Cross Helicity Correlations in the Solar Wind

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Abstract. Over the last decade, several magnetohydrodynamic models of the solar wind proposed a two component structure for the fluctuations: a "slab" (Alfvénic) component with wavenumbers parallel to the ambient dc magnetic field and a quasi two-dimensional (turbulent) component with wavenumbers mostly perpendicular to the magnetic field. Initial support and motivation for these models was given in part from the study of three dimensional correlation functions for the magnetic field from solar wind data (W.H Matthaeus, M.L. Goldstein and D.A. Roberts 1990, JGR 95, 20673). We extend here this study to the analysis of the cross-correlation between the velocity and the magnetic field. The cross-correlation function is simply related to the cross helicity power spectrum, a quantity of great interest for solar wind models. This quantity provides, on one hand, a measure of the relative importance of outgoing and incoming Alfvénic fluctuations. On the other hand, the turbulent properties of the system are greatly influenced by the amount of cross helicity present in it. We analyze ACE data and present preliminary results for the three dimensional cross-correlation function. Special emphasis is given to the implications for solar wind models.

INTRODUCTION

The solar wind is a privileged scenario where in-situ observations unveil many aspects concerning magnetohydrodynamic (MHD) turbulence in a magnetized plasma. Over the last two decades, much progress has been made by several authors by means of spacecraft observations of solar wind turbulence (see for instance [1] and references therein).

As opposed to the hydrodynamic (HD) case, two dynamic fields (velocity and magnetic fields) interplay to determine the evolution of an incompressible MHD turbulent system. A new rugged invariant, namely the cross helicity (or cross correlation between these two fields), gets into the scene. A useful dimensionless expression for this quantity is the normalized cross helicity (σ_c), defined as the ratio of the cross helicity to the total energy, and ranging from -1 to 1. Monopropagating Alfvén waves have maximum $|\sigma_c|$ ($\sigma_c = \pm 1$, depending on the sense of propagation). The cross helicity can be obtained as the difference of the energy of outgoing to incoming Alfvén waves, thus giving a measure of the imbalance between the two. The structure of the incompressible MHD equations is such that the non-linear terms vanish when the cross helicity is maximum. To develop turbulence it is necessary to have counterpropagating fluctuations along the mean magnetic field. These fluctuations are thought to interact non-linearly to produce an energy cascade in perpendicular wavenumbers (see Dmitruk et al. [2] and references therein). High levels of turbulence in the solar wind are usually accompanied by a value of σ_c close to zero (e.g., see [3] and references therein).

The normalized cross helicity and its spectrum have been determined from single-point measurements, and show the dominance of outgoing Alfvénic fluctuations [4]. However, there is to present no observational study of possible anisotropy in the solar wind cross helicity. The solar wind magnetic fluctuations are not isotropic, and strong evidence of the presence of two populations have been provide in [5] and [6]. The former shown the presence of: (a) Alfvénic (slab) fluctuations with correlation lengths stretched in the direction transverse to the background field (B_0) and (b) quasi-two dimensional (turbulent) fluctuations with elongated correlation lengths parallel to B_0.

We present hereafter our preliminary efforts to analyze possible anisotropy of the normalized cross helicity (σ_c), in the same spirit of the analysis of anisotropy in magnetic field self correlations performed in Ref.[4]. The second section describes the technique we use to process the interplanetary data in order to calculate two-dimensional correlation functions. The results are summarized in third section. Finally, we conclude in the last section.

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DATA PROCESSING

We analyze magnetic and bulk velocity fields measured by the Advanced Composition Explorer (ACE) spacecraft, from January 23, 1998 to March 3, 1999. The data have been analyzed with a cadence of one minute. The solar wind observations we analyze here correspond to a distance of ~ 1 AU from the Sun, and essentially on the ecliptic plane.

We group our whole set of data in 4-day intervals, thus obtaining N sub-series (or intervals). For every interval I ($I = 1,...,N$) and from the observed magnetic ($\mathbf{B}_j^I$) and velocity ($\mathbf{V}_j^I$) fields, we define the fluctuation fields ($\mathbf{b}_j^I$, $\mathbf{v}_j^I$, and the Els*ä*sser variables $\mathbf{z}_j^{I,\pm}$), where the index j labels the time t_j^I from the beginning of every sub-series I (i.e., $t_0^I = 0$), as follows:

$$\mathbf{v}_j^I = \mathbf{V}_j^I - \mathbf{U_0}^I$$
$$\mathbf{b}_j^I = \frac{\mathbf{B}_j^I}{\sqrt{4\pi\rho^I}} - \mathbf{V_A}^I$$
$$\mathbf{z}_j^{I,\pm} = \mathbf{v}_j^I \pm \mathbf{b}_j^I \tag{1}$$

Here, $\mathbf{U_0}^I = \langle \mathbf{V}_j^I \rangle$ and $\mathbf{V_A}^I = \langle \mathbf{B}_j^I \rangle / \sqrt{4\pi\rho^I}$ are respectively the time average for the plasma velocity and for the Alfvén velocity, within the interval I; ρ^I is the mean density of mass for the interval I.

In order to compute statistics from the observed data, we need to normalize the fluctuating fields so that the amplitude of the fluctuations in the different intervals be comparable. This is a rather drastic step that requires justification. It has been established that different time intervals in the solar wind have similar statistics, but when comparing one interval to any other there is usually a scaling factor relating the amplitude of the fluctuation in one with respect to the other. For example, the magnetic field on each interval usually is roughly a Gaussian variable, but with varying widths (see, for instance, [7] and references therein). Yet, if all the data were combined together, then a skewed distribution would result since adding two Gaussian distributions together can yield a non-Gaussian distribution. Thus, we choose to normalize the fields so that the energy in each interval is the same, and it is equal to the mean energy (kinetic plus magnetic) of the whole (raw) dataset E. That is, if E^I is the mean energy of the (raw) data in the interval I, we rescale the fluctuating fields ($\mathbf{v}$ and $\mathbf{b}$) with a similarity factor $\lambda^I = \sqrt{(E/E^I)}$. Others, such as Sorriso-Valvo et al. [8] have also used similar normalization schemes.

The two-point velocity correlation function is defined as

$$R_{vv}(\mathbf{r}) = \langle \mathbf{v}(\mathbf{x}) \cdot \mathbf{v}(\mathbf{x}+\mathbf{r}) \rangle \tag{2}$$

Analogous definitions hold for R_{bb}, R_{vb}, and for the correlations in the Els*ä*sser variables: R_{++} and R_{--}.

The ACE spacecraft provides time series of velocity and magnetic field, thus the correlation functions constructed from these data are essentially two-time single-point. However, due to the fact that the mean speed ($\mathbf{V_{sw}}$) of the solar wind is super-Alfvénic, it is possible calculate the spatial correlation functions from the measured temporal fluctuations using the relationship $R(0,t) = R(-\mathbf{V_{sw}}t, 0)$ [4]. These approximations are the MHD analogues of the Taylor 'frozen-in-flow' hypothesis [9].

For a given interval I, the mean speed of the solar wind $\mathbf{U_0}^I$ gives the direction of the lag $\mathbf{r}$, which is almost along the radial direction as measured from the Sun. So, we calculate $R^I(r)$, where r is a distance along $\mathbf{U_0}^I$. An in-house numerical code, which employs the 'Blackman-Tukey' technique [10] was used to calculate the different correlation functions $R^I(r)$. The maximum lag taken, when $R^I(r)$ is calculated, corresponds to two days.

In order to analyze the anisotropy of the fluctuations, we label each interval according to the value of the angle (θ^I) between the direction of the mean field ($\mathbf{V_A}^I$) and $\mathbf{U_0}^I$, and study variations in several statistical quantities as a function of θ, as shown below.

As mentioned before, the Els*ä*sser variables give information on the level of the activity of waves traveling either parallel or anti-parallel to the background magnetic field. To give physical meaning to our analysis, we have grouped the fluctuations according to whether they are traveling outwards from the Sun ("out"), or towards the Sun ("in"), and consistently re-labeled the Els*ä*sser variables as $\mathbf{z}_{out}$ and $\mathbf{z}_{in}$ in each interval. The reduced energy spectra for kinetic and magnetic energy ($E_v(k)$ and $E_b(k)$) , and for the Alfvén waves activity 'outward' ('inward') propagating, $E_{out}(k)$ ($E_{in}(k)$), are obtained by means of a Fast Fourier Transform of the corresponding correlation functions [3]. The spectra are normalized to give the total energies as usual:

$$E_{\{v,b\}} = \frac{1}{2}\langle \{v,b\}^2 \rangle = \int E_{\{v,b\}}(k)dk \tag{3}$$

$$E_{\{in,out\}} = \frac{1}{4}\langle \{z_{in}, z_{out}\}^2 \rangle = \int E_{\{in,out\}}(k)dk \tag{4}$$

From these energy spectra, the reduced cross helicity spectrum (in the direction of the mean wind velocity) can be obtained from

$$H_c(k) = (E_{out}(k) - E_{in}(k))/2, \tag{5}$$

and the normalized cross helicity from

$$\sigma_c(k) = \frac{E_{out}(k) - E_{in}(k)}{E_{out}(k) + E_{in}(k)}. \tag{6}$$

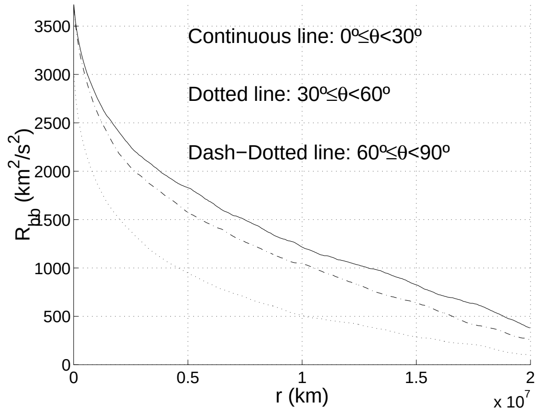

FIGURE 1. Conditioned average of the magnetic field self correlation function $R_{bb}(r)$, in Alfvén units. The averages have been conditioned to that intervals of the whole dataset where the value of the angle θ between the direction of the mean field $\mathbf{V_A}$ and mean wind velocity $\mathbf{U_0}$ is in a selected range. Continuous, dotted, and dash-dotted lines correspond to $0 < \theta < 30$, $30 < \theta < 60$, and $60 < \theta < 90$, respectively. The larger scale shown corresponds to $\sim$ 0.1 AU.

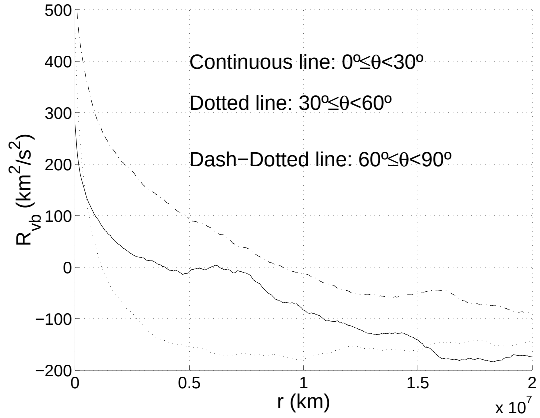

FIGURE 2. Cross helicity correlation function $R_{vb} = (R_{out} - R_{in})/4$. Different curves correspond to different ranges of θ as in Figure 1. Correlation decay more slowly in the direction perpendicular to the mean field.

RESULTS

In order to study spectral anisotropy (or alternatively spatial anisotropy for the correlation functions) we define three ranges for θ. The chosen ranges for θ and the number of intervals that correspond to every range are:

- $0 \leq \theta < 30$ (10 intervals),
- $30 \leq \theta < 60$ (53 intervals),
- $60 \leq \theta < 90$ (24 intervals).

Thus, from the correlation functions of every interval, $R^I(r)$, we carry out conditional averages considering only those intervals which correspond to a given range of θ values obtaining $R(r)$.

Figure 1 shows the conditional average of the magnetic field self correlation function according to θ. It is evident that the curve with steeper correlation corresponds to the oblique direction ($30 \leq \theta < 60$), in full consistency with Figure 3 of [4]. The results shown in this figure support the two component (slab + 2D) model.

The global degree of correlation between **v** and **b** is measured by the cross helicity $H_c = R_{vb}(r = 0)/2$. Considering the whole dataset, a value of $H_c \sim 230\text{km}^2/\text{sec}^2$ is obtained. The total (kinetic plus magnetic) mean energy resulted $\sim 3\text{x}10^3\text{km}^2/\text{sec}^2$. These numbers yield a normalized global cross helicity value of $\sigma_c = 2H_c/E \sim 0.15$. The two-point cross helicity correlation $R_{vb}(r)$ is shown in Figure 2 for the three different ranges of the angle θ. The figure seems to indicate that the correlations along the intermediate direction decay fastest than the other two components do. An alternative view of the angular distribution of cross correlations is given in figure 3. The figure shows the (reduced) normalized cross helicity power spectrum $\sigma_c(k)$ along different directions (the angle ranges defined above), what allows the analysis of different scales at different angles. Note that at intermediate angles ($\theta \sim 45$), for wave numbers larger than $5\text{x}10^{-7}$ km^{-1}, $\sigma_c(k)$ is larger than for the other two directions (perpendicular and along the mean field), what seems in contradiction with the picture of the two populations: the Alfvenic-slab and the turbulent quasi-2D. However we note that our statistics are still too low at extreme angles as to make any definite conclusions as yet. The opposite occurs at large scales ($k < 5\text{x}10^{-7}$ km^{-1}). It is important to stress the preliminary character of these results, what forces us to be cautious and avoid any physical interpretation until these results are either confirmed or corrected with a more complete analysis (see next section). Finally, figure 4 shows the Alfvén ratio spectrum $r_A(k)$. The three curves for different θs are very close to each other, and overall they seem consistent with the values reported in Table III of Ref.[3].

SUMMARY AND CONCLUSIONS

We present preliminary results from a study of anisotropy in the velocity, magnetic, and cross helicity correlation functions (also power-spectra) by considering spatial lags (wave-vectors) at different angles θ with respect to the background magnetic field $\mathbf{B}_0$.

The magnetic self correlations are consistent with previously published results, supporting the two component model of the solar wind. That is, the presence of two pop-

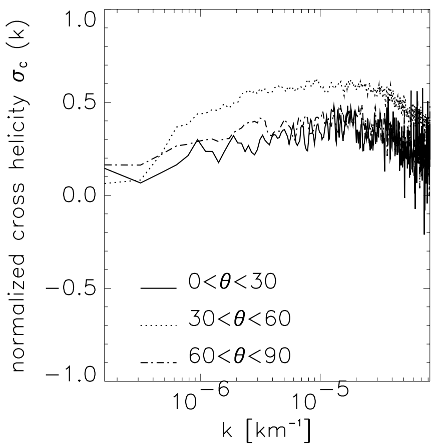

FIGURE 3. Plot of the conditioned average of the normalized cross helicity ($\sigma_c(k) = 2H_c(k)/E(k)$) power spectrum. The average has been conditioned for intervals with different values of θ, as in Figure 1.

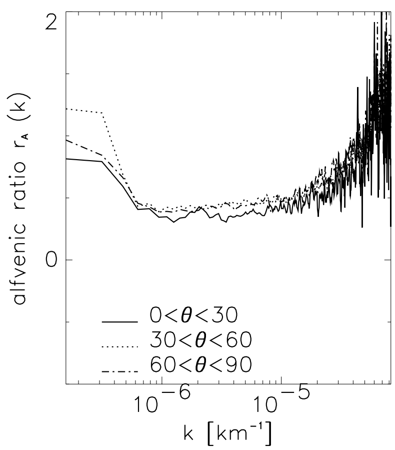

FIGURE 4. Figure showing the reduced spectrum of the Alfvén ratio, $r_A(k) = E_v(k)/E_b(k)$. Different curves correspond to different θ angle ranges, as in Figure 1.

ulations: a "slab" (or Afvénic) population aligned with the main magnetic field and with wavenumbers parallel to it, and a "quasi-2D" (or turbulent) population with almost perpendicular wavenumbers, as it is typical of anisotropic turbulence in the presence of a mean magnetic field.

We have also presented the methodology, techniques and some preliminary results of the study of angular dependence of the power spectrum (or alternatively its self correlation function) for very relevant quantities such as the dimensional and normalized cross helicity and the Alfvén ratio. The progress of our research at this point is still that of an early stage. To achieve more dependable results we need to: (a) extend our temporal base to a larger dataset, where more intervals with extreme angles $\theta \sim 0$ and $\theta \sim 90$ can be found; (b) consequently include more bins in our angular discretization (currently we can only use 3 because of lack of statistics and for simplicity in the analysis); (c) implement a stationary test and exclude data intervals with sector crossings; (d) study the noise we see in our spectra at very high k (close to the Nyquist frequency).

It is for all of these reasons that we resist the temptation of drawing any major conclusions out of the present preliminary analysis. We plan to extend our research in a timely fashion and publish our final results elsewhere. In the meantime, we believe that the physics involved in this research project are worth the effort, and will help achieve a better of the nature of the solar wind and MHD turbulence.

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