science will also know the relation of his science to other branches of knowledge. He will realize that physical tests are inherently incapable of disproving the existence of extraphysical reality, and, knowing that physics deals with only a part of experience, he will make no unfounded pronouncements concerning the extraphysical. He will not, without reflection on fundamentals, apply his results to other disciplines which have different limitations and different valid methods. There will be less error and more truth expounded in the name of science.

Survey of the Interpretations of Quantum Mechanics*

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A survey is made of the hitherto proposed answers to the following crucial questions arising in the physical interpretation of the mathematical formalism of wave mechanics: (I) What is the meaning of the dynamical variables and of their eigenvalues? (II) What is the meaning of the wave function? (III) What is the nature and origin of Heisenberg's uncertainty relations? (IV) What is the nature of the systems dealt with by wave mechanics? Taking into account the interpretations that have been proposed in the last few years, a surprisingly large number of answers is found. The present situation in quantum mechanics is compared with similar crises in the past history of physics. The mere multiplicity of consistent interpretations of quantum mechanics is regarded as a warning against the dogmatic adherence to any one of them, to the exclusion of new possibilities.

INTRODUCTION

A SURVEY of the available and a glimpse at the conceivable interpretations of the quantum theory of "particles" (first quantization) might shed some light on the status and value of every one of those that have so far been advanced, and might even suggest new lines of approach—or it may at least help in avoiding the repetition of old mistakes. The object of the present paper is to sketch such a survey. It is the author's conviction that it is far from complete, if only because an almost exclusively Western bibliography has been available to him.

Of the different formulations of quantum mechanics, we shall consider only wave mechanics, that is, the group of formulations containing a wave equation either as a postulate (as happens in most cases) or as a result derived from assumptions regarded as more fundamental (as is the case with Feynman's space-time approach, or with Weizel's classical foundation of quantum mechanics). We shall consequently leave aside matrix mechanics, the S-matrix formulation and other representations which, after all, can be built on the basis of wave mechanics. Our choice is not altogether arbitrary, because wave mechanics is, of all the formulations of quantum mechanics, the richest source of physical interpretation and of philosophical argument, aiming as it does at a considerably detailed description of the objects with which it is concerned.

Hence, we shall deal with different physical *interpretations* of nearly the same mathematical *formulation* of quantum mechanics. Or, if preferred, we shall discuss different sets of rules of physical interpretation of almost the same mathematical framework. Strictly speaking, the formulations of wave mechanics need not be exactly the same in all respects; there may be some shift in the ordering of the basic axioms, and some of these may be replaced by new ones; moreover, it is a fact that fresh interpretations are often suggested by partial reformulations of the available set of mathematical symbols. But all the interpretations that we are going to

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consider in the following are attempts to understand in physical terms the common mathematical ground consituted by (a) the Schrödinger equation (with any sort of Hamiltonian, provided it does not contain the wave function itself); (b) the usual mathematical restrictions imposed upon the wave function (continuity, uniformity, vanishing at infinity, and some sort of integrability); (c) the definition of matrix element and, in particular, of the (space) average of a dynamical variable; and (d) Heisenberg's "uncertainty relations."

We shall assume that every fairly complete interpretation of quantum mechanics in its Schrödinger representation should answer at least the four following fundamental questions:

(I) *What is the meaning of the dynamical varibles and of their eigenvalues?* This is, do $\mathbf{x}$, $\mathbf{p}$, $\boldsymbol{\sigma}$, $\boldsymbol{\alpha}$, etc., represent objective properties of matter, or are they just symbols obeying formal rules and used exclusively for the economical correlation of empirical data, or are they something else?

(II) *What is the meaning of the wave function?* Is ψ a mere mathematical symbol without any physical meaning, or is it a field strength, or perhaps neither of them? And does it afford a statistical information only, or a description of individual atomic systems as well?

(III) *What is the nature and origin of Heisenberg's uncertainties?* Do they consist of uncertainties originating in the incompleteness of our present theory, or in an indeterminacy rooted in the nature of things or in the nature of experiment?

(IV) *What is the nature of the microsystems dealt with by wave mechanics?* That is, are they particles, or waves, or wavicles, or are they whatever the experimenter decides to conjure up?

The examination of any particular formulation and interpretation of wave mechanics shows that the four above-mentioned questions imply one another, there being no linear relationship of precedence or logical priority among them—a simple type of relation that can be found in some chapters of logic and mathematics, not however in a science which, like theoretical physics, is not a purely deductive framework, since at every step a reference to experiment must or may be made in it.

There are certainly many other important questions, and every fairly satisfactory interpretation should answer them or eliminate them; but it seems that the ones we have raised are the most interesting, at least at present.

As will appear from the following, what is at stake in all of these questions is essentially an epistemological problem, since what is asked about certain entities is, whether they exist in mind alone, or whether they arise only in experiment (or in our account of experiment), or whether they are conceptual reflections of objects having an autonomous existence in the objective or external world. As a consequence, every one of our four questions admits a subjectivist (or idealist), an empiricist (or positivist), and a realistic (or materialistic) answer. Whether we like it or not, as soon as we ask fundamental scientific questions we get entangled in philosophical arguments—or at least the philosopher is entitled to regard them as such.[1]

I. INTERPRETATION OF THE DYNAMICAL VARIABLES AND THEIR EIGENVALUES

The most important answers to question (I) can be grouped in two broad classes, the first of which denies that dynamical variables symbolize properties of matter, while the second group asserts that they do.

(I.1) *First group of answers:* The dynamical variables are constructs, i.e., conceptual entities, representing observable quantities in a symbolical way; their eigenvalues are the possible values that these observables may assume upon a precise measurement.

Two varieties of this thesis can be recognized:

(I.1.a) A dynamical variable is a mathematical object that represents symbolically (i.e., indirectly and conventionally) an observable quantity; it does not represent a property of material objects having an autonomous existence in the external world. The eigenvalues

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[1] The philosophical character of the debate over the physical interpretation of quantum mechanics has been pointed out, among others, by N. Bohr, Erkenntnis **6**, 293 (1936); H. Reichenbach, *The Rise of Scientific Philosophy* (University of California Press, Berkeley and Los Angeles, 1951), pp. 175–176; W. Heisenberg, Naturwissenschaften **38**, 49 (1951).

of a dynamical variable are the possible values that can be found when a measurement of the corresponding observable is performed, whether actually or in a conceivable experiment.[2] And the mean value of a dynamical variable is the (space) average of the potential sequence of all the experimentally obtainable values of the corresponding observable, at a given instant of time. Thus, for example, atomic systems do not have a position in space and time under all circumstances, but they can be attributed a position (at the price of giving up the precise definition of their momentum and energy) by a position measurement. An electron is indeed where it is measured; but when no position measurement is performed it is meaningless to ascribe a position to it. As Heisenberg[3] wrote, "The 'trajectory' arises only when we observe it."

On this interpretation, then, quantum-mechanical operators have a purely mathematical status, while their eigenvalues and averages have an empirical status as well—but not a material status, that is, they are not regarded as corresponding to anything really existing in the material world, independently of experience. This is the usual interpretation advanced by Bohr and Heisenberg; it has been adopted in most textbooks, notably in von Neumann's and Dirac's classical treatises, and it has been embraced, with a few exception, by logical empiricism or neopositivism.[4]

No decisive physical argument has apparently been advanced in support of this interpretation; its main basis is the positivist doctrine according to which physics is not concerned with understanding the objective material world with the help of experiment and theory, but is concerned only with possible measurements and with their economical description.[5] That this position may constitute a serious obstacle to scientific progress, is suggested by a few examples drawn from the recent history of physics: Hertz sought electromagnetic waves in the objective world because they had been predicted theoretically; atoms were assumed to exist before their actual existence was experimentally confirmed, and atomic physics was built in spite of the positivvistic interdictions of Comte, Mach, and their followers; Yukawa assumed that mesons existed before they were actually found in the cosmic radiation; and quantum mechanics contains quantities, like ψ, which are not operationally defined.

But technical objections as well can be raised. There are operators, such as the one representing the position of a particle, which are assigned a continuous spectrum, although no precise measurement of the corresponding observable is possible—whence the rigorous conclusion should be drawn that $\mathbf{x}$ has either a discontinuous spectrum of eigenvalues, or has no eigenvalues at all.[6] In the second place, in the laboratory we daily perform measurements of lengths and angles to which no Hermitean and linear operators are associated—as Schrödinger[7] has pointed out. Thirdly, there is, on the other hand, a large number of linear and Hermitean operators to which no known "observables" correspond—while if the theory were consistenly operational, it should assert that an arbitrary linear and Hermitean operator represents a measurable quantity.[8] Besides, this is an empirically unverifiable statement, hence the operationalist requirement in question leads to a contradiction.

A second variant of thesis (I.1) is the following:

(I.1.b) Dynamical variables are constructs by means of which other constructs (called atoms, electrons, etc.) are described.

This extreme subjectivistic position has no basis in the existing physical theories; it could

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[2] Let us recall that 3 entities are here involved, namely, the observable O, its symbolic operator representative A, and the latter's eigenvalues α_i. A and α_i are assumed to be related to each other through the linear equation $Au_i=\alpha_i u_i$, where (in the nondegenerate case) u_i is the eigenfunction of A corresponding to the eigenvalue α_i.

[3] W. Heisenberg, Z. Physik **43**, 172 (1927), p. 185. See also J. v. Neumann, *Mathematische Grundlagen der Quantenmechanik* (Verlag Julius Springer, Berlin, 1932; Dover Publications, New York, 1943), p. 224 and *passim.*

[4] See, e.g., Ph. Frank, Erkenntnis **6**, 303, 445 (1936). M. Schlick, Erkenntnis **6**, 317 (1936). P. Jordan, Naturwissenschaften 22, 485 (1934).

[5] For a clear and short exposition of the positivist philosophy of physics, see D. Halliday, *Introductory Nuclear Physics* (John Wiley and Sons, Inc., New York, 1950), pp. 1–5.

[6] V. Rojansky, Phys. Rev. **97**, 507 (1955).

[7] E. Schrödinger, Nature **173**, 442 (1954).

[8] P. A. M. Dirac, *The Principles of Quantum Mechanics* (Clarendon Press, Oxford, 1947), third edition, p. 37. For a criticism of this assumption, see E. P. Wigner, Z. Physik **133**, 101 (1952).

only be justified and criticized in the philosophical context.

(I.2) *Secound group of answers:* The dynamical variables are abstract representatives of concrete properties of material systems; their eigenvalues are the values which those properties may assume.

The following varieties may be distinguished:

(I.2.a) Dynamical variables refer always and only to the microsystem under consideration.

According to this interpretation, quantum mechanics deals in most cases with objects O that are not under actual observation, as suggested by the fact that they are adequately described by using Hamiltonians in which the energy of interaction of O with the measurement apparatus M does not appear explicitly. In other words, in most cases the treatment of physical problems by means of quantum mechanics does not attain to the empirical plane; but, just as in classical physics, it is concerned with establishing correspondences with material objects (existing whether we observe them or not), and their conceptual reflections. As soon as the material system includes measuring devices—i.e., as soon as the system under consideration is $O+M$—the interactions of M with O have to be taken into account in the Hamiltonain (as is done in the quantum theory of measurements), especially since such interactions are quantized and can consequently not be neglected in the way classical physics had assumed.

This argument, which may be described as naive, is inconsistent with the fact that the results calculated without taking the $O-M$ interaction into account, are compatible with experimental data, in spite of the fact that we always establish a coupling of O with M when performing a measurement on O. A partisan of the usual interpretation would argue as follows: Observables are potentially observable, so that the theory always deals with what might be observed potentially. Thus, even if the Hamiltonian does not contain explicitly the perturbation energy introduced by the apparatus M, it actually describes the total system $O+M$; hence, the automatic evolution of the dynamic variable A[9] does not refer to O alone, but to $O+M$.

There is nothing wrong with these two refutations of thesis (I.2.a). But the following assumptions, usually connected with such refutation, are highly controversial: namely, that the aforementioned coupling $O-M$ proves that electrons have no existence apart from observers, and that dynamical variables have consequently nothing to do with material systems, but refer exclusively to acts of observation in themselves. These assertions are based on the unwarranted identification of the macroscopic systems M with which microsystems are actually connected, with the observer, or subject in the sense of the theory of knowledge.[10] And this identification is in turn based on the operationalistic belief that things can be said to exist only insofar as they are observed or measured.[11]

Neither the naive nor the subjectivistic interpretation is convincing. The reasonable agreement of calculations (in which usually no account is taken of the $O-M$ coupling) with experiment (which consists in the establishment of such a coupling) presumably only shows that, at the quantum level of accuracy, there are no isolated systems. This, far from meaning that an observer must always be taken for granted, only means that other material systems, macroscopic ones, are always in interaction with the microsystem under consideration.

(I.2.b) Dynamical variables (the so-called observables) refer ambiguously and indirectly to the microsystem; they pertain both to the microsystem and to the measuring device, they describe a "mutual property" of O and M. As a consequence, the description attained with the sole help of the usual dynamical variables and their eigenfunctions is incomplete. Such a description must be completed with the introduction of new variables, the ones describing

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[9] Let us recall that the evolution of A may be described either by its Heisenberg equation

$$\frac{dA}{dt}=\frac{\partial A}{\partial t}+\frac{i}{\hbar}[H,A],$$

or by means of the unitary transformation

$$A\rightarrow A_t=e^{(i/\hbar)Ht}A\ e^{-(i/\hbar)Ht}.$$

[10] For a criticism of this identification, see B. Fogarasi, Deutsche Z. Philosophie 1, 640 (1953).

[11] See the author's attempt to refute this belief, Philosophy and Phenomenological Research 15, 192 (1954).

the true properties of the object, and which in the usual interpretation are called "hidden parameters." These new variables, e.g., the c-numbers $\mathbf{x}(t)$ and $\mathbf{p}(t)$, refer to the system alone, whether it is under observation or not; they are not subjected to uncertainty relations but have always precisely defined values, which suffer deep changes when the system is under observation. A property is always changed under observation, save in the very particular case in which the wave function of the system is an eigenfunction of the operator representing that property (and only in this case does the dynamical variable refer to the system alone). This is Bohm's[12] first interpretation.

In this second group of interpretations, dynamical variables have both a conceptual and an ontological status, and eventually an empirical condition as well. When the quantum theorist interprets the results of his calculations, he usually regards dynamical variables not only as symbolic representations of observables, but also as conceptual entities related in some way to objective properties of a material system. It is only when the theorist states the basic axioms of the usual interpretation, or embarks explicitly in a philosophical crusade in the defense of subjectivistic philosophies, that he tends to restrict the meaning of his quantities to the empirical plane—with which he adopts an anthropocentric world view.[13] He then forgets that in the quantum theory of fields (second quantization), which is built on the quantum theory of particles, the opposite philosophy is adopted, since one assumes the objective reality of unobservable processes (such as virtual transitions, which contribute to observable phenomena without being observable themselves); moreover, one admits the objective meaning of quantities which are usually regarded as unobservable in principle, like the (infinite) self-mass and self-charge of the electrons[14]—which on an operationalist philosophy should not be regarded as belonging to the scientific discourse.

II. INTERPRETATIONS OF THE WAVE FUNCTION

(II.1) *First group of answers:* The ψ function is a mathematical symbol without any physical meaning.

Two varieties of this thesis have to be distinguished:

(II.1.a) The ψ function refers indirectly to empirically-defined quantities; it enables us to make calculations yielding numbers that can be compared with experiment; or, again, de Broglie's waves are nothing but "waves of knowledge" about experimental situations. The sole meaning of the wave function is that of a probability amplitude (Born's statistical interpretation). For example, in the case of Schrödinger's, Pauli's, and Dirac's theories, $|\psi(\mathbf{x},t)|^2 \cdot d\mathbf{x}$ is the probability of finding the microsystem within the volume element $d\mathbf{x}$ centered at the point $\mathbf{x}$ at the instant of time t, when a position measurement is made—not the probability of the existence of the material system within $d\mathbf{x}$. In principle, the wave function (eigenfunction of the energy operator) is not more important than the eigenfunctions of other operators: as shown in the general transformation theory, the energy representation is only one among many; the importance of the solution of Schrödinger's equation is merely a computational one, and nothing prevents us from using other representations, in which $\mathbf{x}$ and t do not appear as independent variables, since the aim of the theory is not the detailed description of processes in space and time. The descriptions and predictions attained by means of the wave function are probabilistic and at the same time they are complete, in the sense that nothing more than probability statements can be made at the quantum level. This is the orthodox, positivistic interpretation set forth by Bohr and Heisenberg, on the basis of Born's hypothesis.

The opinion that the wave function describes nothing but an "information field," that it does not refer to physical objects existing in the

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[12] D. Bohm, Phys. Rev. **85**, 166 (1952); **85**, 180 (1952).

[13] The anthropocentric character of the usual interpretation of quantum mechanics has been pointed out, among others, by E. May, *Kleiner Grundriss der Naturphilosophie* (Westkulturverlag Anton Hain, Meisenheim/Glan, 1949), Sec. 74.

[14] See, e.g., W. Heitler, *The Quantum Theory of Radiation* (Clarendon Press, Oxford, 1954), third edition, p. 277. As regards first quantization, Heitler adopts the usual, positivistic or operationalistic philosophy; see his contribution on "The Departure from Classical Thought in Modern Physics," in P. A. Schilpp, editor, *Albert Einstein: Philosopher-Scientist* (The Library of Living Philosophers, Evanston, 1949).

material world, that it is merely a mathematical auxiliary (a *Rechengrösse*) by means of which possible results of possible experiments can be calculated, has been supported with the following facts: (a) the ψ function is not directly measurable in detail (only its modulus is); (b) it is a complex function, since it satisfies Schrödinger's equation, which contains the imaginary unit; (c) in the general case, in which ψ refers to a system of n mass points, it is defined in an abstract space, namely, the $3n$-dimensional configuration space.

The vitality of these arguments is astonishing. The first of them is valid only on the extra assumption—which is of a philosophical nature—that what has not been observed does not exist. The second argument, concerning the complex character of ψ, loses its strength as soon as one realizes that a complex function is nothing but a couple of real functions united in a convenient way. From the fact that the impedance can often be written in the form $Z=R+iX$, nobody has inferred that R and X are not the representatives of real characteristics of real circuits; and nobody would say that the electromagnetic field is only a field of information because we can write $\mathbf{E}+i\mathbf{H}=\mathbf{F}$ and consequently put together Maxwell's two triplets (for the vacuum) in the form $\mathrm{curl}\mathbf{F}=(i/c)\partial\mathbf{F}/\partial t$. By decomposing ψ in the form

$$\psi(\mathbf{x},t)=R(\mathbf{x},t)\cdot e^{(i/\hbar)S(\mathbf{x},t)}, \qquad (1)$$

where $R(\mathbf{x},t)$ and $S(\mathbf{x},t)$ are real functions, Schrödinger's single equation gets split into two real equations. If wave mechanics had started by postulating these equations, nobody would have said that the synthesis of the real functions R and S in the form (1) proves the lack of physical reality of ψ. Moreover, the complex nature of ψ is not essential in two important cases, namely, the stationary states of the nonrelativistic equation, and the Klein-Gordon equation; in the latter case, the simple change of gauge[15]

$$\left.\begin{array}{l} A_\mu\rightarrow A_\mu-\dfrac{c}{e}\dfrac{\partial S'}{\partial x^\mu} \\ \psi\rightarrow\psi\cdot e^{(i/\hbar)S'} \end{array}\right\};\quad S'=(i/2)\ln(\psi/\psi^*) \qquad (2)$$

leaves the equation invariant, but transforms the complex wave function into the real function

$$\psi\cdot e^{-\frac{1}{2}\ln(\psi/\psi^*)}=|\psi|=R. \qquad (3)$$

As to the argument regarding the abstract nature of the configuration space, it seems as weak as the others. In the first place, does not statistical mechanics use the $6n$-dimensional phase space, and does it not define functions in this space—and do not many of these functions correspond to properties of real systems? Of course, in this case, as in the case of the wave function, no direct and simple correspondence exists between physical properties and the concepts that reflect them; direct and simple correspondences, like that obtaining between the values of the variable $\mathbf{x}$ and the successive positions of a pointmass in space, are the exception rather than the rule in modern theoretical physics; think of the correspondence between the potential four-vector A_μ and the electromagnetic field strengths. In the second place, the method of second quantization, which is in some respects equivalent to the elementary quantum theory, treats microsystems and their associated ψ functions in ordinary space. Finally, de Broglie[16] has recently shown that the usual formulation in configuration space is (at least in the case of bosons) equivalent to a new formulation in 3-dimensional space, in which the wave field accompanying each particle depends on the locations of the remaining particles; this formulation (as well as the quantum theory of fields) is the "authentic field theory" demanded many years ago by Ehrenfest.[17]

A second variant of the thesis that the wave function is nothing but a mathematical auxiliary reads thus:

(II.1.b) The wave function refers indirectly to objectively existent ensembles of similar systems; it does not characterize individual systems but only their membership in a statistical ensemble. The descriptions attained by means of the wave function are accordingly

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[15] E. Schrödinger, Nature **169**, 538 (1952).

[16] L. de Broglie, Compt. rend. **235**, 1345, 1372 (1953), and *La physique quantique, restera-t-elle indéterministe?* (Gauthier-Villars, Paris, 1953). The restriction of the validity of de Broglie's proof to bosons has been pointed out to me by Dr. Hans Freistadt (private communication).

[17] P. Ehrenfest, Z. Physik **78**, 555 (1932). See the rejoinder of W. Pauli, Z. Physik **80**, 573 (1933).

statistical, hence incomplete, since there are elements of physical reality (such as the disintegration time of an individual uranium atom) that have no counterpart in the present theory. References to individual microsystems can consequently have a probability meaning only. This is the statistical (not probabilistic) interpretation of quantum mechanics, proposed by Slater[18] and defended by Einstein,[19] Kemble,[20] Blochinzew,[21] and others.

Against this interpretation it has been argued that in some cases a single microsystem is in question, as happens in diffraction experiments in which electrons impinge upon a crystal individually, one after the other, so that the diffraction pattern is the cumulative results of individual hits.[22] But this argument does not support the usual interpretation, which says nothing of the detailed behavior of the individual electrons—save that they will fall on one of the "clear" diffraction fringes or rings. Until some years ago, Einstein's belief that the present quantum theory is exclusively statistical was a serious rival of the orthodox interpretation (which is not concerned with statistical ensembles of material systems, but with statistical ensembles of observational data that may refer to a single system). But since the causal interpretation of wave mechanics was completed,[12,16] that evaluation of the quantum theory, while still interesting, has ceased to be appealing, since it is now possible to think of every single microsystem as having a precisely defined position $\mathbf{x}(t)$ and an equally precisely defined momentum $\mathbf{p}(t)=\mathrm{grad}S(\mathbf{x},t)$, where S is the phase of the ψ wave [see Eq. (1)].

The hypothesis that ψ has only a probability meaning has been regarded as one of two facts demonstrating that wave mechanics is both subjectivist and indeterministic—the interpretation of Heisenberg's uncertainty relations in terms of the cognitive subject being the second strong argument advanced in favor of that thesis. The probability interpretation of the wave function would actually support subjectivism on the following conditions: (a) if it were the only possible one—which is far from being the case, and (b) if the subjective interpretation of probability were maintained, that is, if every probability statement were regarded as a subjective estimate, as a judgment entailing uncertainty and ranking slightly higher than total ignorance. This is not, however, the way probability is used in physics, be it classical or quantal. In physics the frequency interpretation of probability is used[23]—not however the frequency definition, which is both mathematically faulty and philosophically controvertible. A probability statement, such as "The event E occurs with probability P," when interpreted in physical terms reads "The event E occurs with a frequency f that fluctuates around the probability P." [24] If a transition to the empirical plane is performed, this sentence will have to be read as "The frequency for finding the event E upon observation or measurement is f, a number varying irregularly around P and approaching, also irregularly, to P as the number of observations increases." Our acquaintance with or ignorance of individual details does not change the objective frequency ratios, which reflect a collective behavior (whether of actual or of potential collections); our informational state only alters our estimate of such objective frequency ratios, not the frequencies themselves. Therefore a statistical approach to a physical theory is not necessarily a consequence of lack of information,[25] nor is it usually a dispensable

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[18] J. C. Slater, J. Franklin Inst. **207**, 449 (1929).

[19] Einstein, Podolsky, and Rosen, Phys. Rev. **47**, 777 (1935). See also Einstein's contributions to P. A. Schilpp, editor, *Albert Einstein: Philosopher-Scientist* (The Library of Living Philosophers, Evanston, 1949), and to *Scientific Papers Presented to Max Born* (Stechert-Hafner, Inc., New York, 1953).

[20] E. C. Kemble, Phys. Rev. **47**, 973 (1935).

[21] D. I. Blochinzew, *Grundlagen der Quantenmechanik* (Deutscher Verlag der Wissenschaft, Berlin, 1953), Sec. 14.

[22] It should be noticed that the assumed independence of the electrons or photons passing through the diffraction apparatus is not sufficient to ensure the independence of the hits in the recording screen. Indeed, the "particles" may be independent from one another and still the successive states of the apparatus might not be independent; actually they will not be altogether independent, unless the lapse between two successive passages is larger than the relaxation time of the apparatus.

[23] See, e.g., H. Cramér, *Mathematical Methods of Statistics* (Princeton University Press, Princeton, 1946).

[24] e.g., the statement, "The probability that the atom A will radiate a quantum of frequency ν in the next second is $P(\nu)$," means only that, out of a large number N of atoms of the sort and in the state specified by the symbol A, a fraction about $NP(\nu)$ will radiate in the next second.

[25] The propounders of the subjectivistic interpretation of quantum mechanics generally adopt the classical, Laplacian doctrine on the nature of probability, according to which this concept is essentially related to the amount

luxury when complete information is available[26] but, as Poincaré remarked long ago, it is consistent with a detailed knowledge of the parts; it is not only consistent but also indispensable, since it refers to a whole that emerges out of the interplay of the parts, and which has peculiar qualities of its own. In short, probability statements may but need not be subjective, so that Born's probability interpretation of the wave function does not support subjectivism.

(II.2) *Second group of answers:* The wave function is a physical symbol having as much relationship to material systems as other symbols have.

The following varieties of this interpretation must be distinguished:

(II.2.a) The ψ function refers exclusively to a real matter field associated with each individual system; ψ represents a smeared individual system, $|\psi(\mathbf{x},t)|^2 d\mathbf{x}$ being the actual amount of matter contained in the volume element $d\mathbf{x}$.

This hypothesis, originally proposed by Schrödinger, was very popular in the early days of wave mechanics. The following arguments have been advanced against it: (a) it does not apply in the general case, i.e., for many-particle systems; (b) electrons and other microsystems can be localized in point-like regions, so that (c) upon localization the matter field would have to contract instantaneously to a point (the so-called instantaneous collapse of the wave function upon position measurement)—which would contradict relativity. These arguments are not as strong as they look at first sight. The first of them can be disposed of by means of de Broglie's[16] recent formulation of the many-particle theory in ordinary space. As to the second argument, it can be rejoined with the remark that particles are never localized in points, but always to within an atomic diameter. And the assumed instantaneous character of the contraction of the wave packet would be difficult to test experimentally; besides, it follows from the present theory of quantum measurements, which is obviously too schematic and which moreover is not intended to describe processes unfolding in space and time. As Jánossy has remarked, nothing warrants the impossibility of building up a theory describing nonclassical processes of rapid contraction of the ψ field—and nothing guarantees that relativity is valid in all domanis. Anyhow, the hypothesis that ψ refers only to a field and accounts for no particle characteristic, seems to contradict the narrow spatial localizability of microsystems, that can be achieved experimentally.

(II.2.b) The wave function describes a real all-pervasive fluid, acting as a substratum, which in the simplest case (nonrelativistic theory of spinless particles) is compressible and moves with irrotational flow; particles are either irregularities or inhomogeneities in such a background, which is itself unobservable; local vortices may account for the spin. This hydrodynamical picture was originally proposed by Madelung[27] and it has since been completed by Takabayasi,[28] Schönberg,[29] and Bohm and coworkers.[30]

At the moment it seems difficult to ascertain whether the hydrodynamical analogy is more than a mere metaphor. Anyhow, it is hard to believe that nature should repeat certain privileged patterns at all levels, so that we should expect to find again, at the atomic and at the subatomic levels, entities behaving like large-scale fields, solid bodies, or fluids; at any rate, experience heretofore has shown that our stock of images, born out of our intercourse with macroscopic systems, is miserably reduced in comparison with nature's variety of patterns.

(II.2.c) While the ψ function has a purely statistical meaning, and is useful only when we ignore the actual trajectories of the particles, there exists a second solution, with the same phase but with a different amplitude, which represents the particle. This second solution has

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of information. See N. Bohr, J. Chem. Soc. (London), (1932), p. 349; M. Born, *Natural Philosophy of Cause and Chance* (Clarendon Press, Oxford, 1949), p. 110 and *passim;* W. Heisenberg, Naturwiss. **38**, 49 (1951); L. Rosenfeld, Science Progr. (London) No. 163, 393 (1953).

[26] J. v. Neumann, reference 3, p. 107, has described the statistical treatment of classical phenomena as "*ein Luxus, eine Zutat,*" since in principle we can know the exact values of the initial position and momentum of every particle of an ensemble.

[27] E. Madelung, Z. Physik **40**, 332 (1926).

[28] T. Takabayasi, Progr. Theoret. Phys. (Japan) **8**, 143 (1952); **9**, 187 (1953).

[29] M. Schönberg, Nuovo Cimento **12**, 103 (1954).

[30] D. Bohm and J.-P. Vigier, Phys. Rev. **96**, 208 (1954); Bohm, Schiller, and Tiomno, Suppl. Nuovo Cimento **1**, 48 (1955); D. Bohm and R. Schiller, Suppl. Nuovo Cimento **1**, 67 (1955).

the form

$$u(\mathbf{x},t) = f(\mathbf{x},t) \cdot e^{(i/\hbar)S(\mathbf{x},t)}, \tag{4}$$

where the new amplitude, $f(\mathbf{x},t)$, represents a moving "singularity," i.e., a small moving region where f takes on very high values. This is the theory of the double solution, proposed by de Broglie and worked out by him and Vigier[16] in analogy with Einstein's conception of particles as condensations of fields.

Now, in order to obtain "singular" solutions having the same frequency as that of the continuous solution, the wave equation would have to change substantially in the vicinity of the particle's world-line; for instance, it would have to become nonlinear, so that this theory of the double solution presupposes the existence of two different wave equations rather than the existence of two solutions of a single equation—whence it seems that it should be called the theory of the double equation. Hence, it does not seem to be a possible interpretation of the available mathematical skeleton, which is essentially linear. Although the program of the theory of the double equation is a most interesting proposal, it should be realized that it is based on the conviction, critized above, that ψ cannot represent any physical entity just because it is a complex function defined, in general, in an abstract space.

(II.2.d) The ψ function has two meanings: on the one hand, it represents an objectively existent field attached to every particle and acting on it with the quantum-mechanical force

$$\mathbf{f} = -\nabla[-(\hbar^2/2m)\Delta R/R], \quad R = |\psi|. \tag{5}$$

On the other hand, ψ is also the probability amplitude defining a distribution in position of similar systems (i.e., of systems described by the same ψ function but starting with different positions and momenta). Statistical ensembles appear not only as a result of our ignorance of the exact initial values, but also as a result of the complicated, nearly *quasi*-ergodic character of the motions; this permits the statistical treatment to provide a good reflection of the mean behavior. In other words, Born's relation $P = |\psi|^2$ between the probability density P and the field amplitude is the result of the complicated motion conditioned by the quantum-mechanical potential; it is an equilibrium distribution reached from other distributions ($P \neq |\psi|^2$) corresponding to initial conditions differing from the usual ones, and brought about by the laws of motion themselves.[31] If the initial position (configuration) $\mathbf{x}(t_0)$ and phase $S(\mathbf{x},t_0)$ (boundary condition) are known, this distribution in position is not changed, but in addition to it we are able to determine the true trajectories $\mathbf{x}(t)$. Indeed, the equation

$$\mathbf{p}(t) = m d\mathbf{x}/dt = \nabla S(\mathbf{x},t) \tag{6}$$

can then be integrated to yield the exact location of the particle at each instant of time. (Note that the "field problem" for R and S must be solved before the dynamical problem can be approached.) This is Bohm's first interpretation of wave mechanics in terms of the hidden variables $\mathbf{x}(t)$ and $\mathbf{p}(t)$.

It should be noticed that the average quantum theorist regards the ψ function, at least in his daily work, not as a mere "wave of knowledge," but as representing some sort of reality at some level. Even Bohr[32] has illustrated his own interpretation with drawings showing the diffraction and interference of ψ waves in ordinary space—which would be consistent with the mentalist interpretation of ψ only on condition that ψ were regarded as an ideal component of material objects, i.e., as an Aristotelian "form" (εἶδος).[33]

III. INTERPRETATIONS OF HEISENBERG'S UNCERTAINTIES

The various interpretations of Heisenberg's uncertainty relations fall again in two broad classes:

(III.1) *First group of answers:* Heisenberg's uncertainty relations are indeterminacies.

At least the following varieties of this thesis must be considered:

(III.1.a) The Heisenberg uncertainty relations are neither theoretical uncertainties that might be surmounted by a deeper theory, nor

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[31] The approach to equilibrium has been discussed by D. Bohm, Phys. Rev. 89, 458 (1953) in a special case.

[32] N. Bohr, contribution to the collective volume cited in reference 14, pp. 212, 214, and 216.

[33] Such an Aristotelian interpretation of quantum mechanics has been proposed by C. Bialobrzeski, Rev. métaph. et morale (Paris) 41, 83 (1934).

objective indeterminacies inherent in the nature of things—as there is no such thing as a thing in itself existing autonomously in the real world. The uncertainties are empirical indeterminacies, they are irreducible and uncontrollable traits of the results of observation, and they originate in the impossibility of drawing a definite line of separation, or cut, between subject and object, i.e., a sharp and unambiguous cut between the "object of observation" and the observer. The Heisenberg uncertainty relations constitute unsurpassable boundaries of knowledge; nothing is to be found beyond them, and they cannot be assigned to anything else. This is the orthodox interpretation.

According to this view, the uncertainties have a wholly empirical status: they refer neither to a surmountable paucity of information regarding the behavior of matter, nor to an objective indeterminateness or haziness of the latter. Hence, the usual interpretation of quantum mechanics, when properly understood, supports neither determinism nor indeterminism of the ontological type (although it does assert an empirical indeterminacy), as both *isms* usually entail the assumption that something does exist beyond the empirical plane.[34] Since it is meaningless to discuss the behavior of things independently of their observation, the correct answer to question III is the doctrine of complementarity. Most physicists pay lip service to this orthodox interpretation, while simultaneously believing that it definitely asserts an ontological indeterminism, whereas what it asserts might rather be termed an empirical indeterminism, for it holds that the indeterminacies refer exclusively to the results of observation.

This interpretation is usually made plausible by deriving Heisenberg's relations with the help of ideal experiments (*Gedankenexperimente*), such as the localization of electrons by means of gamma rays in Heisenberg's microscope. In such ideal experiments it seems obvious that the uncertainties always concern results of observation. But the correct and general way of deriving the Heisenberg relations is by starting from the general principles of wave mechanics, without any appeal to measurements; this procedure enables us consequently to understand why Heisenberg's relations are valid even inside the sun, where no observations are made at present. The case of the Heisenberg microscope is then seen to be nothing but a particular case of the interaction of microsystems with their macroscopic environment. When the observer does play a role at all, it is not as a cognitive subject, but only through his material means of observation; the subject of the theory of knowledge has nothing to do with such physical processes, which have nothing psychical in them.

(III.1.b) Heisenberg's uncertainties are objective and irreducible indeterminacies originating solely in the perturbation produced by the measurement apparatus on the observed system; they are irreducible because of the indivisibility of the quantum.

This interpretation is quite popular among physicists and philosophers of physics,[35] although it is at variance with the positivistic theory of knowledge as well as with the doctrine of complementarity, since the assertion that the apparatus produces a disturbance on the microsystem entails the "metaphysical" assumption that the microsystem existed, in an unperturbed state, before being observed. Bohr[36] and other upholders[37] of the usual interpretation of quantum mechanics have repeatedly maintained that the measuring apparatus and the "object of observation" form a whole, an unanalyzable, irrational totality, so that it is meaningless to

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[34] This position seems to have been attained only around 1936, when the usual interpretation of quantum mechanics was recognized as a piece of the logical empiricist philosophy. See N. Bohr,[1] P. Frank,[4] M. Schlick.[4] Until then, Bohr had supported indeterminism, asserting that nature can make a free choice among several possibilities; see N. Bohr, *La théorie atomique et la description des phénomènes*, translated by A. Legros and L. Rosenfeld (Gauthier-Villars, Paris, 1932), p. 4 and *passim*.

[35] See, e.g., V. F. Lenzen, "Philosophical problems of the statistical interpretation of quantum mechanics," J. Neyman, editor, in *Second Berkeley Symposium on Mathematical Statistics and Probability* (University of California Press, Berkeley and Los Angeles, 1951), p. 567. Lenzen draws the inference that on both sides of the partition between observer and observed object strict causality holds, being only at the partition that indeterminacy appears. Also Dirac, *op. cit.*, p. 4, concludes that causality may still be assumed to apply to closed undisturbed systems.

[36] N. Bohr, Phys. Rev. 48, 696 (1935); see also Dialectica (Neuchâtel) 2, 312 (1948) and reference 32, p. 317.

[37] L. Rosenfeld, Progr. Sci., No. 163, 393 (1953); C. F. v. Weizsäcker, in *Martin Heideggers Einfluss auf die Wissenschaft* (A. Francke Ag. Verlag, Bern, 1949); W. Pauli, Dialectica (Neuchâtel) 8, 112 (1954).

trace the statistical distribution of the results of observation to an interaction between the object and the measuring device; the quantum, which is indivisible, is deemed to belong to both the object and the observer.

But quite apart from this sort of criticism, it is obvious that the interpretation in terms of a perturbation by the measuring apparatus is open to the same objection we had advanced against the usual interpretation, namely, that it does not account for the fact that Heisenberg's uncertainties exist even in the absence of any measurement—although they are obviously supposed to be tested or at least testable by experiment.

(III.1.c) Heisenberg's uncertainties are objective and irreducible indeterminacies evidencing the operation of absolute chance at the atomic level; that is, the uncertainties have an ontological status. This belief, which is very popular among physicists, has been defended by Born[38] and by Landé,[39] for whom atoms and the like are "small scale gambling devices."

It is a fact that most quantum-mechanical quantities in general fluctuate in an irregular way; but this irregularity need not be indeterminate (i.e., produced out of nothing) and irreducible, as is assumed in this interpretation. Thus, de Broglie-Bohm's quantum-mechanical force enables us to explain quantum fluctuations as an irregular but precisely determined effect. Besides, if chance were absolute, the particular case of zero scatter would be difficult to explain.

(III.1.d) Heisenberg's spreads are objective indeterminacies, but they are not irreducible; they arise in some lower level motions in analogy with the Brownian motion of middle-sized particles immersed in a fluid composed of smaller particles (molecules). This hypothesis has been suggested by Weizel and Bohm and co-workers.

According to Weizel,[40] motion at the quantum level can be regarded as a classical diffusion process brought about by the random action of the constituents of an all-pervasive background (the elements of which he calls *Zeronen*), which are external to the system under consideration. The latter moves in accordance with the laws of classical mechanics, but it is disturbed by the random collisions of the *Zeronen*, which act as the cause of the quantum-mechanical fluctuations. The successive actions of the *Zeronen* are independent of one another, and it is not indispensable to think of them as particles of an aether, since they may represent random perturbations coming from outside the system. Bohm and Vigier,[41] on the other hand, have suggested that microsystems can be regarded as inhomogeneities embedded in a lower level fluid that undergoes turbulent motion, i.e., spontaneous fluctuations of its density and velocity, so that the values obtained in the quantum theory are only mean values of such random variables.

The unsatisfactory feature of these interesting attempts is that they push chance a step below but retain it as an ultimate, instead of explaining it. In the case of Weizel's theory Newtonian mechanics is regarded as another ultimate (on which random perturbations are superimposed) —which is equally unconvincing. Anyhow, it is important to have shown that the Schrödinger equation can be deduced on the basis of definite models, instead of postulating it as the starting point for a phenomenological theory, as is usually done.

(III.2) *Second group of answers:* Heisenberg's inaccuracies are uncertainties, i.e., they refer to our imperfect knowledge of things, not to the things themselves.

The following varieties have been proposed:

(III.2.a) Heisenberg's scatters originate in the wave aspect of matter; since the concept of position entails that of point-particle, it cannot be rigorously applied to waves. In other words, the uncertainties arise from the application of concepts of particle dynamics to the description of wave phenomena. This hypothesis, involving the reality of the ψ waves, has been defended by Laue[42] and others. It would be difficult to dispute that at least a part of our difficulties stem from the application, to an entirely new field, of concepts that have grown up in classical dynamics. But the interpretation of Heisenberg's

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[38] M. Born, *Natural Philosophy of Cause and Chance* (Clarendon Press, Oxford, 1949).

[39] A. Landé, Philosophy Sci. **20**, 101 (1953).

[40] W. Weizel, Z. Physik **134**, 264 (1953); **135**, 270 (1953); **136**, 582 (1954).

[41] D. Bohm and J. P. Vigier, Phys. Rev. **96**, 208 (1954). See also references 29 and 30.

[42] M. v. Laue, Naturwiss. **20**, 915 (1932); **22**, 439 (1934).

uncertainties in terms of concepts out of their proper context is rather negative, except in so far as it might constitute a stimulus for the search of a different idea.

(III.2.b) Heisenberg's spreads originate in the coarseness of the measuring apparatus (which is macroscopic), as compared with the microsystem; each macrostate of the apparatus corresponds to many different microstates of the observed system, so that instead of measuring single values we actually measure whole ranges of values of the variables attached to the microsystem. In the above description the term "apparatus" designates in general the macroscopic environment of the microsystem.

This interpretation does not seem to have been carried through; it would be interesting to develop it, if only to recall that the definition of eigenvalues as the sharp results of precise measurements is anything but operational.

(III.2.c) Heisenberg's uncertainties are a particular case of a universal relation appearing in Markov chain phenomena (such as Brownian motion) when the specification of the variables determining the exact state of the system is incomplete. Hence, Heisenberg's relations are exclusively a consequence of the statistical approach, and are independent of the perturbations caused by the measurement. In the present theory, which is incomplete, quantum processes may be regarded as special Markov chains, i.e., as chains of events having interdependent probabilities; the quantum potential turns out to be just a statistical result of such stochastic processes. This interpretation was originally suggested by Fürth[43] and worked out in detail by Fényes[44]; the root of it is the formal analogy existing between the Schrödinger equation and the Fokker diffusion equation.

As in the case of the hydrodynamical models, it is doubtful whether more than mere outward resemblances are involved in this approach. But it is a doubtless interesting one, as it suggests that hidden variables can be introduced in quantum mechanics (thereby falsifying again von Neumann's theorem), so that quantal statistics may not be essentially different from classical statistics; besides, this interpretation shows that the Heisenberg spreads need not be regarded as arising exclusively in connection with measurements.

(III.2.d) Heisenberg's uncertainties refer to the "observables" (q-numbers), not however to the "hidden variables," which have precise values at any given instant. The statistical distribution of values is produced by the usually large and rapid fluctuation of the quantum-mechanical force, which causes particles to deviate from their classical path in an irregular but in principle causal way. Quantum-mechanical fluctuations are present even in the absence of measuring apparatus, but they are obviously enhanced by their presence. This is Bohm's[12] first interpretation.

Whatever be the interpretation that will finally prevail, one thing is beyond dispute, namely, that the present state of the quantum theory offers no secure basis to the doctrines asserting that Heisenberg's relations show the haziness of reality, or the ultimate indeterminacy of becoming, or the material basis of free will—nor the free will of the material basis.

IV. NATURE OF MICROSYSTEMS

This paper will not discuss particular properties such as mass and charge, the nature of which is still poorly understood, especially since it was realized that "particles" cannot be regarded as isolated but must be viewed as immersed in oceans of virtual particles, in the zero-point electromagnetic field, and other substitutes of aether, with which they interact. The subject of controversy in connection with the interpretations of elementary quantum mechanics is much simpler, namely, whether matter at the microscopic scale is something definite or not and, in case it is, whether it behaves as particle, as wave, as wavicle or as something different.

(IV.1) *First group of answers:* Microsystems are nothing definite.

The following variants can be recognized:

(IV.1.a) Microsystems are neither corpuscular nor undulatory, they simply are not in themselves, since they do not exist apart from the experimental setups designed to measure observables. The words particle and wave designate neither material objects nor properties of ma-

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[43] R. Fürth, Z. Physik 81, 143 (1933).

[44] I. Fényes, Z. Physik 132, 81 (1952).

terial objects, but concepts, images, or models that we use in the complementary descriptions of certain possible experiments. Outside the descriptions of experiments (which descriptions are equivalent, whether they are couched in the corpuscular or in the undulatory language), there are neither particles, nor waves, nor wavicles. It is meaningless to ask about the properties of something which, like matter, is only a fiction. As Heisenberg[1] wrote, the quantum theory is not concerned with nature—but with our cognition of nature. And as Frank[45] explained, "There are experimental setups which can be described by using the term 'position of a particle' and others which can be described by using the terms 'momentum' or 'wavelength.' All the confusion is produced by speaking of an object instead of the way in which some words are used." This is Bohr's doctrine of complementarity, embraced by most positivists.

The doctrine of complementarity is both phenomenalist (since it renounces the explanation of the mechanism of quantum processes) and dualistic, as it proclaims an irreducible duality—not however in nature but in the description of experiments. As Margenau[46] writes, "Bohr does not ask science to make a choice—he asks science to resign itself to an eternal dilemma. He wants the scientist to learn to live while impaled on the horns of that dilemma, and that is not philosophically healthy advice." Such a dualism "relieves its advocates of the need to bridge a chasm in understanding by declaring that chasm to be unbridgeable and perennial; it legislates a difficulty into a norm."[47]

(IV.1.b) Microsystems are just vibrations or, if preferred, vibrations of nothing into nothing. This view, which has occasionally been uttered, is unlikely to attract people unprepared to receive the nihilism to which it leads.

(IV.2 *Second group of answers:* Microsystems are something definite.

The main varieties of this thesis are:

(IV.2.a) Microsystems are particles; particles are basic and irreducible, whereas waves are derived: they emerge in the random notion of particles. This interpretation is quite widespread, and can be supported, e.g., by the derivation of the features of electron diffraction from Heisenberg's relations ,with the assumption that the latter refer to randomly moving point-particles.[48] This interpretation has been carried through by Bopp.[49]

The hypothesis that mircosystems are ultimately corpuscular is, of course, justified by all those experiments in which they are actually localized in point-like regions. Some of the arguments that have been advanced against this view are: (a) no experiment has so far detected the position of a particle with an accuracy greater than one atomic diameter; (b) microsystems can interfere, though not with one another but, in a way, with themselves. For example, if an electron wave impinges on a two-slit system, the wave emerging from the first slit, ψ_1, combines with the wave emerging from the second slit, ψ_2, in the form

$$|\psi_1+\psi_2|^2=|\psi_1|^2+|\psi_2|^2+(\psi_1^*\psi_2+\psi_1\psi_2^*). \quad (7)$$

Now, in the right-hand side only the first two terms can be interpreted as particle densities, whereas the last two represent the interference effects; the latter cannot be accounted for in terms of interactions of particles, since ψ_1 and ψ_2 can interfere only because they refer to the same microsystem; that is to say, the dark spots in the recording screen are not the result of electrons extinguishing one another (which would contradict charge conservation) but are simply nonaccessible regions in the field.

(IV.2.b) Microsystems are waves; that is, waves are basic, particles being derived.

This interpretation is suggested by the following mathematical facts: (a) the basic equation of wave mechanics is a "wave" equation, i.e., it is formally similar to the wave equations of classical physics; (b) in the quantum theory of fields (second quantization) particles appear as quanta of fields. But the most impressive

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[45] Ph. Frank, Foundations of Physics, being No. 7, Vol. I of the *International Encyclopedia of Unified Science* (University of Chicago Press, Chicago, 1946), p. 55.

[46] H. Margenau, *The Nature of Physical Reality* (McGraw-Hill Book Company, Inc., New York, 1950), p. 422.

[47] H. Margenau, Phys. Today, **7**, No. 10, 9 (1954). For other criticisms of the doctrine of complementarity, see Bohm, reference 12, and L. Jánossy, Acta Phys. Hung. **1**, 423 (1952).

[48] See, e.g., A. Sommerfeld, *Wellenmechanik* (F. Ungar Publishing Company, New York, s.d.), pp. 200 ff.

[49] F. Bopp, Z. Naturforsch. **9a**, 597 (1954).

argument in favor of this view is, of course, the diffraction of "particles." However, this interpretation is inconsistent with the mass of experiments in which microsystems are fairly well localized and stable, i.e., in which they manifest themselves as just the opposite of a spreading wave packet. To solve this difficulty without appealing to complementarity, it has been suggested that microsystems are stable (i.e., nonspreading) and localized pulses in a wave field, or regions in which the ψ field bunches up. But stability and bunching-up do not seem attainable with a linear theory, so that this is not a possible interpretation of the available mathematical formalism. On the other hand, in nonlinear wave fields there are fairly stable and localized pulse-like concentrations behaving as particles[50]; hence, the theory that particles are nothing but manifestations of fields could be developed with the help of nonlinear equations, as was pointed out long ago by Einstein.

Although this is one of the most interesting and least explored possibilities, it must be realized, firstly, that there is an infinity of mathematically possible nonlinearities and no obvious physical criterion to limit the choice among them; secondly, that the hidden source of this important program is the monistic belief that there must be some ultimate substance out of which all material objects are made—a Milesian belief that, after all, is of an ontological character, and which should not be easily accepted by those who, far from wishing to level down everything in the world, regard nature as a sort of level structure, every one of its levels having its own, specific, irreducible qualities.

(IV.2.c) Microsystems are both corpuscular and undulatory; that is, the field aspect and the particle aspect are both basic and simultaneously real. The most complete and consistent elaboration of this hypothesis seems to be Bohm's.[12] Renninger,[51] in an unduly neglected paper, has suggested an experiment in order to test the duality (in the case of light) with a single setup.

This hypothesis that microsystems are at the same time corpuscular and undulatory has often been regarded as self-contradictory, since according to classical ideas a particle is a bit of impenetrable matter concentrated in a small and precisely definite region, whereas a wave acts throughout a whole region. An entity both concentrated and extended seemed as fabulous as a centaur, and Bohr's complementarity was aimed precisely at avoiding this seeming contradiction—though at the price of enthroning an unsolved paradox. However, there is nothing contradictory in the assumption that some aspects of microsystems (e.g., the mass and/or the charge) are concentrated in point-like regions behaving as cores of the whole, while other attributes (such as the energy) spread over a large region. In Bohm's interpretation, an electron within a box occupies the whole volume of the latter; although the electron mass and charge are here assumed to be concentrated in a definite point in space, the accompanying ψ field spreads over the whole available volume. Even if no causal interpretation is explicitly used, it can be shown that Dirac's electron can be regarded as constituted by a point-like and massless charge moving within a region of dimensions of one Compton wavelength, over which the mass is smeared[52]; in this model, too, some attributes are regarded as concentrated while others are assumed to be spread over space, so that the principle of contradiction is not violated.

The unsatisfactory feature of the interpretation described above is not of a logical nature, but rather that it takes for granted that the wave aspect is associated to the particle aspect without explaining in detail the form of such a symbiosis; but this is to ask too much of an interpretation of a theory which, like ordinary wave mechanics, assigns no structure to the "fundamental" particles. A further unsatisfactory feature of this interpretation is that the field-aspect influences the particle motion, but the latter is in turn without influence on the field aspect—a one-sidedness that leaves an uncomfortable feeling as to the nature of the quantum-mechanical "force"; a similar objection can be made against the hydrodynamical models

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[50] See, e.g., Finkelstein, Lelevier, and Ruderman, Phys. Rev. **83**, 326 (1951); L. de Broglie, Compt. rend. **236**, 1453 (1953); L. Jánossy, reference 47.

[51] M. Renninger, Z. Physik **136**, 251 (1953).

[52] M. Bunge, Nuovo cimento **13**, 977 (1955).

in which the inhomogeneities are dragged by the fluid without influencing the latter.

It seems, in sum, that we still do not understand satisfactorily how the two aspects, the corpuscular and the undulatory, are linked to each other; but at least we may now assume, without contradiction, that microsystems are neither merely particles nor pure waves, nor waves trying to imitate particles or vice versa. We seem to begin to understand that microsystems are very rich and dynamic entities in close interconnection with their environment, and exhibiting different properties in accordance with the latter's changes. Is it unreasonable to require future theories to explain in greater detail how the wave aspect gets enhanced at the cost of the particle aspect and vice versa? It looks as reasonable as not to impose upon them the condition of remaining trapped in the wave-particle sandwich, since after all it is not unlikely that wave and particle turn out to be extreme manifestations of some radically new entity, of which we have not the slightest idea.

V. HOW MANY INTERPRETATIONS ARE THERE?

We have agreed to call an interpretation of wave mechanics that body of theory which answers at the very least questions (I) to (IV) on the basis of the available mathematical framework and the available empirical evidence. According to the above survey a surprisingly large number of different interpretations have been or might be set forth. Is this multiplicity of possible interpretations of wave mechanics not a sign of crisis? It recalls at least two critical periods of theoretical physics: the first is that of the 13th and 14th centuries, in which Aristotelian dynamics was critized and patched up by the physicists of the schools of Oxford and Paris, notably Buridan and Oresme; the second is the end of the 19th century, in which several reinterpretations of Newtonian mechanics were advanced, chiefly by Hertz and Mach. None of these reforms survived, because none of them succeeded in discovering the keys for solving the riddles they approached—which keys were the principle of inertia in the former case, and the connection of mechanics with electromagnetism in the latter. But these attempts to reformulate and reinterpret mechanics were effective to the extent to which they contributed to destroy the prestige of established authorities and the fear of them. The medieval nominalists contributed to shake the prevailing confidence in the peripatetic dogmas, and the late 19th century critics of Newtonian mechanics contributed to create an atmosphere of criticism that must have helped in the birth of relativity and the quantum theory. Buridan could, of course, not suspect what Galileo and Newton would discover and invent three centuries later, and Mach indignantly rejected the epithet of the forerunner of relativity, which he had not deserved. The main contributions of Buridan and Mach, as far as mechanics was concerned, were not to foreshadow the ensuing developments—they definitely did not—but to shake the prevailing confidence in established dogmas.

Perhaps something similar is happening at present in connection with quantum mechanics. Maybe none of the interpretations listed above will survive entirely, but a radically new theory will appear in the next future; at least let us hope that it will, as it is so badly needed to cope with nuclear and meson physics. Still, the multiplicity of interpretations of wave mechanics may favor the creation of a freer climate of discussion of fundamental problems, a climate that should help in the building of deeper and more inclusive theories.

The mere multiplicity of more or less consistent interpretations is of course not enough for that; physicists must first become aware of such a multiplicity, if only to realize that the usual, positivistic, interpretation of quantum mechanics is not the sole possible one, and that its crystallization into a set of dogmas of an assumed perennial validity may become a serious obstacle to progress in physics.[53]

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[53] For a criticism of the usual interpretation of quantum mechanics, see Mario Bunge, Brit. J. Phil. Sci. **6**, 1 (1955); **6**, 141 (1955).