Teaching the photon gas in introductory physics

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The ideal gas is often the only thermodynamic system for which equations of state are studied in introductory physics. The photon gas can be a rich supplement to the ideal gas, and a vehicle for introducing 20th century physics concepts. © *2002 American Association of Physics Teachers.*
[DOI: 10.1119/1.1479743]

I. INTRODUCTION

The monatomic classical ideal gas, which is called the ideal gas in this paper, is a staple of introductory physics. It is understandable in the context of a system of noninteracting point particles,[1] and its equations of state are tractable mathematically. These virtues of the ideal gas are summarized in Table I. With them come several less desirable characteristics. First, it is based on classical physics or the semiclassical limit of a quantum ideal gas, and hence does not provide insights on quantum or relativistic phenomena. Second, the internal energy is independent of volume, a property that holds only in the low density limit for real gases. Third, the ideal gas gives no insight into changes of state such as the vapor to liquid transition. Finally, because it is usually the only system for which equations of state are encountered, students tend to come away with a sense that every system behaves as an ideal gas.

In contrast, the photon gas is a quantum mechanical system of particles (quanta of the electromagnetic field) called photons.[2] The thermal behavior of photons in blackbody radiation has played a pivotal role in the development of quantum mechanics. In addition, because photons move with the speed of light, the photon gas is a relativistic system. Thus it reflects two major developments of 20th century physics: quantum mechanics and relativity. Over a decade ago, the Introductory University Physics Project called for more 20th century physics in introductory courses.[3] Inclusion of the photon gas would work toward this goal.

The quantum mechanical probability amplitudes for photons can interfere constructively or destructively with one another, but photons do not ordinarily affect one another's energies, momenta, or polarizations, which simplifies their thermodynamic behavior. Unlike the ideal gas, the internal energy function for the photon gas is volume dependent. Remarkably, despite its nonatomic nature, the photon gas can provide insights into the liquid–vapor phase transition. Finally the very notion of the photon gas disabuses students of the thought that every thermodynamic system behaves as an ideal classical gas.

A key feature of the photon gas is that it has a variable particle number, N. Consider a container of volume V, whose walls are maintained at temperature T. Suppose it has been emptied of matter by a vacuum pump. It cannot be entirely "empty" because the walls radiate photons into the container. Some photons scatter off the walls, with some being absorbed and new ones being emitted continually. A dynamic equilibrium exists when the average absorption and emission rates are equal. Thus, an apparently empty container actually is filled with a photon gas, a fact that can intrigue students.

Unlike the ideal gas, for which there are three independent variables, N, T, and V, the photon gas has just two independent, controllable variables, T and V. We can envisage building a photon gas from energy stored in the container walls. Consider the container in Fig. 1. Imagine purging it of all atoms with a vacuum pump, and then moving the piston to the left until it touches the left wall. The volume is then zero and the walls (including the piston) have temperature T. Now slowly move the piston to the right, keeping the wall temperature constant using a reservoir. Photons will pour out of the walls as the volume increases, until the dynamic equilibrium described above occurs. In this way, we mentally construct a photon gas of volume V and temperature T, with average photon number $N(T,V)$. Building the photon gas using this thought experiment can help develop an understanding of the nature of the variable particle photon gas.

The most straightforward approach for introducing the photon gas in introductory physics is to define it in a way similar to that in the preceding paragraphs, and to then display the relevant thermodynamic equations of state and examine their implications. The extent to which the equations of state are used can vary. At minimum, the equations can be presented and interpreted. If time allows and the interest level is sufficient, they also can be used to analyze isothermal and adiabatic processes for the photon gas, as is done in the body of this paper.

A more ambitious approach is to use kinetic theory to establish the connection between internal energy and pressure for the photon gas, and to use calculus to derive the equations of state. This procedure is presented in the Appendix as a resource for teachers.

A rich literature, mainly related to blackbody radiation, exists in books on modern physics,[4] quantum physics,[5] optics,[6] and classical and statistical thermodynamics.[7–13] Numerous citations to the literature are given in Ref. 14. Nevertheless, the photon gas has not found its way into introductory physics textbooks. The main purpose of this paper is to encourage teachers of introductory physics and textbook authors to adopt the photon gas as a supplement to the ideal gas. Some of the ideas here might also be useful to teachers of modern physics and junior-senior level thermal physics.

II. PHOTON GAS EQUATIONS OF STATE AND PROCESSES

The equations of state for the internal energy $U(T,V)$ and pressure $P(T)$ are

Table I. Summary of monatomic classical ideal gas properties.

Property type	Description
Independent variables	N, V, T
System	Collection of N noninteracting point particles, each with mass m, described by classical mechanics
Temperature-independent result from kinetic theory, relating the internal energy U and pressure P	$U = \frac{3}{2}PV$
Internal energy	$U = \frac{3}{2}NkT$
Pressure	$P = NkT/V$
Entropy[a]	$S = Nk[\ln(T^{3/2}V/N) + \ln(2\pi mk/h)^{3/2} + \frac{5}{2}]$
Work W on the gas for isothermal volume change by ΔV	$W = -NkT\ln(1 + \Delta V/V)$
Energy Q added to gas by heat process for isothermal volume change by ΔV	$Q = NkT\ln(1 + \Delta V/V)$
Entropy change of gas for isothermal volume change by ΔV	$\Delta S = Nk\ln(1 + \Delta V/V)$
Reversible adiabatic condition, where the heat capacity ratio $C_p/C_V = 5/3$	$PV^{5/3} = \text{constant}$

[a]The expression for the entropy is the Sackur–Tetrode equation, the classical limit for Bose–Einstein and Fermi–Dirac quantum ideal gases with atoms of mass m. Planck's constant h connotes the Sackur–Tetrode equation's quantum origin, and k reflects the thermodynamic nature of the gas.

$$U(T,V) = bVT^4 \tag{1}$$

and

$$P(T) = \tfrac{1}{3}bT^4. \tag{2}$$

The constant b cannot be determined from thermodynamics, but its value can be borrowed from statistical physics or experimental results. It is given by

$$b = \frac{8\pi^5 k^4}{15h^3 c^3} = 7.56 \times 10^{-16} \text{ J K}^{-4}\text{ m}^{-3}. \tag{3}$$

Note that b depends on Planck's constant h, which reflects the quantum mechanical nature of the photon gas, the speed of light c, which reflects its relativistic nature, and Boltzmann's constant k, which reflects its thermodynamic nature. At the introductory level, we can introduce Eqs. (1) and (2) and give the numerical value of b, without broaching the formula in Eq. (3).
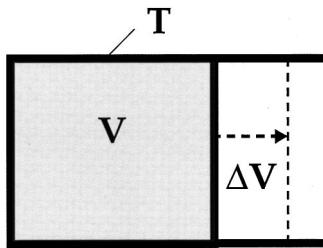

Fig. 1. Photon gas in a container with wall temperature T and volume V. The right wall is movable and its quasistatic movement can alter V reversibly by ΔV.

Given the foregoing, it is straightforward to analyze isothermal processes for the photon gas. Consider a quasistatic volume change from V to $V + \Delta V$. From Eq. (1), if T is constant,

$$\Delta U = bT^4 \Delta V. \tag{4}$$

From the first law of thermodynamics, $\Delta U = Q + W$, where $W = -\int P(T)dV$ is the work done *on* the photon gas. When T is constant, so is $P(T)$, and Eq. (2) implies that

$$W = -\tfrac{1}{3}bT^4 \Delta V. \tag{5}$$

Thus the energy gained by the photon gas from the concomitant increase or decrease in the number of photons, which constitutes a radiative heat process, is

$$Q = \tfrac{4}{3}bT^4 \Delta V. \tag{6}$$

The slow isothermal volume change under consideration is reversible, and the entropy change, Q/T, of the photon gas is

$$\Delta S = \tfrac{4}{3}bT^3 \Delta V. \tag{7}$$

This entropy change, which is linear in ΔV, is very different from the logarithmic volume dependence of the corresponding entropy change for an ideal gas, which is shown in Table I.

Now suppose we build the photon gas as described earlier by choosing the initial volume in Eq. (7) to be zero and the final volume to be V. We then allow the piston to move to the right slowly to volume V, creating the photon gas. Because at zero volume, the photon number $N = 0$, evidently $S = 0$; that is, there can be no entropy if there are no photons. Equation (7) then implies that at volume V,

$$S = \tfrac{4}{3}bVT^3. \tag{8}$$

Notice that Eqs. (1) and (2) imply that the enthalpy[15] $H = U + PV$ is

$$H = \tfrac{4}{3}bVT^4. \tag{9}$$

Clearly for an isothermal volume change, $\Delta H = Q$ and for an expansion from zero volume to volume V, $H = Q = TS$. This example makes clear that enthalpy is the energy needed to form the photon gas and to do the work needed to make available the volume V it occupies.[16]

Next, consider a slow adiabatic volume change. Adiabatic means that no photons are emitted or absorbed by the container walls. For a photon gas, the *only* possible type of heat process is via radiation; that is, energy can be exchanged with the container *only* by the emission and absorption of photons. An adiabatic volume change requires that the container walls be perfectly reflecting mirrors. Under such a process the photon number cannot change because a perfect reflector is also a nonemitter of photons. Because the number of photons cannot change, $N(T,V) = \text{constant}$. Furthermore, a slow, reversible adiabatic process leaves the entropy of the photon gas unchanged, so $S(T,V) = \text{constant}$. The constancy of both $N(T,V)$ and $S(T,V)$ implies that $N(T,V) = \text{constant} \times S(T,V)$, which along with Eq. (8), leads to the conclusion that
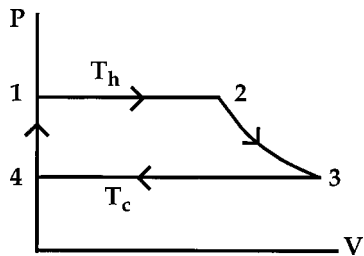
$$N(T,V) = rVT^3, \tag{10}$$

Fig. 2. A plot of pressure P versus volume V for a reversible Carnot cycle using a photon gas as the working fluid. The horizontal segments 1–2 and 3–4 are isothermals at temperatures T_h and T_c, respectively. Segment 2–3 is a reversible adiabatic expansion. Segment 4–1 occurs at zero volume and therefore entails only the container walls and not the photon gas.

where r is a constant. As was the case for b, r cannot be evaluated within the domain of thermodynamics, but its value can be borrowed from statistical physics,[17]

$$r = 60.4 \left(\frac{k}{hc} \right)^3 = 2.03 \times 10^7 \text{ m}^{-3} \text{ K}^{-3}. \qquad (11)$$

As before, the constants h, c, and k illustrate the quantum mechanical, relativistic, and thermodynamic nature of the photon gas. Also as before, to keep the discussion elementary, the value of r can be stated without broaching the formula in Eq. (11).

From Eqs. (8) and (2), it is clear that the condition of constant entropy is

$$T^3 V = \text{constant} \quad \text{or} \quad PV^{4/3} = \text{constant}. \qquad (12)$$

Interestingly, the second form in Eq. (12) is similar to the corresponding condition in Table I for an ideal gas, for which $PV^\gamma = \text{constant}$, where $\gamma = C_p/C_V = 5/3$. However, the similarity is only formal because for the photon gas, C_p does not even exist, because one cannot vary the temperature at constant pressure for a photon gas.

Having found Eqs. (2) and (12), we can sketch a reversible Carnot cycle on a pressure–volume diagram. We take advantage of the fact that the photon gas can be brought to zero volume, so that the Carnot cycle looks as shown in Fig. 2. The horizontal isotherms 1–2 and 3–4 come from the fact that $P = P(T)$. The adiabatic segment 2–3 is qualitatively similar to reversible adiabatic curves for the ideal gas. The vertical segment 4–1 corresponds to heating the container walls from T_c to T_h, with zero photons in the zero volume container. Along this segment, the walls undergo a heat process, but there is no photon gas present and thus $Q = 0$ for a photon gas working fluid.

Figure 2 makes it clear that the pressure–volume representation of a Carnot cycle does not necessarily appear as it does for an ideal gas. The purpose here is not to exhibit a *real* working fluid, but rather to illustrate that a Carnot cycle's $P–V$ plot can differ from that obtained for an ideal gas. It is a good exercise for students to use the photon gas equations to show that, as expected, the thermal efficiency is $\eta = 1 - T_c/T_h$, the reversible Carnot cycle efficiency for *any* working fluid.

III. COMPARISON: IDEAL AND PHOTON GASES

After introducing the photon gas and examining its behavior under isothermal and adiabatic processes, it is useful to

Table II. Comparison of equations for classical ideal and photon gases.

Classical ideal gas	Photon gas
N is specified and fixed	$N = rVT^3$
$U = \frac{3}{2}NkT$	$U = bVT^4 = 2.7NkT$
$P = NkT/V$	$P = \frac{1}{3}bT^4 = 0.9NkT/V$
$S = Nk[\ln(T^{3/2}V/N) + \ln(2\pi mk/h)^{3/2} + \frac{5}{2}]$	$S = \frac{4}{3}bVT^3 = 3.6Nk$

compare it with the ideal gas. One comparison examines corresponding equations of state of the two gases. Such a comparison is shown in Table II, using the fact that $N = 2.03 \times 10^7 \ VT^3$. In this view, the pressure and internal energy functions are remarkably similar if the dependent variable N is displayed explicitly for the photon gas. Thus for example, the average energy per photon in a photon gas is 2.7 kT, compared with 1.5 kT for the ideal gas. Similarly, the pressure P of the photon gas is 0.9 NkT/V compared with NkT/V for the ideal gas. Keep in mind, however, that N is *not* an independent variable for the photon gas, so the similarities are strictly formal. The entropy functions are very different looking for the photon and ideal gases. Notably, the entropy per photon is $3.6k$, independent of temperature.

A numerical comparison is shown in Table III. The ideal gas is taken to have the mass of monatomic argon and is at 300 K and normal atmospheric pressure. The numerical values of N, U, P, and S for the photon gas are all approximately 10 or more orders of magnitude smaller than for the ideal gas, which is why we can ignore the photon gas when discussing the thermodynamics of an ideal gas in the vicinity of room temperature and atmospheric pressure.

On the other hand, for sufficiently high temperatures, the number of photons can exceed the number of ideal gas atoms in an equal volume V. Indeed, it is straightforward to show that if 1.00 mol of argon ideal gas is at standard atmospheric pressure, 1.01×10^5 Pa, the corresponding average number of photons exceeds the number of atoms for any V if $T > 1.38 \times 10^5$ K. The equations in Table II imply that for $T \approx 1.41 \times 10^5$ K, the ideal and photon gases have nearly the same internal energies and pressures, while the ideal gas entropy is still significantly larger than the photon gas entropy. Of course, because of the T^3 behavior of N and S, and the T^4 behavior of P and U, the photon gas will dominate the ideal gas in *all* respects for sufficiently high T. In this discussion we have ignored the ionization of the ideal gas atoms that would occur at such high temperatures.

Other comparisons are possible. For example, the ideal gas entropy becomes negative for sufficiently small T and diverges to negative infinity in the limit $T \rightarrow 0$. This inad-

Table III. Numerical comparison of classical ideal and photon gas functions. Here the ideal gas is 1.00 mol of monatomic argon at $P = 1.01 \times 10^5$ Pa, $V = 2.47 \times 10^{-2}$ m^3, and $T = 300$ K.

Function	Classical ideal gas	Photon gas
N	6.02×10^{23} atoms	1.35×10^{13} photons
U	3.74×10^3 J	1.51×10^{-7} J
P	1.01×10^5 Pa	2.04×10^{-6} Pa
S	155 J/K	6.71×10^{-10} J/K

equacy of the ideal gas model comes from its classical character. In contrast, the photon gas entropy approaches zero in the $T=0$ limit. Similarly, the ideal gas constant volume heat capacity is constant and remains so in the zero temperature limit, unlike real gases, whose heat capacities approach zero, consistent with the third law of thermodynamics. The photon gas heat capacity at constant volume, $C_V = 4bVT^3$, approaches zero in this limit, as the number of photons approaches zero. The behavior of the photon gas reflects its quantum mechanical nature.

The ideal gas gives no indication of the condensation phenomenon that a real gas experiences at temperatures below its critical temperature. As a real gas is compressed isothermally at a temperature below its critical temperature, part of it begins to condense into the liquid state, keeping the pressure constant. Although it is a very different kind of system, with very different physics, the photon gas can shed light on this phenomenon because under isothermal compression, photons get absorbed, becoming part of the energy of the walls, providing an analogy to the vapor-to-liquid transition: the photons play the role of the vapor molecules, and the absorbed energy of the walls are the analog of the liquid molecules. The energy of compression to zero volume for the photon gas is the rough analog of the heat of condensation for the real gas.

For the photon gas, the pressure remains constant as the volume decreases at constant temperature because the number of photons decreases while the energy of the walls increases. For a real gas, the pressure remains constant during isothermal compression because the number of gas molecules decreases as gas molecules become liquid molecules. Condensation phenomena occur because of attractive forces between molecules, while photon absorption occurs because atoms and molecules continually absorb and emit radiation, as their electronic energies increase and decrease. Although the physics differs for the two phenomena, both are characterized by constant temperature, constant pressure, and a variable number of gas particles. Discussion of these matters can shed light not only on radiation, but on the phenomenon of condensation.

IV. TWO BRIEF EXAMPLES

Perhaps the most exciting example of a photon gas is the cosmic microwave background radiation.[18] The latter is in essence a gas of "old" photons that was created in the early, hot Universe approximately 13 billion years ago, and which has cooled to 2.7 K. Inserting the latter temperature and the value of r in Eq. (11) into Eq. (10) gives $N/V = 416$ photons/cm^3. Such photons, which make even dinosaur bones seem rather young, are in our vicinity all the time. An awareness of the photon gas opens the door to an understanding of this remarkable phenomenon.

A second example uses a well-known result from kinetic theory[19] together with Eq. (10). Suppose a photon gas exists in a cavity within a solid that is at temperature T, and that photons can leak out through a small opening in the walls. The kinetic theory result for the particle flux is $\frac{1}{4}(N/V)c = \frac{1}{4}rcT^3$, and the energy flux, measured in watts/m^2, is $\frac{1}{4}(N/V)(U/N) = \frac{1}{4}cbT^4$. This result is the well-known energy flux from a blackbody, where $cb/4 = \sigma = 5.67 \times 10^8$

W m^{-2} K^{-4}, is the Stefan–Boltzmann constant. In appropriate contexts, one can move on to study Planck's radiation law and, ultimately, quantum mechanics.

V. CONCLUSIONS

The photon gas can enrich the introduction to thermodynamics. Its basic equations lead students into new territory involving creation and annihilation of photons, which provides a thought provoking introduction to modern physics ideas.

An important related point is that photons are everywhere. That is, because all matter radiates, it is literally impossible to have a region of space that is free of photons. In this sense, the photon gas has the distinction of being ubiquitous, another point that can pique the intellectual curiosity of students.

Despite the evident richness of the photon gas, its equations of state are tractable and have straightforward interpretations. In addition to its potential for enriching the study of thermal physics, the photon gas serves as a good foundation for subsequent introduction to cavity radiation.

In summary, the photon gas has much to offer teachers and students. Its study can supplement the ideal gas or can be initiated in a course on modern physics. A more in-depth treatment is appropriate for junior or senior level thermal physics.

APPENDIX

The objective of the Appendix is to derive Eqs. (1) and (2). Kinetic theory enables us to deduce a simple relationship between the internal energy U and pressure P of a photon gas,

$$P(T) = \frac{1}{3} \frac{U(T,V)}{V} = \frac{1}{3} u(T). \tag{A1}$$

The right-hand side defines the energy density $u(T)$.

To obtain Eq. (A1), we make several assumptions. First, we assume isotropy, namely, the average number of rightward moving photons within a specified range of velocity is the same as the corresponding average number of leftward moving photons. Denote the number of photons per unit volume with x components of velocity between c_x and $c_x + dc_x$ by $n(c_x)dc_x$. The assumed isotropy implies $n(c_x) = n(-c_x)$. (Because photons all have speed c, c_x varies solely because of differing velocity directions.) Integration over c_x gives

$$\int_{-c}^{c} n(c_x)dc_x = 2 \int_{0}^{c} n(c_x)dc_x = \frac{N}{V}, \tag{A2}$$

where N is the (average) number of photons. The function $Vn(c_x)/N$ is a probability distribution function that can be used to calculate averages such as

$$\overline{c_x^2} = \int_{-c}^{c} \frac{Vn(c_x)}{N} c_x^2 dc_x = 2 \int_{0}^{c} \frac{Vn(c_x)}{N} c_x^2 dc_x. \tag{A3}$$

The last step follows because $n(c_x)c_x^2$ is an even function of c_x. Equation (A3) will be used in our derivation of Eq. (A1).

Our second assumption is that there is a well-defined average photon energy $\langle e \rangle$, which depends solely on the wall temperature and not on the system volume V. This assumption is motivated by the expectation that the energy distribution of the emitted photons depends on the wall temperature. In contrast, the average number of photons N must depend on both temperature T and volume V because in equilibrium, the absorption and emission rates can be equal only if N achieves a sufficiently large value, which increases with V. Our last assumption is that N is proportional to the system volume; that is, $N(T,V) = n(T)V$. We now proceed with the kinetic theory derivation.

Fix the right wall in Fig. 1 and denote the container's horizontal length by L and its cross sectional area by A. Choose a small time interval $\Delta t \ll L/c$, and consider rightward moving photons with x-components of velocity between c_x and $c_x + dc_x$, located within distance $c_x \Delta t$ from the right wall. The latter region has spatial volume $Ac_x \Delta t$, and an average number of photons, $(Ac_x \Delta t)n(c_x)dc_x$, within it will collide with the right wall in time interval Δt. The momentum magnitude for a photon with energy e is e/c and the x component of its momentum is $(e/c)(c_x/c)$. In an elastic collision with the right wall a photon's momentum change is $2(e/c)(c_x/c)$, and the average force it exerts on the wall during time Δt is $2ec_x/(c^2 \Delta t)$.[20] The average force for all such collisions by photons with average energy $\langle e \rangle$ and x component of velocity between c_x and $c_x + dc_x$ is $(2\langle e \rangle c_x/c^2)(Ac_x)n(c_x)dc_x$.

If we integrate over c_x from 0 to c and divide by area A, the average pressure on the wall from photons impinging at all angles is

$$P = \frac{2\langle e \rangle}{c^2} \int_0^c n(c_x)c_x^2 \, dc_x = \frac{\langle e \rangle N}{Vc^2}\overline{c_x^2} = \frac{1}{3}\frac{\langle e \rangle N}{V}. \tag{A4}$$

The penultimate step follows from Eq. (A3) and the last step follows from the isotropy condition, $\overline{c_x^2} = c^2/3$. In Eq. (A4), $N\langle e \rangle = U$, the internal energy of the photon gas. With our assumptions that $\langle e \rangle$ is solely a function of T and $N = n(T)V$, Eq. (A4) reduces to Eq. (A1), $P(T) = \frac{1}{3}u(T)$, where $u(T) = U/V$. It is clear from Eq. (A1) that pressure is solely a function of temperature and thus, the pressure–volume isotherms for a photon gas form a family of constant–pressure curves. We now use the forms $P = P(T)$ and $U = u(T)V$, along with the Carnot cycle in Fig. 2 to derive Eq. (1). Along segments 1–2, 2–3, 3–4, and 4–1, the internal energy changes of the photon gas are $\Delta U_{12} = u(T_h)V$, $\Delta U_{23} = -\frac{1}{3}\int_V^{V+\Delta V} u(T)dV$, $\Delta U_{34} = -u(T_c)(V+\Delta V)$, and $\Delta U_{41} = 0$, respectively. Because U is a state function, these changes must add to zero along the cycle. Simplification occurs when $T_h - T_c \ll T_c$ and $\Delta V \ll V$, in which case we replace ΔV by dV. Then the addition of the four internal energy changes gives $u(T_h)V - \frac{1}{3}u(T)dV - u(T_c)(V+dV) = 0$, where $T_c \leqslant T \leqslant T_h$. Replacement of $u(T)$ with $u(T_c)$ induces an additive error $\leqslant du \equiv u(T_h) - u(T_c)$. Thus to first order in du and dV the sum of the internal energy changes around the cycle reduces to

$$V \, du - \tfrac{4}{3}u \, dV = 0. \tag{A5}$$

The individual entropy changes of the photon gas along the Carnot cycle are $\Delta S_{12} = \frac{4}{3}u(T_h)V/T_h$, $\Delta S_{23} = 0$, $\Delta S_{34} = -\frac{4}{3}u(T_c)(V+dV)/T_c$, and $\Delta S_{41} = 0$, respectively. Because S is a state function, these changes also must add to zero along the cycle. Using $dV \ll V$ and $dT \equiv T_h - T_c \ll T_c$, we have $1/T_h \approx (1 - dT/T_c)/T_c$, and the condition for zero entropy change along the cyclic path becomes

$$\frac{4}{3}\left[\frac{(u(T_h)-u(T_c))V}{T_c} - \frac{u(T_c)dV}{T_c} - \frac{u(T_h)V \, dT}{T_c^2} \right] = 0. \tag{A6}$$

Replacing T_c with T and T_h with $T + dT$, and retaining only first order terms, Eq. (A6) becomes

$$V \, du - u \, dV - \frac{uV \, dT}{T} = 0. \tag{A7}$$

We can eliminate dV by combining Eqs. (A5) and (A7) to obtain

$$\frac{1}{4}du = \frac{u}{T}dT. \tag{A8}$$

Finally, integration of Eq. (A8) gives $u(T) = bT^4$, which is equivalent to Eq. (1). As mentioned, the numerical value of b is obtained from statistical mechanics. The combination of Eqs. (1) and (A1) gives Eq. (2).

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[1]Weak interactions must exist in order for the gas to achieve thermodynamic equilibrium. It is assumed that these interactions have negligible effects on the equations of state.

[2]The term "photon" was coined by Gilbert N. Lewis, "The conservation of photons," Nature (London) **118**, 874-875 (1926).

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[14]Harvey S. Leff (in preparation).

[15]Students of introductory physics do not typically encounter enthalpy in their courses, and the discussion in the text can be skipped. On the other hand, students *are* introduced to enthalpy in their chemistry courses, and the discussion in the text might help them appreciate, if not fully understand, enthalpy in a wider context.
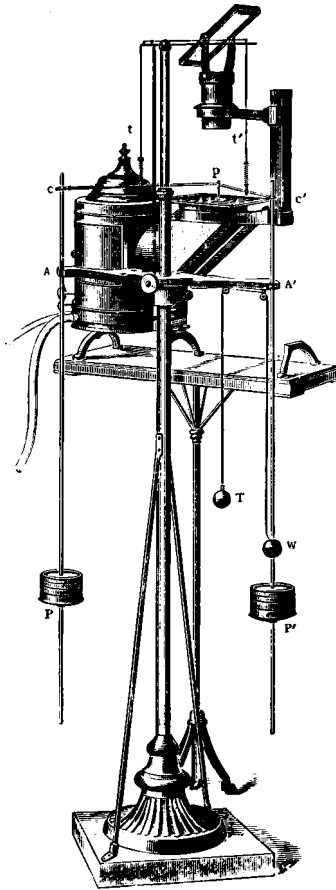
[16]See Ref. 10, p. 33.

[17]In Eq. (11), the prefactor 60.4 in r arises from an integral, which gives $8\pi\zeta(3)\Gamma(3) = 60.4$. Here $\zeta(3)$ and $\Gamma(3)$ are zeta and gamma functions, respectively.

[18] Edward Harrison, *Cosmology: The Science of the Universe*, 2nd ed. (Cambridge University Press, Cambridge, 2000), Chap. 17.

[19] E. H. Kennard, *Kinetic Theory of Gases* (McGraw-Hill, New York, 1938), Sec. 37.

[20] The assumption of elastic collisions does not hold for walls that can absorb and emit photons. In that case, however, isotropy guarantees that Eq. (A1)

still holds because the average number of photons coming from a wall section in any time interval must equal the number approaching the wall, and the average momentum approaching must equal the average momentum leaving. The mathematical treatment used for elastic collisions thus holds in an average sense when inelastic collisions occur, and isotropy holds.

Lissajous Figure Drawing Device. Lissajous figures are the resultant of two simple harmonic motions at right angles to each other. In this dedicated device, the SHMs are provided by two pendula swinging at right angles to each other. Horizontal rods coupled to the tops of the pendula drive a stylus P that scrapes the soot from a smoked glass plate held on the stage of the overhead projector. The audience thus sees a black screen with the figure being traced out in white. The illustration is from J. A. Zahm, *Sound and Music*, second edition (A. C. McClurg & Co., Chicago, 1900), p. 409 (Notes by Thomas B. Greenslade, Jr., Kenyon College)