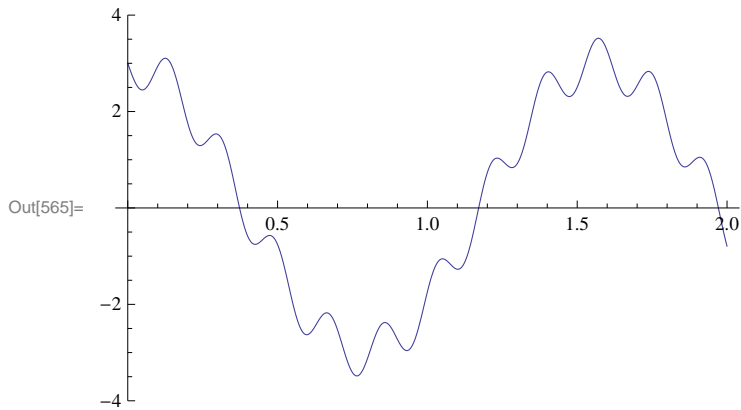


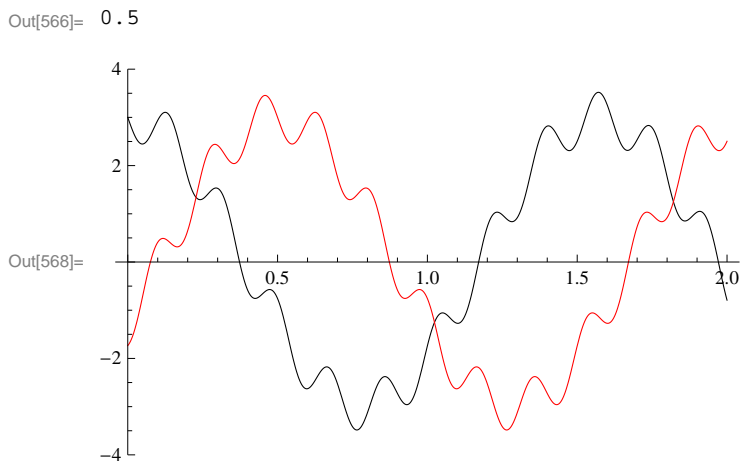
Paquetes de Onda

Ondas

```
In[564]:= A[x_] := x^2 / 125 - Sin[35 * x] / 2 + 3 * Cos[4 * x]  
Plot[A[x], {x, 0, 2},  
PlotRange -> {-4, 4}  
]
```



```
In[566]:= v = 0.5  
t = 1;  
Plot[{A[x], A[x - v * t]}, {x, 0, 2},  
PlotRange -> {-4, 4},  
PlotStyle -> {RGBColor[0, 0, 0], RGBColor[1, 0, 0]}  
]
```

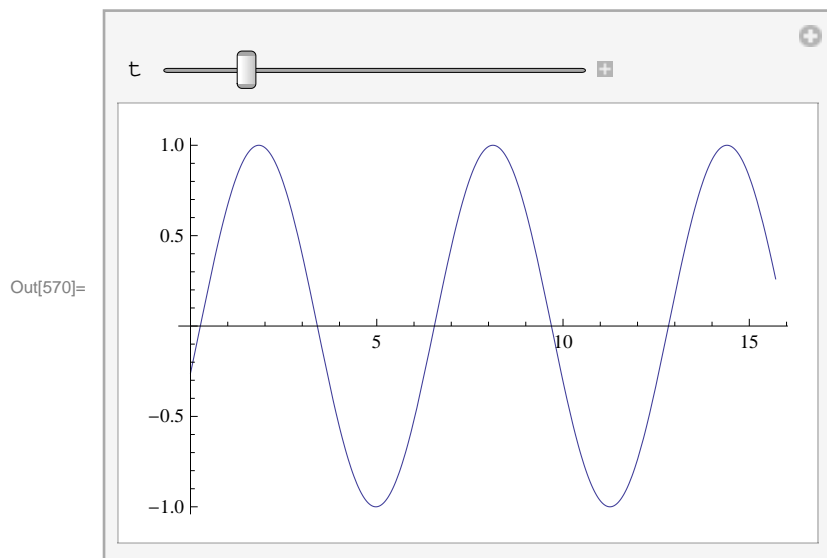


Dibujo de Ondas Armonicas

```
In[607]:= Onda[x_, t_] := Sin[x - v * t];
```

```
In[485]:= v = 2;  
T = 2 * Pi / v;  
Nf = 4;  
Delta t = 2 * T / Nf;
```

```
In[570]:= Manipulate[Plot[Onda[x, t], {x, 0, 5 π}], {{t, "t"}, 0, T}]
```



Ejercicios:

1. Hallar las velocidades de avance de la onda (grupo o fase?)
2. Que pasa si en lugar de Sin[x] se pone Cos[x] ?

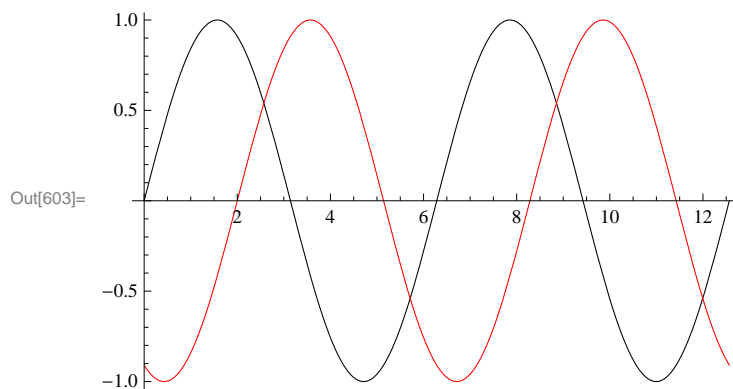


Construccion de paquetes de ondas

```
In[597]:= Clear[Onda];
Onda[k_, ω_, x_, t_] := Sin[k * x - ω * t];
```

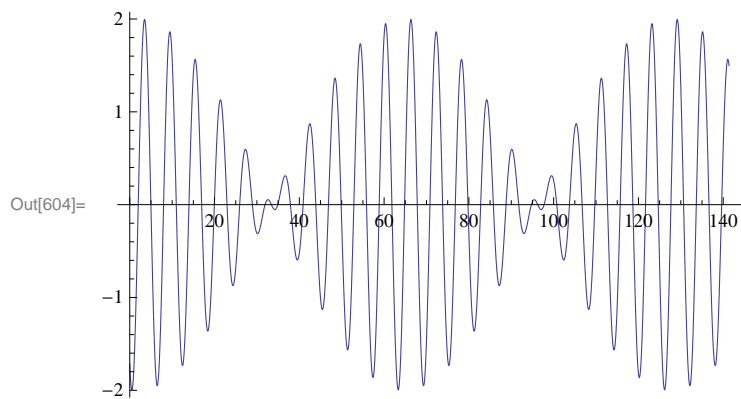
```
In[599]:= Δk = k / 10;
Δω = ω / 10;
```

```
In[601]:= k = 1;
ω = 2;
Plot[ {Onda[k, ω, x, 0], Onda[k, ω, x, 1] }, {x, 0, 4 π},
PlotStyle -> {RGBColor[0, 0, 0], RGBColor[1, 0, 0]}
]
```



```
In[578]:= Δk = k / 10;
Δω = ω / 10;
```

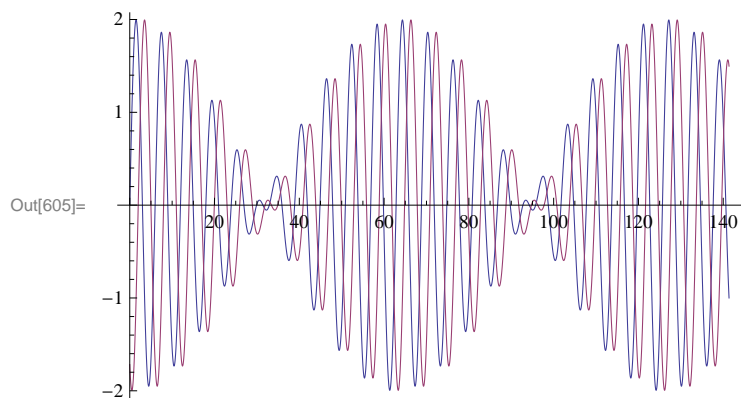
```
In[604]:= Plot[ Onda[k, ω, x, 1] + Onda[k + Δk, ω + Δω, x, 1] , {x, 0, 45 π} ]
```



Ejercicios

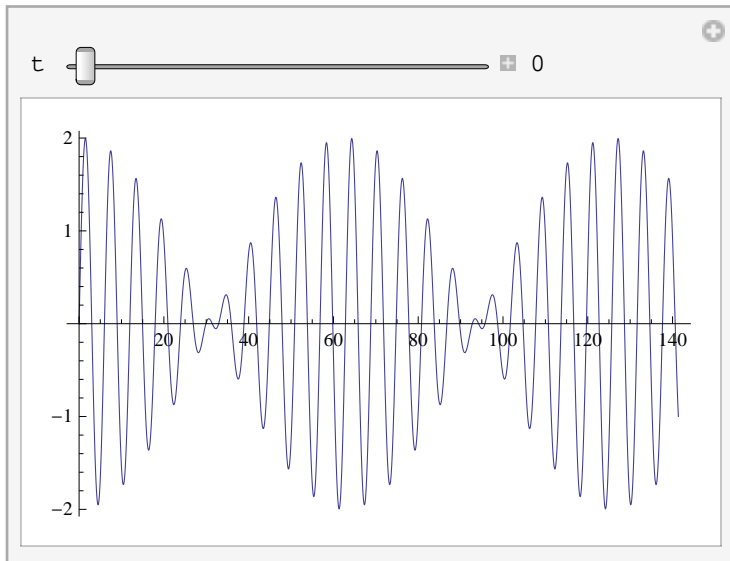
1. Hallar las velocidades de avance de la onda (grupo o fase?)
2. Construir un paquete de ondas sumando muchas ondas
3. Avanza el paquete? A que velocidad?

```
Plot[ {Onda[k, ω, x, 0] + Onda[k + Δk, ω + Δω, x, 0],
      Onda[k, ω, x, 1] + Onda[k + Δk, ω + Δω, x, 1] }, {x, 0, 45 π} ]
```



```
In[606]:= Manipulate[Plot[ Onda[k, ω, x, t] + Onda[k + Δ k, ω + Δω, x, t] , {x, 0, 45 π} ],
  {{t, 0, "t"}, 0, T, Δ t/100, Appearance → "Labeled"}]
```

Out[606]=



In[504]:= **Paquete de Ondas**

```
In[583]:= ClearAll[k, ω, x, t, wavepacket]
```

```
In[584]:= nondas = 10;
k = 1;
ω = 2;
Δ k = k / nondas;
Δω = ω / nondas;
```

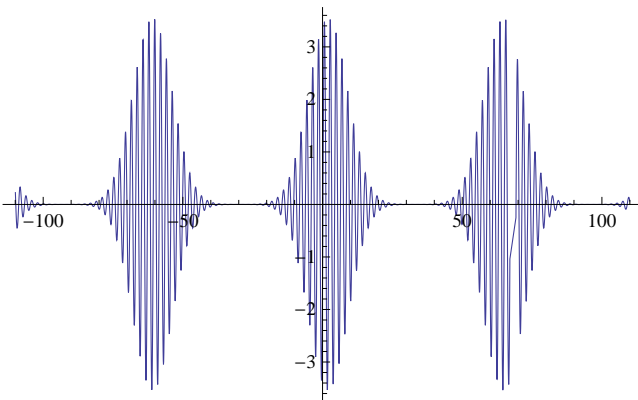
```
In[589]:= δ k = 0.2;
```

```
In[590]:= Onda[k_, ω_, x_, t_] := Exp[I (k * x - ω * t)];
```

```
In[591]:= wavepacket[k_, ω_, x_, t_] :=
  Sum[Exp[- (i Δ k / δ k)^2] Onda[k + i Δ k, ω + i Δω, x, t], {i, -1 nondas, 1 nondas, 1}]
```

```
In[514]:= Plot[Re[wavepacket[3, 2, x, 1]], {x, -35 π, 35 π}, PlotRange → All]
```

Out[514]=



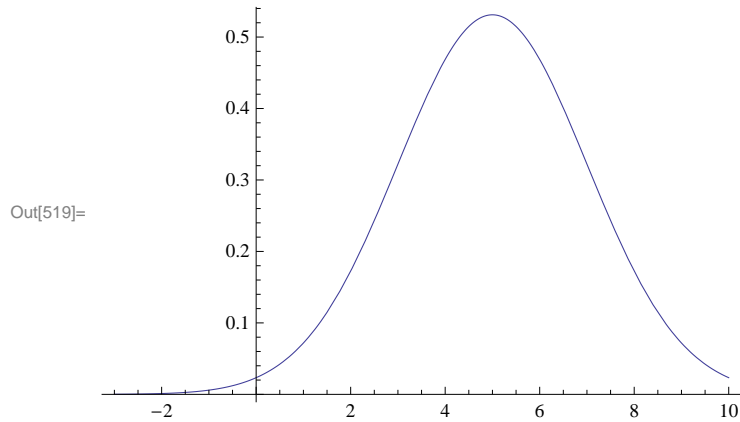
Paquete de Ondas Gaussiano

```
In[516]:= G[x_] := 
$$\frac{1}{\sqrt{\sigma} \sqrt{\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}}$$

```

```
 $\sigma = 2;$   
 $a = 5;$ 
```

```
Plot[G[x], {x, -3, 10}]
```



Ejercicios

1. Comprobar que G^2 esta normalizado
3. Que significan σ y a

Transformacion Fourier

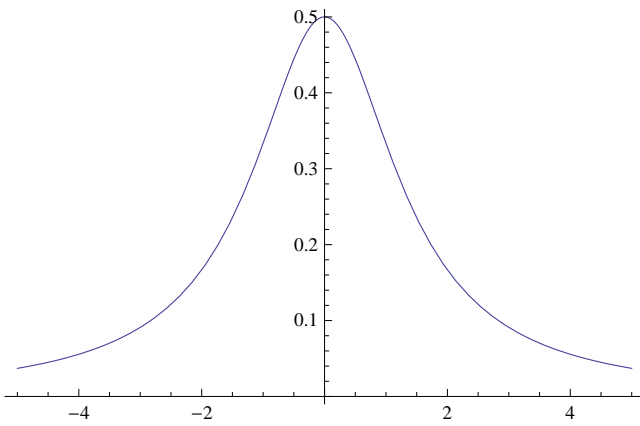
```
In[524]:= Clear[x, k, Yx, Fk];  
Yx[x_] = 1 / (x^2 + 2);
```

```
In[526]:= Yk[k_] = Integrate[Yx[x] * Exp[I k x], {x, -Infinity, Infinity}]
```

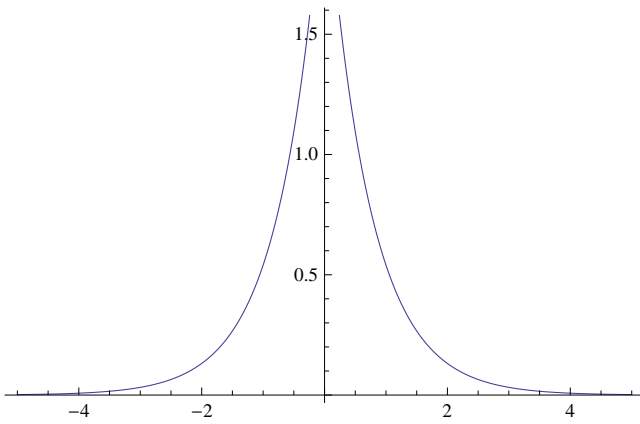
```
Out[526]= If[k ∈ Reals,  $\frac{e^{-\sqrt{2} \text{Abs}[k]} \pi}{\sqrt{2}}$ , Integrate[ $\frac{e^{i k x}}{2 + x^2}$ , {x, -∞, ∞}, Assumptions → k ∉ Reals]]
```

```
In[527]:= Plot[Yx[x] , {x, -5, 5}]  
Plot[Yk[k] , {k, -5, 5}]
```

Out[527]=



Out[528]=

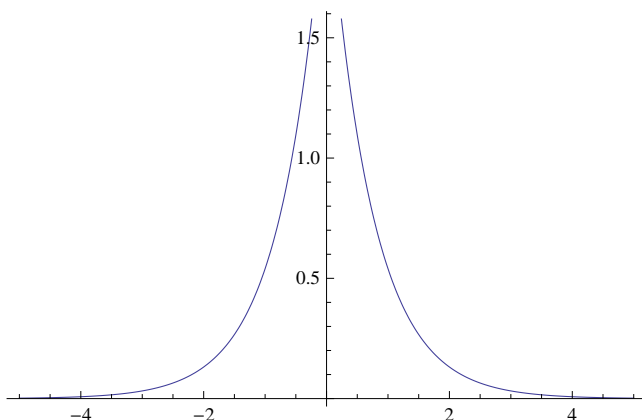


```
In[529]:= Yk[k_] = FourierTransform[Yx[x] , x, k]
```

Out[529]= $\frac{1}{2} e^{-\sqrt{2} \text{Abs}[k]} \sqrt{\pi}$

In[530]:= `Plot[Yk[k], {k, -5, 5}]`

Out[530]=



In[532]:= `Clear[x, k, σ, a, Gk];`

$$G[x_] := \frac{1}{\sqrt{\sigma} \sqrt{\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}}$$

In[534]:= `Gk[k_] = FourierTransform[G[x], x, k]`

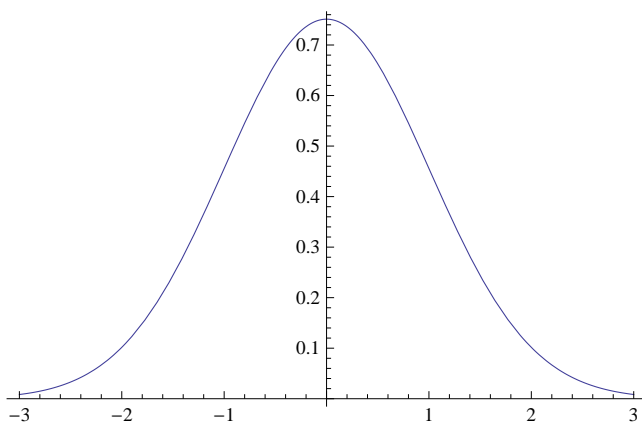
$$\text{Out[534]= } \frac{e^{i a k - \frac{k^2 \sigma^2}{2}} \text{Abs}[\sigma]}{\pi^{1/4} \sqrt{\sigma}}$$

In[535]:= `Clear[x, k, σ, a, Gk];`

`Gk[k_] := FourierTransform[G[x], x, k]`
`σ = 1;`
`a = 0;`

In[539]:= `Plot[G[x], {x, -3, 3}]`

Out[539]=



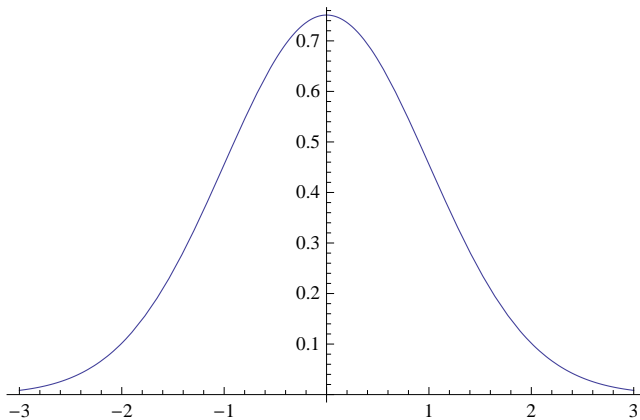
In[540]:=

`Gk[1.5]`

Out[540]= 0.243855

```
In[541]:= Plot[ Re[Gk[k]] , {k, -3, 3}]
```

Out[541]=



Ejercicios



1. Comprobar que Gk^2 esta normalizado
3. Como varian los anchos y posiciones de la transformada?

Valores Medios

```
In[547]:= ClearAll[σ, a]
```

```
In[548]:= amed :=
  Assuming[{σ > 0, Element[σ, Reals]}, Integrate[G[x] * x * G[x], {x, -Infinity, Infinity}]]
```

```
In[549]:= a2med :=
  Assuming[{σ > 0, Element[σ, Reals]}, Integrate[G[x] * x^2 * G[x], {x, -Infinity, Infinity}]]
```

```
In[550]:= amed
```

Out[550]= a

```
In[551]:= a2med
```

Out[551]= $a^2 + \frac{\sigma^2}{2}$

```
In[552]:= σ x := a2med - (amed) ^ 2
```

```
In[553]:= σ x
```

Out[553]= $\frac{\sigma^2}{2}$

```
In[554]:= P[x_] := -I h * G'[x]
```

```
In[555]:= P[2]
```

Out[555]= $\frac{i (2 - a) e^{-\frac{(2-a)^2}{2\sigma^2}} h}{\pi^{1/4} \sigma^{5/2}}$

```
In[556]:= pmed := Assuming[{σ > 0, Element[σ, Reals]}, Integrate[G[x] * P[x], {x, -Infinity, Infinity}]]
```

In[557]:= **pmed**

Out[557]= 0

In[558]:= **P2[x_] := -I h * P'[x]**

In[559]:= **p2med := Assuming[{σ > 0, Element[σ, Reals]}, Integrate[G[x] * P2[x], {x, -Infinity, Infinity}]]**

In[560]:= **p2med**

Out[560]=
$$\frac{h^2}{2 \sigma^2}$$

In[561]:= **σ p := p2med - (pmed)^2**

In[562]:= **σ p**

Out[562]=
$$\frac{h^2}{2 \sigma^2}$$

In[563]:= **σ x σ p**

Out[563]=
$$\frac{h^2}{4}$$