1995. The convention for the curves and the data is the same as in Fig. 1.

Using the expressions derived for the relative magnetic helicity and the values of the parameters coming from the fitting, we compute the relative helicity per unit length (L) and per unit of volume (Vol). The value of a χ^2 test and the characteristics of the flux rope for every case are:

- Linear force-free field ($\chi^2_{Lfff} = 1.7$ nT):

$$|\vec{\mathbf{B}}(\vec{\mathbf{r}}=0)| = 7.2 \text{ nT}, R = 0.035 \text{ AU},$$
$$\alpha = 65.8 \text{ AU}^{-1}$$
$$H_{r,Lfff} = 0.0016 \text{ nT}^2 \text{ AU}^3 L$$
$$H_{r,Lfff}/Vol = 0.40 \text{ nT}^2 \text{ AU}$$

- Uniform twist ($\chi^2_{Ut} = 1.5$ nT):

$$|\vec{\mathbf{B}}(\vec{\mathbf{r}}=0)| = 7.5 \text{ nT}, R = 0.035 \text{ AU}, b = 46.2 \text{ AU}^{-1}$$
$$H_{r,Ut} = 0.0015 \text{ nT}^2 \text{ AU}^3 L$$
$$H_{r,Ut}/Vol = 0.39 \text{ nT}^2 \text{ AU}$$

- Constant current model: Results taking the orientation of the tube as computed with the minimum variance method (see [12]):

$$\chi^2_{ConstJ} = 2.1 \text{ nT},$$
$$|\vec{\mathbf{B}}(\vec{\mathbf{r}}=0)| = 8.8 \text{ nT}, R = 0.035 \text{ AU},$$
$$J_z = 0.0435 \text{ nT/s}, J_\phi = 0.0401 \text{ nT/s},$$
$$H_{r,ConstJ} = 0.0013 \text{ nT}^2 \text{ AU}^3 L$$
$$H_{r,ConstJ}/Vol = 0.34 \text{ nT}^2 \text{ AU}$$

Results fitting simultaneously the orientation and the physical parameters of the tube:

$$\chi^2_{ConstJ} = 1.9 \text{ nT},$$

$$|\vec{\mathbf{B}}(\vec{\mathbf{r}}=0)| = 8.7 \text{ nT}, R = 0.020 \text{ AU},$$
$$J_z = 0.12 \text{ nT/s}, J_\phi = 0.07 \text{ nT/s},$$
$$H_{r,ConstJ} = 0.00041 \text{ nT}^2 \text{ AU}^3 L$$
$$H_{r,ConstJ}/Vol = 0.31 \text{ nT}^2 \text{ AU}$$

In this latter case, the best fit is found when both the orientation and physical parameters are fitted together; this is evident from both the values of χ^2_{ConstJ} and Figures 1-3. This may imply that the minimum variance method does not give the best results for the flux tube orientation (and consequently radius). It is the aim of a future paper to explore the range of validity of this method. However, when the same orientation is used for the three models, the values of the relative magnetic helicity per unit length are in a very good agreement; being the same true for its value per unit volume.

DISCUSSION AND CONCLUSIONS

Coronal mass ejections are originated by an instability of the solar coronal field. The plasmoid ejected from the Sun will carry part of the magnetic helicity of its original field. This structure will consequently appear as an interplanetary magnetic flux rope. There is increasing evidence that this is the case from observations showing that the helicity sign in magnetic clouds matches that of their source regions (see e.g., [15, 16, 17]). Therefore, the computation of the magnetic helicity is an important tool to compare interplanetary and solar phenomena.

We have estimated the relative magnetic helicity values for a hot interplanetary flux tube observed by Wind on October 24-25, 1995. The measured magnetic field components of the structure have been fitted using three different approaches: a linear force-free field, a uniform twist and constant current model. The three models fit relatively well the observations being the quality of the fit, according to a χ^2 test, in ascending order: the uniform twist, the linear force-free field and the constant current model. The relative magnetic helicity per unit length derived for the three cases is very similar when the orientation of the tube and, therefore, its radius is the same. The latter is computed using a minimum variance method, as discussed in [12]. Only in the case when the orientation is computed independently, by fitting it together with the physical parameters depending on the model, we find a value which is a factor $\sim 4-5$ lower. We suggest that this is an indication of the limitation of the minimum variance method, what we intend to quantify in a following paper. However, when we compute the helicity per unit of volume, that is to say when the results are independent of the radius, we find again very similar results for the three models, $H_r/Vol \sim 0.3-0.4$ nT^2 AU.

We conclude that magnetic helicity is a quantity that can be used to study the link between coronal and interplanetary phenomena, in the sense that, no matter the model used, its value is well-determined. The case studied here is an example. Our next step is to extend our analysis to a large dataset of interplanetary phenomena, in particular, magnetic clouds.

ACKNOWLEDGMENTS

This research has made use of NASA's Space Physics Data Facility (SPDF). This work was partially supported by the Argentinean UBA grant UBACYT X059. S.D. is a fellow of CONICET and C.H.M. is a member of the Carrera del Investigador Científico, CONICET. C.H.M. and P.D. thank ECOS (France) and SETCIP (Argentina) for their cooperative science program A01U04.

REFERENCES

1. Berger, M. A., *Geophysical and Astrophysical Fluid Dynamics*, **30**, 79–104 (1984).
2. Lundquist, S., *Ark. Fys.*, **2**, 361+ (1950).
3. Priest, E. R., *Solar magneto-hydrodynamics*, Dordrecht, Holland ; Boston : D. Reidel Pub. Co. ; Hingham,, 1982.
4. Hidalgo, M. A., Cid, C., Medina, J., and Viñas, A. F., Sol. Phys., **194**, 165–174 (2000).
5. Sonnerup, B. U., and Cahill, L. J., J. Geophys. Res., **72**, 171–183 (1967).
6. Berger, M. A., and Field, G. B., *J. Fluid. Mech.*, **147**, 133+ (1984).
7. Finn, J. H., and Antonsen, T. M., *Comments Plasma Phys. Contr. Fusion*, **9**, 111+ (1985).
8. Burlaga, L., Sittler, E., Mariani, F., and Schwenn, R., J. Geophys. Res., **86**, 6673–6684 (1981).
9. Burlaga, L. F., J. Geophys. Res., **93**, 7217–7224 (1988).
10. Lepping, R. P., Burlaga, L. F., and Jones, J. A., J. Geophys. Res., **95**, 11957–11965 (1990).
11. Démoulin, P., Mandrini, C. H., van Driel-Gesztelyi, L., Thompson, B. J., Plunkett, S., Kovári, Z., Aulanier, G., and Young, A., A&A, **382**, 650–665 (2002).
12. Farrugia, C. J., Janoo, L. A., Torbert, R. B., Quinn, J. M., Ogilvie, K. W., Lepping, R. P., Fitzenreiter, R. J., Steinberg, J. T., Lazarus, A. J., Lin, R. P., Larson, D., Dasso, S., Gratton, F. T., Lin, Y., and Berdichevsky, D., "A Uniform-Twist Magnetic Flux Rope in the Solar Wind," in *AIP Conf. Proc. 471: Solar Wind Nine*, 1999, pp. 745–748.
13. Hidalgo, M. A., Cid, C., Medina, J., Viñas, A. F., and Sequeiros, J., J. Geophys. Res., **107**, 10.1029+ (2002).
14. Press, W. H., Teukolsky, S. A., Vetterling, W. T., and Flannery, B. P., *Numerical Recipes*, Cambridge University Press, 1992.
15. Bothmer, V., and Schwenn, R., *Space Science Reviews*, **70**, 215–+ (1994).
16. Rust, D. M., Geophys. Res. Lett., **21**, 241–244 (1994).
17. Marubashi, K., "Interplanetary Magnetic Flux Ropes and Solar Filaments," in *Coronal Mass Ejection, Geophys. Monograph 99*, 1997, pp. 147–156.