Spectral Distribution of the Cross Helicity in the Solar Wind

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There are a variety of theoretical and observational indications that fluctuation energy in astrophysical and space plasma turbulence is distributed anisotropically in space relative to the magnetic field direction. The cross helicity, represented by correlations between velocity and magnetic field fluctuations, enters a magnetohydrodynamic description on equal footing with the energy, but its anisotropy has not been examined in the same degree of detail. Here we employ Advanced Coronal Explorer data to examine the rotational symmetry of the cross helicity. We find that the normalized cross helicity is associated more or less equally with all angular components of the fluctuations. This favors turbulence models that allow for cross communication between parallel and perpendicular wave numbers, suggesting that "wavelike" and "turbulencelike" fluctuations are strongly coupled.

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The magnetic field induces a preferred direction that influences the evolution and spatial correlations in magnetohydrodynamic plasma turbulence. This effect has important implications for the basic physics of the turbulent plasma, including heating, instabilities, wave particle interactions, energetic particle (cosmic ray) scattering, and thermal conduction. Significant observational constraints regarding turbulence have been provided by more than three decades [1–3] of spacecraft data, making solar wind fluctuations the most completely studied case of astrophysical magnetohydrodynamic (MHD) turbulence, and the only one extensively studied using *in situ* methods. Two paradigms have arisen. In the first, fluctuations are described as noninteracting Alfvén waves propagating away from sources near the Sun [2]. These fluctuations have a distinctive correlation between velocity and magnetic fluctuations, or cross helicity, that signifies dominance of outward propagation and that decreases with increasing heliocentric distance [4–6]. In the second perspective, the fluctuations represent an active, evolving turbulent medium, displaying properties similar to Kolmogoroff hydrodynamic turbulence, along with MHD features such as the quasiequipartition of kinetic and magnetic energy [1,5,7,8]. The decrease in Alfvénic correlation is often attributed to the effects of driving by large scale shear, which injects zero cross helicity turbulence [9]. In this way shear driving by turbulence leads to a general reduction of cross helicity with increasing heliocentric distance, although there are observations that suggest the sporadic presence of Alfvénic correlations in highly oblique wave vectors at about 8 AU (astronomical units) [10]. In spite of the dissimilarity of these descriptions, certain classes of composite spectral models in which both wave and turbulence properties are embodied seem to reconcile the two viewpoints [11–13], and explain many observed features that cannot be accounted for by either waves or turbulence alone [14,15]. While models based on "two components" of these types are used in simplified kinematic descriptions, there are a number of studies that indicate that wavelike and turbulencelike fluctuations cannot evolve independently (e.g., [16–18]). Thus it is entirely unclear on theoretical grounds where one anticipates cross helicity to reside in the spectrum. Are the parallel wave vectors more Alfvénic because they are more wavelike? Is the low frequency, more turbulent part of the spectrum necessarily less Alfvénic? Or, alternatively, do couplings between high and low frequency spectral components blur this distinction? Here we examine these questions through systematic observational analysis, using a direction-sensitive correlation method [19]. We conclude that the normalized cross helicity, and its characteristic Alfvénic correlation, is about equally present in all the analyzed spectral components at 1 AU.

In MHD there are two interacting dynamic vector fields (velocity $\mathbf{v}$ and magnetic field $\mathbf{b}$). The correlation between $\mathbf{v}$ and $\mathbf{b}$, the cross helicity, written here as the mean value $H_c = \langle \mathbf{v} \cdot \mathbf{b} \rangle$, plays an important role in MHD theory. In linear and in nonlinear Alfvén wave theory [20] cross helicity is a signature of propagation effects. The total cross helicity is a "rugged invariant" of ideal MHD for suitable boundary conditions, surviving arbitrary cutoffs of the representation in Fourier modes [21,22], and therefore has been employed in minimum energy calculations of stable MHD configurations [23,24]. Various studies found [25–27] that cross helicity often grows in time relative to energy, leading ultimately to a purely correlated "Alfvénic" state. However, this dynamic alignment process is not observed in the solar wind; instead [6] the degree of correlation is seen to decrease as the turbulence ages. This cannot be accounted for by wave based "WKB" theory [28]. The resolution appears to be related to the fact that dynamic alignment is not universal [29] and, in particular, that a strongly sheared velocity field can drive the turbulence towards other asymptotic states. Thus, in the solar wind, shear associated with high speed stream interfaces [9] may drive the turbulence away from Alfvénic states, consistent with observations.

Paralleling the development of MHD theory of cross helicity, there has been an evolution on thinking about the characteristics of energy spectra in MHD turbulence and in the solar wind. The observation of preferential excitation of gradients perpendicular to the applied magnetic field in laboratory devices [30] led to the theoretical suggestion that turbulence operates mainly in a low frequency "reduced MHD" mode in the presence of a strong guide field [31,32] and that the associated preferential excitation of perpendicular wave vectors is generated naturally by MHD turbulence [33,34]. In response to this, multicomponent models were implemented for solar wind studies [12,19] where it has been found that neither quasi-2D nor parallel-wave spectral components by themselves are sufficient to explain, for example, both energetic particle scattering [14] and proton heating [15]. Astrophysical adaptations of these ideas have focused on the steady state quasi-2D component [35] and ignored the wavelike component, with sometimes puzzling consequences, as in cosmic rays scattering [36]. In any case it is completely clear that the solar wind cannot be described by a model with a single simple symmetry [13,19,37] and that therefore the issue of where in wave vector space one finds the cross helicity, usually taken as an indicator of wave effects, is crucial in the advancement of models of astrophysical turbulence. It is this question that we address here.

We analyze magnetic and bulk velocity fields measured by the Advanced Composition Explorer (ACE) spacecraft, from January 23, 1998, to June 30, 2002. The data have been analyzed with a cadence of 1 min. A preliminary report on the technique and goals of our research efforts has been presented elsewhere [38]. The solar wind observations we analyze here correspond to a distance of $\sim$1 AU from the Sun, and essentially on the ecliptic plane. The data are grouped in four-day intervals, thus obtaining N subseries (or intervals). Intervals showing sector crossings are identified by visual inspection and removed. We then shift our data set by two days and repeat the procedure, thus maximizing the data utilization. For every interval I, from the observed magnetic ($\mathbf{B}^I$) and velocity ($\mathbf{V}^I$) fields we define the fluctuation fields $\mathbf{b}^I$, $\mathbf{v}^I$, and the Elsässer variables $\mathbf{z}^{I,\pm} = \mathbf{v}^I \pm \mathbf{b}^I$, as follows: $\mathbf{v}^I = \mathbf{V}^I - \mathbf{U}_0^I$, $\mathbf{b}^I = \mathbf{B}^I - \mathbf{B}_0^I$. Here, $\mathbf{U}_0^I$ and $\mathbf{B}_0^I$ denote linear fits (except for the highly structured radial component of $\mathbf{V}$, for which we employ a cubic) to $\mathbf{V}$ and $\mathbf{B}$ (respectively) in the interval I, intended to remove coherent trends in the data [5]. Magnetic fields are in velocity units: $\mathbf{B}^I \to \mathbf{B}^I/\sqrt{4\pi\rho^I}$, using the mean mass density for the interval I, ρ_I. Our main goal is to compute two-point correlations of the form $R_{vv}(\mathbf{r}) = \langle \mathbf{v}(\mathbf{x}) \cdot \mathbf{v}(\mathbf{x}+\mathbf{r})\rangle$. Analogous definitions hold for R_{bb}, for R_{vb}, and for the correlations in the Elsässer variables: R_{++} and R_{--}.

The single spacecraft ACE data that we employ provide two-time single-point correlations. However, because of the super-Alfvénic and supersonic character of the solar wind, we construct spatial correlation functions in the usual way [5] by making use of the MHD analogues of the Taylor "frozen-in-flow" hypothesis [39]. For a given interval I, the mean solar wind velocity $\mathbf{V}_{sw}^I \equiv \langle \mathbf{V}^I \rangle$ gives the associated spatial lag $\mathbf{r} = \mathbf{V}_{sw}^I t$ at time t; this is almost along the heliocentric radial direction. In this way we employ a "Blackman-Tukey" technique [40] to compute the correlation functions $R^I(r)$. The maximum computed lag corresponds to two days. In order to analyze the anisotropy of the fluctuations, we label each interval according to the value of the angle θ^I between the direction of the mean field $\mathbf{V}_A^I \equiv \langle \mathbf{B}^I \rangle$ and $\mathbf{V}_{sw}^I$, and study variations in several statistical quantities as a function of θ. More specifically, we define five ranges for θ, as shown in Fig. 1. The number of data intervals in each angular range (ordered by increasing θ) is 30, 106, 111, 122, and 89. Thus, from the correlation functions of every interval, $R^I(r)$, we carry out conditional averages considering only those intervals that correspond to a given range of θ values obtaining $R(r)$.

It has been established that different data intervals spanning a few days in the solar wind have similar statistics, but when comparing one interval to any other there is usually a scaling factor relating the amplitude of the fluctuation in one with respect to the other (see, for instance, [41,42] and references therein). Thus, we choose

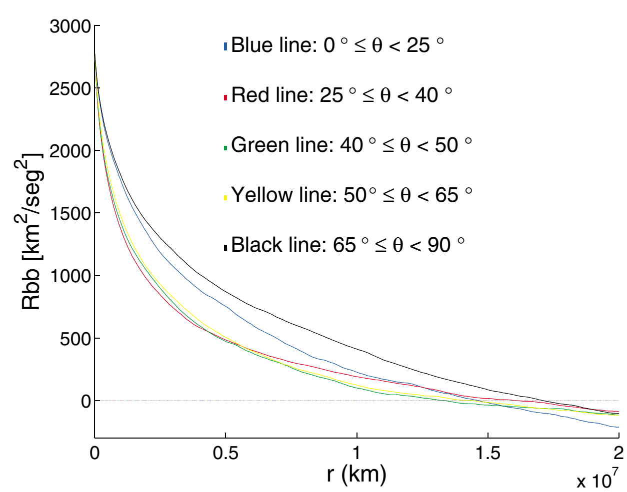

FIG. 1 (color). Magnetic field self-correlation function $R_{bb}(r)$, computed at different angles θ between the direction of the mean magnetic field and the displacement vector $\mathbf{r}$. Error bars are shown beside the legends. They represent the statistical error, evaluated at a separation r equal to the correlation length. The largest scale shown corresponds to $\sim$0.1 AU.

a normalization scheme that takes this effect into account. We simply compute, in each data interval, normalized correlation functions of the form $R_{fg}^{\mathrm{norm},I}(r) \equiv \lambda^I \langle \mathbf{f}^I(\mathbf{x}) \cdot \mathbf{g}^I(\mathbf{x}+\mathbf{r})\rangle$, where $\lambda^I \equiv \langle \mathbf{f} \cdot \mathbf{g}\rangle / \langle \mathbf{f}^\mathbf{I} \cdot \mathbf{g}^\mathbf{I}\rangle$, and $\mathbf{f}$ and $\mathbf{g}$ represent any of the fluctuating fields defined above. Note that the chosen normalization implies $R_{fg}^{\mathrm{norm},I}(0) = \langle \mathbf{f} \cdot \mathbf{g}\rangle$ for all intervals I. For simplicity in the notation, we omit the "norm" label hereafter.

To give physical meaning to our analysis, we have grouped the fluctuations according to whether they are traveling outwards from the Sun ("out"), or towards the Sun ("in"), and consistently relabeled the Elsässer variables as $\mathbf{z}_{\mathrm{out}}$ and $\mathbf{z}_{\mathrm{in}}$ in each interval. The reduced energy spectra for kinetic and magnetic energy $[E_v(k)$ and $E_b(k)]$, and for the Alfvénic fluctuations $[E_{\mathrm{out}}(k)$ and $E_{\mathrm{in}}(k)]$, are obtained by means of a fast Fourier transform of the corresponding correlation functions [8]. From these energy spectra, the reduced cross helicity spectrum (in the direction of the mean wind velocity) can be obtained from $H_c(k) = [E_{\mathrm{out}}(k) - E_{\mathrm{in}}(k)]/2$, and the normalized cross helicity from $\sigma_c(k) = [E_{\mathrm{out}}(k) - E_{\mathrm{in}}(k)]/[E_{\mathrm{out}}(k) + E_{\mathrm{in}}(k)]$. Unidirectionally propagating Alfvén waves have maximum $|\sigma_c|$, with $\sigma_c = \pm 1$, depending on the sense of propagation relative to the mean magnetic field. The structure of the incompressible MHD equations is such that the nonlinear terms vanish when the cross helicity is maximum. High levels of turbulence in the solar wind are usually accompanied by a value of σ_c close to zero (e.g., see [8] and references therein).

Figure 1 shows the conditional average of the magnetic field self-correlation function according to θ. It is evident that for $r < 10^7$ km, the curves with more extended correlations correspond to the quasiparallel $(0 \leq \theta < 25)$ and quasiperpendicular $(65 \leq \theta < 90)$ directions, in full consistency with Fig. 3 of [5]. In this regard, the results shown in this figure support the "Maltese cross" anisotropic structure for magnetic correlations. The angular dependence of the two-point cross helicity correlation $R_{vb}(r)$ in turn is shown in Fig. 2. The figure shows that the cross correlation between $\mathbf{v}$ and $\mathbf{b}$ has the same anisotropic structure as the magnetic self-correlation. This is a new important result regarding the three-dimensional structure of solar wind turbulence. As a corollary, one would expect that a normalized quantity such as σ_c would be nearly isotropic. This is indeed the case. Figure 3 shows the (reduced) normalized cross helicity power spectrum $\sigma_c(k)$ along different directions. The behavior displayed by the figure is essentially isotropic. Note that the values for σ_c are similar to the ones reported in Table III of Ref. [8] (see also Ref. [6]). The Alfvén ratio spectrum $r_A(k) = E_v(k)/E_b(k)$ (not shown here for brevity) is also essentially isotropic.

We have presented a study of anisotropy in the velocity, magnetic, and cross helicity correlation functions and power spectra. The magnetic self-correlations are consistent with previously published results [11] that show distinct lobes aligned with the parallel and perpendicular axes. (This Maltese cross pattern motivates the identification of two idealized populations: a "slab" population with wave vectors aligned with the main magnetic field and a "quasi-2D" population with almost perpendicular wave vectors.) The present observations show that the cross helicity too is highly anisotropic, and in almost exactly the same sense as the anisotropy of the energy. Thus, somewhat remarkably, the normalized Alfvénic correlation is about equally present in all the analyzed spectral components. This observational result rules out any model in which the Alfvénic correlation is concentrated in a particular angular part of the spectrum—such as either the wavelike or the quasi-2D component separately. Sometimes Alfvénicity is traced to propagation of outward (pure cross helicity) waves away from the source region of the super-Alfvénic wind [2]. On the other hand, (high perpendicular wave number) turbulence is viewed sometimes as driven and of low cross helicity [43]. Adopting this perspective, some multicomponent models [12] take the slab component to be Alfvénic and the

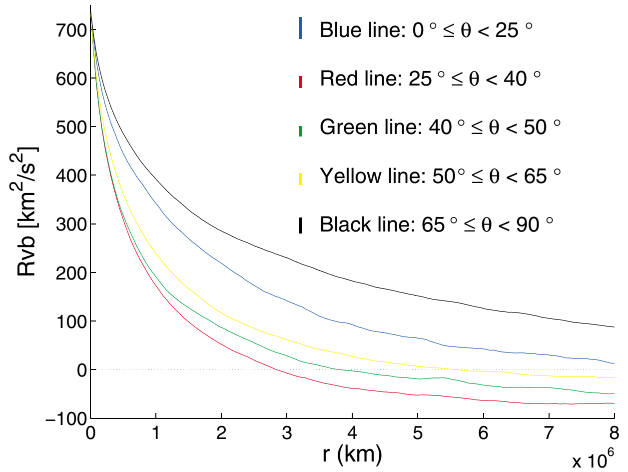

FIG. 2 (color). Cross-helicity correlation function $R_{vb} = (R_{\mathrm{out}} - R_{\mathrm{in}})/4$, computed at different ranges of θ as in Fig. 1. Error bars have the same meaning as in Fig. 1.

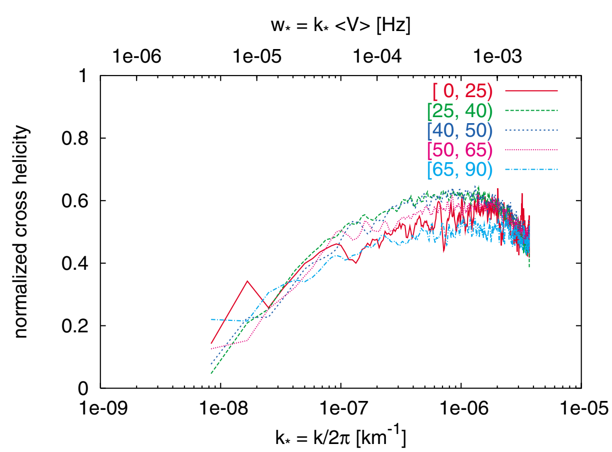

FIG. 3 (color). Reduced power spectrum of the normalized cross helicity $[\sigma_c(k)]$, for different values of θ, as in Fig. 1.

quasi-two-dimensional part to be non-Alfvénic or even static magnetic pressure-balanced "structures." Such interpretations are extreme and, in fact, unrealizable. For example, if we take the normalized cross helicity to be $\sigma_c = 0.6$ at 1 AU, then it is impossible to account for all of the cross helicity in slab modes if the latter comprise only 20%–30% of the energy as various models suggest [13,14,44]. Now we also see that such assumptions are inconsistent with direct observations. Clearly, if the slablike modes are in some sense the source of cross helicity, there must be strong transfer of cross helicity to account for the current observations. This implies a strong coupling between parallel ("wave") and perpendicular ("turbulence") components. A candidate for such a coupling is the presence of inhomogeneities in the solar wind velocity. It has recently been shown [18], by means of numerical solutions of the MHD equations in a solar windlike scenario, how an initial configuration of "parallel waves" can be perturbed by small-scale shear from wind irregularities to produce perpendicular wave numbers, thus transferring high cross helicity from one component to the other. While a component description may be useful for some applications, it seems clear that at a fundamental level solar wind turbulence is acting like a single entity.

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[1] P. J. Coleman, Astrophys. J. **153**, 371 (1968).
[2] J.W. Belcher and L. Davis, Jr., J. Geophys. Res. **76**, 3534 (1971).
[3] J. R. Jokipii, Annu. Rev. Astron. Astrophys. **11**, 1 (1973).
[4] B. Bavassano, M. Dobrowolny, F. Mariani, and N. F. Ness, J. Geophys. Res. **87**, 3617 (1982).
[5] W. H. Matthaeus and M. L. Goldstein, J. Geophys. Res. **87**, 6011 (1982).
[6] D. A. Roberts, M. L. Goldstein, L.W. Klein, and W. H. Matthaeus, J. Geophys. Res. **92**, 12 023 (1987).
[7] M. L. Goldstein, D. A. Roberts, and W. H. Matthaeus, Annu. Rev. Astron. Astrophys. **33**, 283 (1995).
[8] C.-Y. Tu and E. Marsch, *MHD Structures, Waves and Turbulence in the Solar Wind: Observations and Theories* (Kluwer, Dordrecht, 1995).
[9] D. A. Roberts, S. Ghosh, M. L. Goldstein, and W. H. Matthaeus, Phys. Rev. Lett. **67**, 3741 (1992).
[10] D. A. Roberts, L.W. Klein, M. L. Goldstein, and W. H. Matthaeus, J. Geophys. Res. **92**, 11 021 (1987).
[11] W. H. Matthaeus, M. L. Goldstein, and D. A. Roberts, J. Geophys. Res. **95**, 20 673 (1990).
[12] C.-Y. Tu and E. Marsch, J. Geophys. Res. **98**, 1257 (1993).
[13] J.W. Bieber, W. Wanner, and W. H. Matthaeus, J. Geophys. Res. **101**, 2511 (1996).
[14] J.W. Bieber, W. H. Matthaeus, C.W. Smith, W. Wanner, M. Kallenrode, and G. Wibberenz, Astrophys. J. **420**, 294 (1994).
[15] W. H. Matthaeus, G. P. Zank, C.W. Smith, and S. Oughton, Phys. Rev. Lett. **82**, 3444 (1999).
[16] S. Ghosh, W. H. Matthaeus, D. A. Roberts, and M. L. Goldstein, J. Geophys. Res. **103**, 23 691 (1998).
[17] S. Ghosh, W. H. Matthaeus, D. A. Roberts, and M. L. Goldstein, J. Geophys. Res. **103**, 23 705 (1998).
[18] M. L. Goldstein, D. A. Roberts, and A. Deane, in *Solar Wind Ten*, AIP Conf. Proc. No. 679 (AIP, New York, 2003), pp. 405–408.
[19] W. H. Matthaeus, M. L. Goldstein, and D. A. Roberts, J. Geophys. Res. **95**, 20 673 (1990).
[20] A. Barnes, in *Solar System Plasma Physics*, edited by E. N. Parker, C. F. Kennel, and L. J. Lanzerotti (North-Holland, Amsterdam, 1979), Vol. I, p. 251.
[21] L. Woltjer, Proc. Natl. Acad. Sci. U.S.A. **44**, 833 (1958).
[22] L. Woltjer, Proc. Natl. Acad. Sci. U.S.A. **44**, 489 (1958).
[23] U. Frisch, A. Pouquet, J. Léorat, and A. Mazure, J. Fluid Mech. **68**, 769 (1975).
[24] D. Fyfe and D. Montgomery, J. Plasma Phys. **16**, 181 (1976).
[25] M. Dobrowolny, A. Mangeney, and P. Veltri, Astron. Astrophys. **83**, 26 (1980).
[26] W. H. Matthaeus, S. Oughton, D. Pontius, and Y. Zhou, J. Geophys. Res. **99**, 19 267 (1994).
[27] A. Pouquet, M. Meneguzzi, and U. Frisch, Phys. Rev. A **33**, 4266 (1986).
[28] J.V. Hollweg, J. Geophys. Res. **95**, 14 873 (1990).
[29] A. C. Ting, W. H. Matthaeus, and D. Montgomery, Phys. Fluids **29**, 3261 (1986).
[30] D. Robinson and M. Rusbridge, Phys. Fluids **14**, 2499 (1971).
[31] D. C. Montgomery, C.-S. Liu, and G. Vahala, Phys. Fluids **13**, 815 (1972).
[32] H. R. Strauss, Phys. Fluids **19**, 134 (1976).
[33] J.V. Shebalin, W. H. Matthaeus, and D. Montgomery, J. Plasma Phys. **29**, 525 (1983).
[34] S. Oughton, E. R. Priest, and W. H. Matthaeus, J. Fluid Mech. **280**, 95 (1994).
[35] P. Goldreich and S. Sridhar, Astrophys. J. **438**, 763 (1995).
[36] B. D. G. Chandran, Phys. Rev. Lett. **85**, 4656 (2000).
[37] V. Carbone, F. Malara, and P. Veltri, J. Geophys. Res. **100**, 1763 (1995).
[38] S. Dasso, L. J. Milano, W. H. Matthaeus, and C.W. Smith, in *Solar Wind Ten* (Ref. [18]), pp. 546–549.
[39] G. Taylor, Proc. R. Soc. London A **164**, 476 (1938).
[40] R. Blackman and J. Tukey, *Measurements of Power Spectra* (Dover, Mineola, NY, 1958).
[41] N. S. Padhye, C.W. Smith, and W. H. Matthaeus, J. Geophys. Res. **106**, 18 635 (2001).
[42] L. Sorriso-Valvo, V. Carbone, P. Veltri, G. Consolini, and R. Bruno, Geophys. Res. Lett. **26**, 1801 (1999).
[43] D. A. Roberts, M. L. Goldstein, W. H. Matthaeus, and S. Ghosh, J. Geophys. Res. **97**, 17 115 (1992).
[44] G. P. Zank and W. H. Matthaeus, J. Geophys. Res. **97**, 17 189 (1992).