Physics 554

Astronomy 510

Nuclear Astrophysics

Winter 2004

Chapter 1

Big Bang Nucleosynthesis

1.1 Elemental abundances

One of the most important clues about the origin and subsequent evolution of our universe comes from nuclear physics, the elemental abundances we can measure on the sun's surface, on the surfaces of distance stars (including elements dredged up from the interior by mixing), and in the interstellar medium. One can divide the elements into roughly five groups:

a) The dominant elements in our universe are hydrogen and helium, which account for almost all of the known baryons. Their abundances by mass are

^{1}H ~ 0.75

^{4}He ~ 0.25

We will see that these elements owe their abundances primarily to nuclear and weak interaction processes occuring in the first few minutes after the big bang. Because of this, their abundances are powerful probes of cosmology. We will also see that these abundances can also be tied to cosmological physics at quite a different era, the era of recombination, when electrons, protons, and other bare nuclei recombined, forming atoms. At that point the universe became transparent to photons. Measurement of the cosmic microwave background, the radiation that decoupled at this time, provides another test of the baryon abundance in our universe, because that bayon number influences the density fluctuations that grew over the first 400,000 years of the universe's evolution.

Figure 1: The solar system abundances of the elements (these elements were incorporated into the solar system 4.7 b.y. ago) as a function of A=Z+N. Note: a) the large abundances of H and He; b) the deep "hole" coresponding to Li/Be/B; c) a series of peaks, particularly prominent for the α nuclei, corresponding to the products of stellar burning between mass 12 and mass $\sim$ 40; d) a mass peak near Fe, A $\sim$ 56-60; e) rare heavier elements, but with mass peaks near A $\sim$ 130 and A $\sim$ 195. From Hix and Thielemann, astro-ph/9906478/.

Figure 2: The evolution of galactic Li as a function of metallicity. Stellar metallicity serves as a clock, with low-metallicity stars having formed early, high metallicity later (when the interstellar medium was enriched in metals from previous generations of stars). The [Fe/H] is the metallicity measure, relative to solar. Note the Li abundance plateau – called the Spite plateau – at low metallicity, indicating that some baseline of Li existed when the first stars were formed. This is assumed to be the primordial value. Note the great spread of values for stars of solar metallicity. The two circles correspond to the expected standard solar model Li (the high value) and the measured Li. The sun managed to destroy mosts of its Li – most likely by dredging Li to depth (to high temperatures, where it can be burned) – during some past epoch. Also shown are various theoretical mechanisms proposed for synthesizing Li. From Ryan et al., astro-ph/9905211/.

Figure 3: A more detailed view of solar system heavy element abundances. Note that the two mass peaks mentioned earlier are double peaks. We will see that the peak components are produced by different neutron-capture mechanisms. From Truran et al., astro-ph/0209308/.

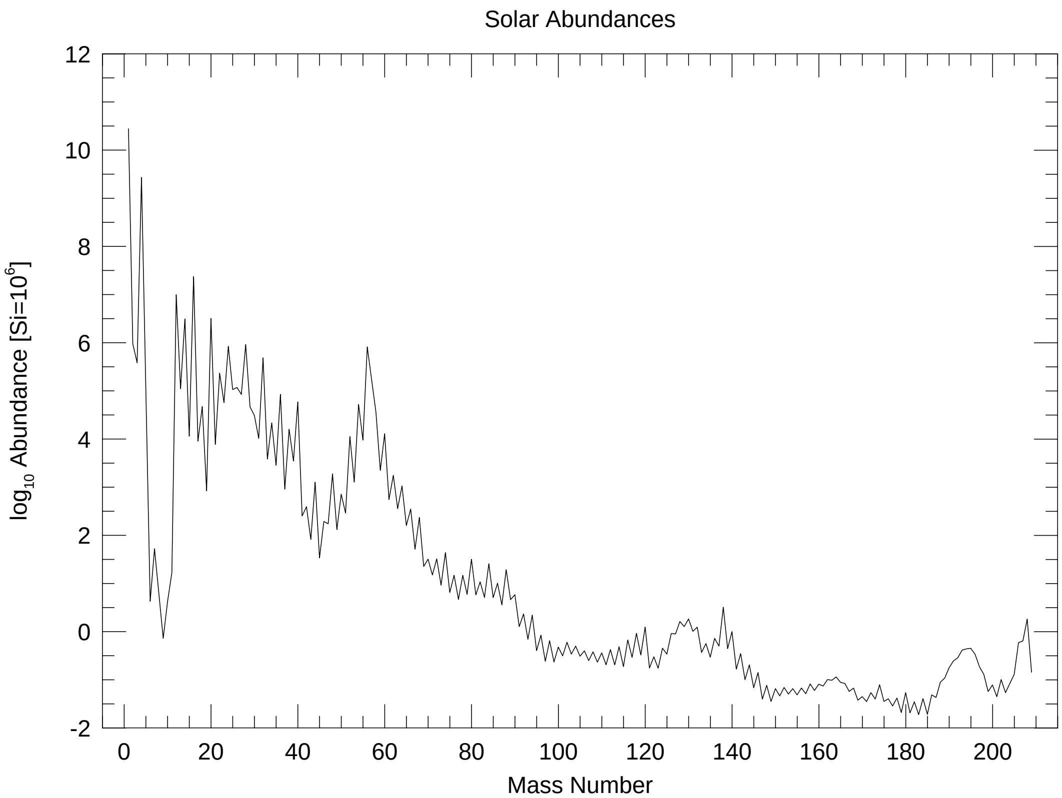
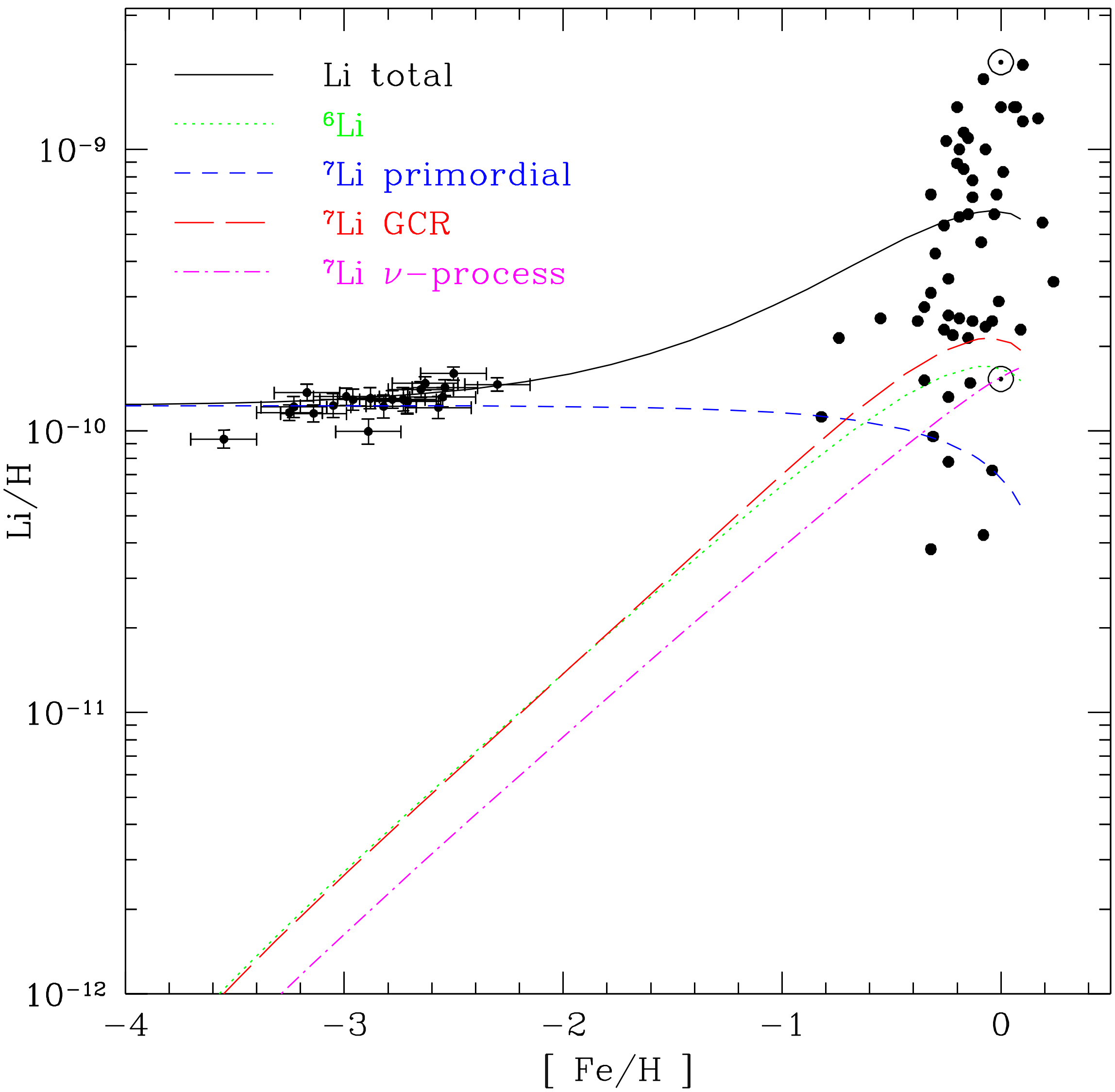


Fig The abundances of isotopes in the solar system as a function of atomic mass [3,4]. The abundances are normalized so that the total abundance of silicon is 10^6.



Li total
⁶Li
⁷Li primordial
⁷Li GCR
⁷Li ν−process
Li/H
[Fe/H]
10⁻⁹
10⁻¹⁰
10⁻¹¹
10⁻¹²
−4
−3
−2
−1
0

Figure

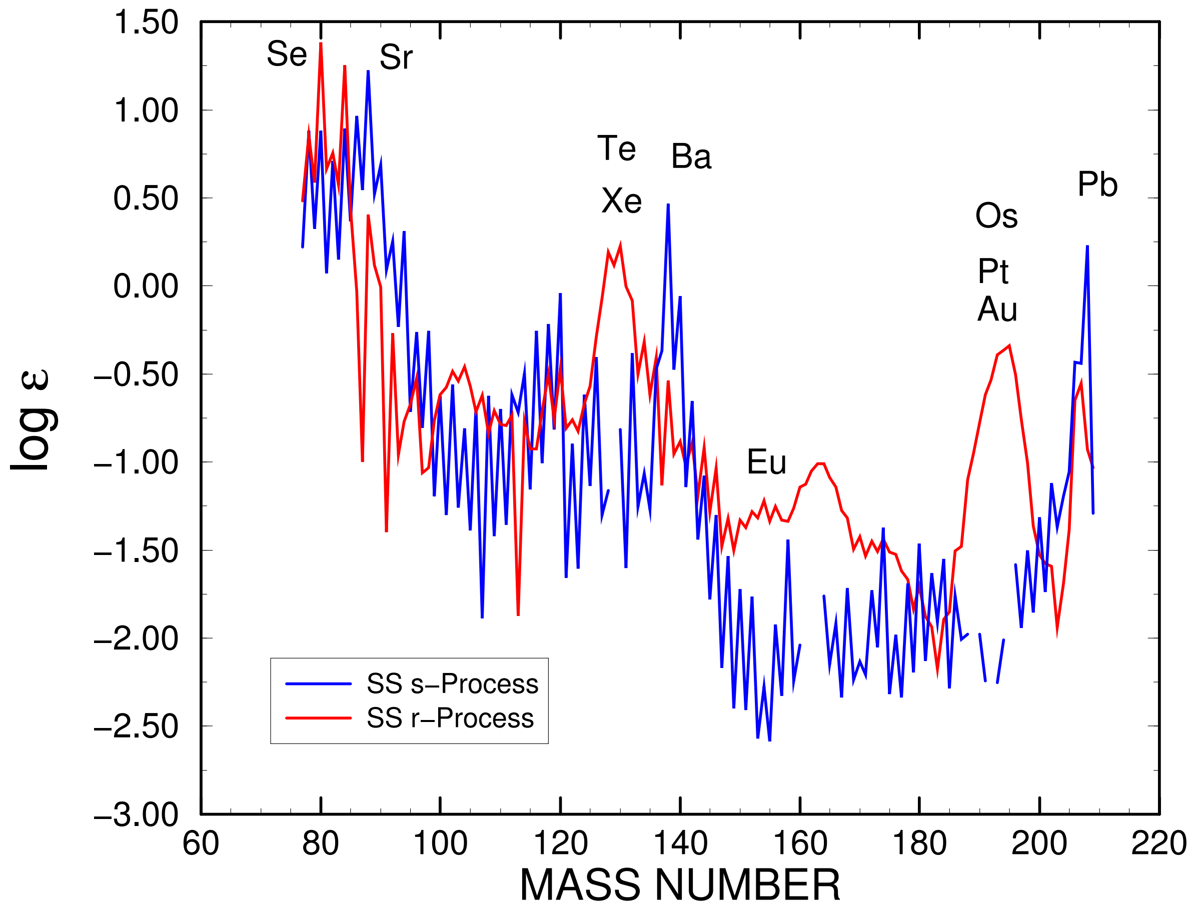


Fig. — The *s*-process and *r*-process abundances in solar system matter, based upon the work by Käppeler *et al.* (1989). Note the distinctive *s*-process signatures at masses A ≈ 88, 138, and 208 and the corresponding *r*-process signatures at A ≈ 130 and 195, all attributable to closed shell effects on neutron capture cross sections. It is the *r*-process pattern thus extracted from solar system abundances that can be compared with the observed heavy element patterns in extremely metal-deficient stars. The total solar system abundances for the heavy elements are those compiled by Anders and Grevesse (1989).

b) In contrast to the abundance of H and He, the lighter "1p-shell" nuclei — those heavier than He but lighter than C – are relatively rare. Relative to H and He, their abundances are lower by 8-10 orders of magnitude:

^{6}Li $\sim$ 7.75E-10

^{7}Li $\sim$ 1.13e-8

^{9}Be $\sim$ 3.13E-10

^{10}B $\sim$ 5.22E-10

^{11}B $\sim$ 2.30E-9

These rare elements have a fascinating heritage. We will see that ^{7}Li can be produced in the Big Bang. Li, Be, and B can also be produced in the interstellar medium, when energetic cosmic-ray protons collide with elements like C, N, and O. This process thus connects to two important chapters in this course – one on the origin of cosmic rays, the other on supernovae (which on exploding enrich the interstellar medium with the products of stellar burning, such as C, N, and O). (Other stars have intense winds that similarly eject elements into the interstellar medium.) But Li, Be, and B can also be synthesized by core-collapse supernovae, directly by the interactions of neutrinos in the carbon shells of such stars. Finally, elements like Li are notoriously fragile, easily burned in stars. (We will see that our sun has only 1/100th the Li we would anticipate, for example.) The result is a very interesting but complicated history for these rare elements. By carefully measuring abundances – in our solar system, in pristine material found in the interstellar medium, and on the surfaces of stars of varying ages – we can deduce their evolution history. The goal of the nuclear astrophysics is then to use that history to understanding how the various processes mentioned above have operated since the Big Bang.

c) The relatively more abundant, and largely α-stable elements C, N, O, Ne, Mg, Si. Here α-stable is slang for a nucleus whose (N,Z) is a multiple of ^{4}He, (N,Z)=(2,2). An important property of the nuclear force is pairing: nucleons are fermions with spin 1/2. As nucleons fill the lowest levels of the nuclear potential, they gain additional energy from the attractive interaction of two like nucleons (two protons or two neutrons) residing in the same level – that is, having the same spatial wave function. Two like-nucleons can exist in the same spatial level because there are two spin choices. The α-stable nuclei are those that, naively, could be envisioned as fully paired. These nuclei tend to be more tightly bound than their neighbors with broken pairs. Thus these nuclei, in the thermal furnances inside the cores of stars, tend to be the thermodynamically-favored products of nuclear burning. These are the final nuclear states that yield the most energy, when daughter nuclei fuse.

^{12}C $\sim$ 3.87E-3

^{14}N $\sim$ 0.94E-3

^{16}O $\sim$ 8.55E-3

^{20}Ne $\sim$ 1.34E-3

^{24}Mg $\sim$ 0.58E-3

^{28}Si $\sim$ 0.75E-3

d) There is an abundance peak near iron-group nuclei. Elements near Fe and Ni have the largest binding energy/nucleon. Thus such elements are the last possible products of a sequence of fusion reactions that produce more tightly bound nuclei, thus liberating energy to maintain the gas pressure of a star. The synthesis of such elements requires hot, dense conditions – rather explosive conditions – because of the large Coulomb barriers that must be overcome for nuclear fusion. High temperatures, and thus high bombarding energies, are needed.

e) The heavy elements, A $\gtrsim$ 100. Many of these elements are very rare. Yet an examination of abundances as a function of A or N shows interesting patterns – several abundance peaks. In fact, on closer examination, each peak is actually a double peak, with the two components split by $\sim$ 10 mass units. The huge Coulomb barriers that inhibit the fusion of heavy nuclei argue another mechanism must be at work. The only reasonable mechanism is (n,γ) reactions. We will see that heavy elements can be produced in ordinary stars with relatively weak neutron sources (thus the capture on a given nucleus occur slowly), or under explosive conditions (such as a supernova or neutron star collision) where extremely high neutron densities can be produced for short times.

So how are the differences in abundances among these groups explained?

First question: Could ^{4}He, 25% by mass, be generated by stellar burning during the lifetime of our galaxy?

Edington first proposed 4p $\rightarrow$ ^{4}He as an energy source for our sun

$$4\text{p} \rightarrow {}^4\text{He} + 2\text{e}^+ + 2\nu_e$$

The mass difference for this reaction is $\sim$ 25.7 MeV, while the typical kinetic energy carried off by neutrinos $\sim$ 0.4 MeV. Therefore about 25 MeV is released per four protons consumed. This is the energy that keeps the electron gas in our sun hot: energy is produced in the solar core at the required rate, just about balancing the energy that is carried off the sun by the photons emitted from the photosphere. Thus we can estimate the rate of fusion in the solar core from the measured solar constant.

solar constant $\sim$ 0.033 cal/sec/cm^2
distance to earth $\sim 1.49 \times 10^{13}$ cm = r
therefore the power output is

$$(4\pi r^2)(.033\text{cal/sec/cm}^2) \sim 0.92 \cdot 10^{26}\text{cal/sec} \sim 2.4 \cdot 10^{39}\text{MeV/sec}$$

but as 4 protons are consumed for every 25 MeV produced

$$\rightarrow 4 \cdot 10^{38} \text{ p/sec consumed}$$

Now the sun has a mass equal to that of $\sim 1.19 \cdot 10^{57}$ protons

So assume 5 billion years (b.y.) (the sun is estimated to be about 4.6 b.y. in age) of burning at the current power level. Then we can estimate the number of protons consumed over that lifetime

$$(3.15 \cdot 10^7 \text{sec/year})(5 \cdot 10^9 \text{years})(4 \cdot 10^{38} \text{protons/sec}) \sim 0.63 \cdot 10^{56} \text{protons}$$

So this is

$$\frac{0.63}{11.9} \sim 5.3\% \text{ of the sun's mass}$$

Thus

$$\text{only} \sim 5\% \text{ of protons converted in 5 b.y.}$$

(Of course, this He is also locked in the core of our sun, not in places like the interstellar medium where it could be counted by those interested in determining abundances.) And many protons are not in stars. Thus the tentative conclusion is that stellar burning contributes to, but cannot account for all, of the ^{4}He. In fact, this conclusion can be put on much firmer ground by looking at the ^{4}He abundance as a function of stellar metallicity, which is kind of a "clock" for the galaxy. Very metal poor stars presumably were formed very early (before supernovae and novae created many of the metals). The surfaces of such stars should not know about the ^{4}He synthesis in the core, but rather be representative of the star at its birth. So if the surface shows a large ^{4}He abundance, one would conclude that that ^{4}He was primordial.

We will learn more about 4p $\rightarrow$ ^{4}He later. It was first described by Bethe and Critchfield $\sim$ 1939; the theory was further developed by Salpeter and by Burbidge, Burbidge, Fowler, and Hoyle in the 1950s.

1.2 Big bang nucleosynthesis

Gamow in the 1940s: proposed a "big bang" cosmology where the universe began as a hot soup, then expanded and cooled. When cooled below about kT $\sim$ 1 MeV, when e^+ - e^- annihilation would occur, that soup would consists of the familiar stable particles like p, n, e^-, and γ.

The basic idea of big bang nucleosynthesis: a nuclear reaction network that begins with n + p $\rightarrow$ d + γ and ends ....??

It is absolute clear this must happen as

$$\tau_{1/2}(\mathrm{n}) \sim 10 \text{ minutes}$$

So if there is no nucleosynthesis, there would be no neutrons now. This is an important qualitative idea: neutrons exist in our present day world only because they bind in nuclei. Free neutrons have enough energy to decay to protons via beta decay. Bound neutrons do not because their binding energy makes this decay energetically impossible. So some kind of "condensation" or "freezeout" of nuclei from the hot big bang must have occurred.

Now we can calculate the n/p ratio. The mass abundance is 75% H and 25% ^{4}He. So this means for every ^{4}He (4 mass units) there must be about 12 protons (12 mass units). Thus there are 2 neutrons and 14 protons in the sum. Therefore

$$\frac{n}{p} \sim \frac{1}{7}$$

in our galaxy.

Challenge: Can we understand this number?

Does the number tell us anything about the early universe?

Recall in a thermal gas a particle has a kinetic energy of E $\sim$ 3kT/2, where k = Boltzman's constant = $0.862 \cdot 10^{-10}$ MeV/K. Here capital K stands for degrees K. So

$$1 \text{ MeV} \sim \text{kT} \Rightarrow \text{T} = 1.16 \cdot 10^{10}\text{K}$$

That is, an MeV is the typical thermal energy of a particle in a heat bath of T = 10 billion degrees K.

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Before we proceed we need to have a few results about cosmology. This will be very quick and without derivation, as it is the subject of another quarter (particle astrophysics etc) of this two-quarter series. We start with the Robertson-Walker metric (that is, measure of distance) for a homogeneous, isotropic universe

$$d\tau^2 = dt^2 - \text{R}^2(\text{t}) \left[\frac{\text{dr}^2}{1 - \text{kr}^2} + \text{r}^2 \text{d}\theta^2 + \text{r}^2 \sin^2\theta \text{d}\phi^2 \right]$$

where

k=+1 ↔ curved space, finite

k= 0 ↔ flat space, infinite

k=-1 ↔ curved space, infinite

Note that R(t) sets the scale of the geometry, and is a function of time. The coordinate r in the metric above is dimensionless: it's not the usual r. The dimension of length is carried by R(t). So if an observer sits at the comoving reference point (r,θ, ϕ) (note t is the time such an at-rest observer would measure), the length of the radius from r=0 to him/her is

$$\text{R(t)} \int_0^{\text{r}} \frac{\text{dr}'}{\sqrt{1 - \text{kr}'^2}}$$

Thus if R(t) increases with t, it would be like riding on the surface of an expanding balloon.

Now the evolution of R(t) is determined by the Einstein equations which, under certain simplifying assumptions, yield

$$\frac{\frac{d\text{R(t)}}{\text{dt}}}{\text{R(t)}} = \text{Hubble“constant”} = \sqrt{\frac{8\pi\text{G}\rho(\text{t})}{3}}$$

where G is Newton's gravitational constant and $\rho(\mathrm{t})$ is the energy density. One can show from energy conservation that when the universe is dominated by relativistic particles ρ satisfies

$$\rho \propto \frac{1}{\mathrm{R}^4}$$

(The assumption about relativistic particles is connected with the need to know the equation of state to figure out how ρ changes when R(t) changes. For relativistic particles, the pressure is just 1/3 of ρ.)

So from this result we note

$$\frac{\frac{d\rho}{dt}}{\rho} = \frac{-4\mathrm{R}^{-5}\frac{\mathrm{dR}}{\mathrm{dt}}}{\mathrm{R}^{-4}} = \frac{-4\frac{d\mathrm{R}}{\mathrm{dt}}}{\mathrm{R}}$$

So we can plug in the expression for the Hubble constant and integrate to get

$$\rho(\mathrm{t}) = \frac{3}{32\pi \mathrm{G}(\mathrm{t}+\beta)^2}$$

where β is an integration constant. Finally, one can write still another expression for ρ by calculating the energy density, which in the early universe is dominated by light, relativistic particles (the photon, electron and positron, and the three neutrinos). This yields

$$\rho(\mathrm{t}) = \mathrm{N}\frac{\pi^2}{30}\mathrm{T}^4$$

where T is the temperature and N is the effective numbers of degrees of freedom (=43/4 if all of the species above contribute). This formula is relatively easy to understand. The density of massive particles should go like T^3 because the momentum states (k_x, k_y, k_z) will fill up roughly to (T, T, T). Then an extra power of T comes from the fact we are calculating the energy density, and each particle has an energy $\sim T$.

So examining these expressions yields

$$\mathrm{R} \sim \mathrm{T}^{-1} \sim \sqrt{\mathrm{t}}$$

We can look up in Weinberg's book some numbers from more careful calculations:

	$T(K)$	R/R_o	$t(sec)$
$\sim 10MeV$	10^{11}	$1.9 \cdot 10^{-11}$	.0108
$\sim 1MeV$	10^{10}	$1.9 \cdot 10^{-10}$	1.103
$\sim 100keV$	10^{9}	$2.6 \cdot 10^{-9}$	182
$\sim 10keV$	10^{8}	$2.7 \cdot 10^{-8}$	19200

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So let's look at some of these epochs

A) t $\sim$ .01 sec T $\sim 10^{11}$K kT$\sim$10 MeV$\gg$ 2$m_e c^2$

Neutrinos and electrons/positrons are kept in chemical equilibrium by neutral- and charged-current interactions:

The neutral current reaction (Z_o exchange) has the same strength for all three neutrino flavors (ν_e, ν_μ, ν_τ). The charged-current reaction involves only electron-family particles: the analogous reactions involve muon-family particles does not occur because of the temperature. The mass of two muons exceeds 200 MeV, and thus cannot be produced at this time.

There are similar charge-changing reactions coupling n $\leftrightarrow$ p. As we will be discussing weak interactions quite a bit (they are involved in most of the stellar reactions of interest to us), let's say a bit more at this time about the rules for charged current reactions. The reactions p $\leftrightarrow$ n are

$$\mathrm{p} + \mathrm{e}^- \leftrightarrow \mathrm{n} + \nu_e$$
$$\mathrm{n} + \mathrm{e}^+ \leftrightarrow \mathrm{p} + \bar{\nu}_e$$
$$\mathrm{p} \leftrightarrow \mathrm{n} + \mathrm{e}^+ + \nu_e$$

$$\mathrm{n} \leftrightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e$$

We will treat the neutrinos as Dirac particles (that is, a particle with a distinct antiparticle). The rules for the reactions above are then simple. First, charge is conserved, which means the sum of the charges going into a reaction are equal to the sum of the outgoing charges. Second, there is a second additively-conserved charge, lepton number. The lepton numbers are defined as follows

particle	*l*
e^+	-1
e^-	$+1$
ν_e	$+1$
$\bar{\nu}_e$	-1

The n and p can be considered two possible states in which to find the nucleon; these states have different masses, m(n) = 939.566 MeV and m(p) = 938.272 MeV. A two state system in thermal and chemical equilibrium - weak interactions n $\leftrightarrow$ p are the mechanism for maintaining chemical equilibrium - has occupation probabilities

$$\mathrm{occup} \propto \mathrm{g_i e^{-E_i/kT}}$$

where g_i is the number of states at energy E_i (the two spin states in this case). Thus

$$\frac{\mathrm{n}}{\mathrm{p}} = \frac{e^{-m_n/kT}}{e^{-m_p/kT}} = e^{-\Delta m/kT}$$

where Δm = 1.294 MeV. At $T = 10^{11}$K, kT=8.62 MeV. Thus at this temperature,

$$\frac{\mathrm{n}}{\mathrm{p}} = 0.86$$

Although this temperature is far above the temperature of nucleosynthesis, the n/p ratio has already begun to drop.

B) t $\sim$ 0.1 sec $T \sim 3 \cdot 10^{10}$ K $\sim$ 3 MeV

Somewhat before this temperature is reached the ν_μ and ν_τ have fallen out of equilibrium. This occurs because the rate for interactions with electrons is too slow to keep up with the rate of expansion of the universe: we will do an example below. Note that the muon and tauon flavor reactions off electrons/positrons are only about 1/7 as strong as for electron neutrinos: that is why these "heavy flavors" decouple first.

At around 3MeV the ν_e also decouple.

We will now explore this question of "falling out of equilibrium" in the context of the n $\leftrightarrow$ p reactions to see whether nucleons are still in equilibrium. To answer this question we need to know:

• What is the time scale for n $\leftrightarrow$ p reactions?

• What is the time scale describing the expansion of the universe that forms the comparison scale?

The time scale for the n $\leftrightarrow$ p reactions can be posed as that for a neutron in our thermal bath to convert to a proton via n + $\nu_e \leftrightarrow$ p + e$^-$. That rate is

$$\Lambda(T) \sim \langle \sigma v \rangle n_\nu(t)$$

were n_ν is the electron neutrino number density and v is the relative velocity of the neutrino and neutron, which we can take as c=1. Now n_ν has the dimensions of 1/cm^3 and v of cm/sec, so the product carries the dimensions of flux, 1/cm^{2}sec. The cross section is roughly

$$\sigma \sim G_F^2 E_\nu^2 \sim G_F^2 (kT)^2$$

What about n_ν? This is our previous argument that the number of states/unit volume for relativistic particles should go like (kT)^3. Thus we conclude

$$\Lambda(T) \sim (kT)^5 G_F^2$$

As discussed in class G_F has the dimensions of $1/\text{mass}^2$, as it represents the exchange of a W boson: so there is a coupling constant on each end and a propagator that goes like $1/M_W^2$ (since all momenta of the scattering particles are much, much less than M_W). So we can look up the numerical value in old papers that measured the strength of β decay of the neutron. A easy way to remember the approximate result is

$$G_F \sim \frac{10^{-5}}{M_N^2}$$

where M_N is the nucleon mass. Thus

$$\Lambda(T) \sim (kT)^5 \frac{10^{-10}}{M_N^4}$$

Note that this has the units of mass (we are free to insert c^2 with each factor of M_N). But Λ must have the dimensions of 1/sec, as it is a rate. So setting

$$M_N \sim 10^3 \text{MeV}$$

and inserting one factor of

$$\frac{1}{\hbar} = \frac{1}{6.6 \cdot 10^{-22} \text{MeVsec}}$$

to provide the needed units leads to

$$\Lambda(T) \sim \frac{0.15}{\text{sec}} (\frac{kT}{\text{MeV}})^5.$$

At 3 $\cdot 10^{10}$ K this gives a rate of 17/sec. That is, the typical lifetime of a neutron in such a thermal bath is about 0.06 sec. Of course, we dropped all sorts of πs and 2's in this calculation. Had we done things more carefully the result would be (see Weinberg's book)

$$\Lambda(T) \sim 0.76/\text{sec}(\frac{\text{kT}}{\text{MeV}})^5$$

So this would give a neutron lifetime at $3 \cdot 10^{10}$K of about 0.011 sec.

Let's pause here for a remark. Even though we haven't yet discussed to what we will compare this rate, it should be clear that weak rates evolve VERY rapidly in the early universe, dropping as T^5. We don't have any other quantity (t, R, etc) that changes so fast. So clearly at some point we will reach a T where weak rates are so slow that they can't keep up with the other changes in the universe. Furthermore, the transition (range of T) over which this "freezeout" occurs should be rather narrow, since the functional dependence on T is steep.

The first guess for a comparison timescale is the age of the universe, which at this epoch is approximately

$$\tau_{universe} \sim 0.81 \mathrm{sec} (\frac{\mathrm{MeV}}{\mathrm{kT}})^2$$

or about 0.12 sec. So this is $\sim$ 10 times longer than our neutron lifetime: we conclude the weak rates are easily keeping the chemical equilibrium between neutrons and protons at this time.

Actually a better comparison rate is the Hubble parameter, which clearly has dimensions of 1/sec and which clearly does describe the rate of change of the universe at any given time. It is given by

$$\sqrt{\frac{8\pi G\rho(t)}{3}}$$

and

$$\rho(T) = N\frac{\pi^2}{30}(kT)^4 \sim 3.5(kT)^4$$

So

$$\mathrm{Hubble\ rate} \sim 5.4(\mathrm{kT})^2\sqrt{\mathrm{G}}$$

The value of G in MeV units is $6.7 \cdot 10^{-45}\mathrm{MeV}^{-2}$. Thus, inserting the needed $\hbar$ we have

$$\mathrm{Hubble\ rate} \sim 4.4 \cdot 10^{-22} \langle\frac{\mathrm{kT}}{\mathrm{MeV}}\rangle^2 \frac{\mathrm{MeV}}{\hbar} \sim \frac{0.67}{\mathrm{sec}}\langle\frac{\mathrm{kT}}{\mathrm{MeV}}\rangle^2$$

We see this is close to our more naive guess based on

$$\frac{1}{\tau_{universe}} \sim \frac{1.23}{\text{sec}} \langle \frac{kT}{MeV} \rangle^2$$

C) Epoch of decoupling

As we have these nice formulas, let's cut to the chase and find the temperature characterizing the epoch of decoupling. This should be when the neutron lifetime and the Hubble rate are comparable. From what we have done above, this is easy:

$$\frac{0.67}{\text{sec}} \langle \frac{kT}{\text{MeV}} \rangle^2 \sim \frac{0.76}{\text{sec}} \langle \frac{kT}{\text{MeV}} \rangle^5$$

which yields

$$kT = 0.96\text{MeV}$$

So that's a nice round number to remember ($\sim$ 1 MeV). Now that we have the temperature at which the n $\leftrightarrow$ p system breaks out of chemical equilibrium, we can evaluate the corresponding n/p ratio at this point:

$$\frac{n}{p} \sim e^{\Delta m/kT} \sim 0.25$$

Note this is not 1/7, but it is also not too far from it. It is also "on the right side": we expect it too continue to drop. Why? First, we've just calculated the point where the Hubble rate and weak rates are comparable. As the temperature drops a bit, the ν-induced reactions continue to push things toward the proton-rich side. Indeed, if we look in Weinberg's book, in the next 10 seconds (1 MeV is about 1 second after the big bang) the n/p ratio will drop to about .17 $\sim$ 1/6. And after that there continues to be a slow decrease in the neutron percentage because of neutron β decay - but this has the timescale of 10 minutes.

So the next issue is to figure out when the nuclei form; and the answer better be in less than 10 minutes, since we can't tolerate much β decay and still get 1/7 for the n/p ratio. The nucleon gas in the early universe is relatively dilute, with the consequence that nuclei must be made by two-body reactions. The nucleon force has a range of only a few fermis ($1\mathrm{f} = 10^{-13}$ cm), and the chance of getting three or more fusing nuclei within this radius is negligible.

The only two-nucleon bound state is the deuteron, with a binding energy of 2.24 MeV. But 2.24 MeV! We just found out that n $\leftrightarrow$ p chemical equilibrium persisted to about 1 MeV. With a 2.24 MeV binding energy, wouldn't all the neutrons want to be in deuterium at or before that time? (therefore freezing the n/p ratio at some value above 1/4)

The answer is no, for reasons having to do with the nucleon to photon ratio. For reasons we don't fully understand, when the very early universe cooled, it left over a net baryon number. That is, we have neutrons and protons in our universe, not antineutrons and antiprotons. Presumably at very early times there were approximately equal numbers of quarks and antiquarks, but not precisely equal numbers. As the universe cools, quarks and antiquarks (nucleons and antinucleons) annihilated each other. At the end - for reasons connected with baryon number violation, CP violation, and nonequilibrium physics - a small residual baryon number excess remained. The resulting baryon/photon number-density ratio is (deduced from nucleosynthesis arguments we are about to describe)

$$\frac{n_N}{n_\gamma} = \eta \sim 10^{-9} - 10^{-10}$$

So consider the two possible states of n+p

$$n + p \leftrightarrow d + \gamma$$

This reaction is electromagnetic and therefore fast, so we can reasonably assume it is in equilibrium in the early universe. The equilibrium condition is

$$(\text{no. of n + p}) \times |< \mathrm{d}\gamma|\mathrm{T}|\mathrm{np} >|^2 \sim (\text{no. of d} + \gamma) \times |< \mathrm{np}|\mathrm{T}|\mathrm{d}\gamma >|^2$$

Now detailed balance tells us that the transition probabilities are equal and cancel. The phase space on the left-hand size is simple: any n+p pair can form a deuteron, emitting a γ. Thus the number of such pairs per unit volume V is

$$V n_n n_p$$

But on the right-hand side only those photons with enough energy to break up a deuteron can initiate reactions – an energy greater than $\Delta m_d = 2.24$ MeV. In a thermal gas photons have a Bose-Einstein distribution

$$n_\gamma = \int_0^\infty \frac{8\pi\epsilon^2 d\epsilon}{e^{\epsilon/kT} - 1}$$

Very crudely, then, if kT is small compared to Δm_d, the number of photons participating in the reaction above is

$$n_\gamma^{eff} \sim \int_{\Delta m_d}^\infty 8\pi\epsilon^2 exp^{-\epsilon/kT} d\epsilon$$

Again very crudely this is about

$$\sim e^{-\Delta m_d/kT} n_\gamma$$

(We will do this calculation correctly in a couple of lectures, using the Saha equation to describe the relative abundances of nuclear species. The point of the current approach is to emphasize that the low temperature of deuterium formation is due to the very large number of photons relative to nucleons.) This then yields

$$n_p n_n \sim n_d n_\gamma e^{-\Delta m_d/kT}$$

Now $n_p/n_\gamma \sim \eta$. The time when deuterium forms can be defined as the time when $n_n \sim n_d$, that is, when half of the neutrons are in deuterium nuclei. It follows that the epoch of nucleosynthesis is given by

$$e^{-2.24\mathrm{MeV/kT}} \sim \eta$$

This equation relates two unknowns, η and the temperature $T = T_d$ for deuterium formation. If one has an independent measurement of η, then this determines T_d and thus the temperature/time at which deuterium forms. This formation triggers rapid formation of other nuclei (mostly ^{4}He), and thus leads to a characteristic pattern of light big-bang nuclei, as we discuss below. Today we have such an independent measurement of η, from fluctuations in the cosmic microwave background radiation temperature (discussed briefly in the next chapter).

Conversely, if one has some way to determine T_d, one can then determine an important cosmological parameter η. This we can do, and is the reason for the importance of BBN. Basically the abundance of ^{4}He – the fact that n/p = 1/7 at T_d – determines T_d.

Unfortunately to do this well requires a computer because the weak interactions that govern the n/p ratio have fallen out of equilibrium. The last simple time was at $T \sim 1$ MeV and $t \sim 1$ sec – when things fell out of weak equilibrium. Again, we view the n/p system as a two-state problem, with the p state being energetically prefered. After weak equilibrium is broken there still are some reactions that transform protons into neutrons: they just operate slowly compared to the Hubble time. And there are reactions that push neutrons into protons. In fact it should be clear that two of these

$$\nu_e + n \rightarrow p + e^-$$

$$n \rightarrow p + e^- + \bar{\nu}_e$$

are likely the most important rates after falling out of weak equilibrium. Both reactions run "downhill" – to the lower energy state. Thus the first reaction can be induced by very low energy ν_es. The second is free decay.

With a bit of work to get these cross sections (plus the neutron-producing ones) and to code them up, we could follow n/p evolution with increasing T until He forms, for a series of η values. The game would be to find that η that gives us the amount of ^{4}He we need to ensure n/p=1/7. This value would be characterized by some (correct) T_d. Wish we had the time! Instead, let's consider the second reaction above, as we know the rate of neutron decay, which will overestimate the time to deuterium formation – but we will keep this in mind.

To reduce n/p = 0.25 to 1/7 requires that the neutron number be reduced to 0.62 of its starting value. As the neutron half life is about 10.4 minutes. Thus the required decay time is given by

$$0.62 = exp(t\ ln2/10.4\text{min})$$

yielding $t \sim 7.1$ minutes. And had we done the correct calculation instead of this partial one, the answer would have been $t \sim 3.0$ minutes, or $T_d \sim 1.1 \times 10^9$K. This is an energy of about 95 keV, *much* below Δm_d.

So we then find

$$\eta \sim e^{-\Delta m_d/0.095MeV} \sim 10^{-10}$$

In fact, the pros do a much more elegant analysis, calculating not only ^{4}He (the end of the chain discussed below) but also the minor productions of d, ^{3}He, and ^{7}Li. A careful fit to the all the observationally deduced abundances gives the best value for η.

The long wait from $T \sim \Delta m_d$ to deuterium formation is called the deuterium bottleneck. Only when the temperature drops to $\sim$ 100 keV does the number of effective photons – those with enough energy to break up deuterium – drop to a low enough level to allow deuterium to exist at an appreciable abundance.

It is clear that more baryons – the larger η – the higher T_d. Alternatively one can state:

the smaller the baryon number of the universe the later the epoch of nucleosynthesis

Figure 4: The predictions of Big Bang nucleosynthesis for light element abundances: ^{4}He, D, ^{3}He, and ^{7}Li as a function of η, the baryon-to-photon number-density ratio. Because these elements trend in different ways, observations that allow one to deduce primordial abundances then determine η. This is traditionally what has been done in BBN simulations. However now there is an independent, even more accurate determination of η: WMAP, a probe of the cosmic microwave background. (This will be discussed in Chapter 2.) Thus there is now a parameter-free BBN prediction of abundances that can be compared to observation. From Cyburt et al., astro-ph/0302431/.

Figure 5: Here the WMAP results are used to determine η, which then produces BBN predictions (no free parameter!) that can be compared to observations. In the upper right good agreement is obtained with one deuterium abundance determination, reasonable agreement with another (the dashed line). WMAP prefers larger values for ^{4}He (Y_p) and for ^{7}Li, relative to observation. The Li tension is significant. From Cyburt.

Figure 6: This figure shows the scatter of recent deuterium abundance determinations. The new method that produced these data – a determination of deuterium abundances in gas clouds believed to contain almost primordial material - will be discussed in the next chapter. From Steigman, astro-ph/0309347/.

Figure 7: The WMAP CMB power spectrum fit with different values of η. (ω_B is proportional to η.) Clearly 20% variations in η destroy any agreement with the WMAP results. WMAP determines η to a precision of about $\pm$ 4%. From Steigman.

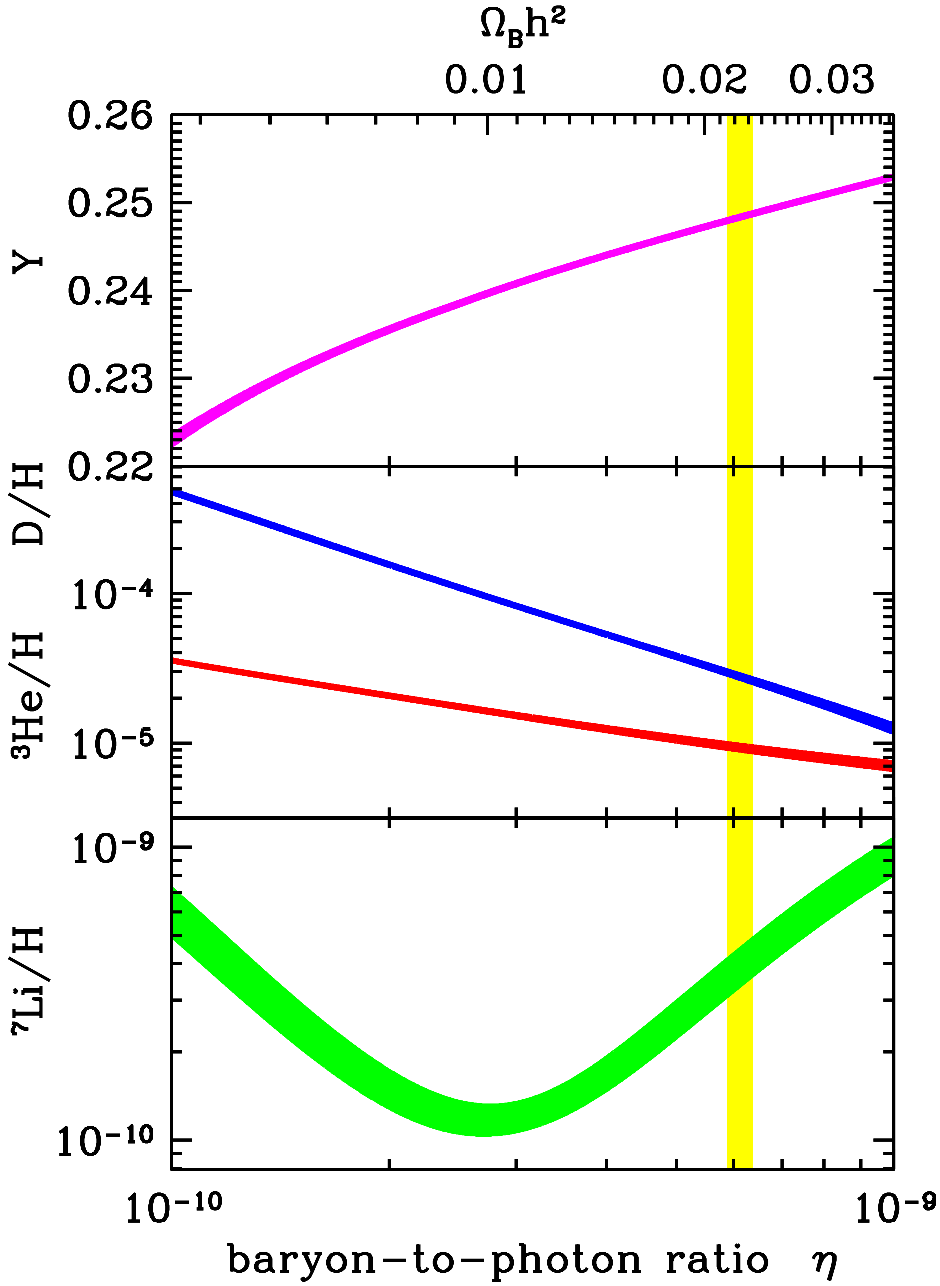

Figure: Abundance predictions for standard BBN [12]; the width of the curves give the $1-\sigma$ error range. The WMAP η range (eq. 1) is shown in the vertical (yellow) band.

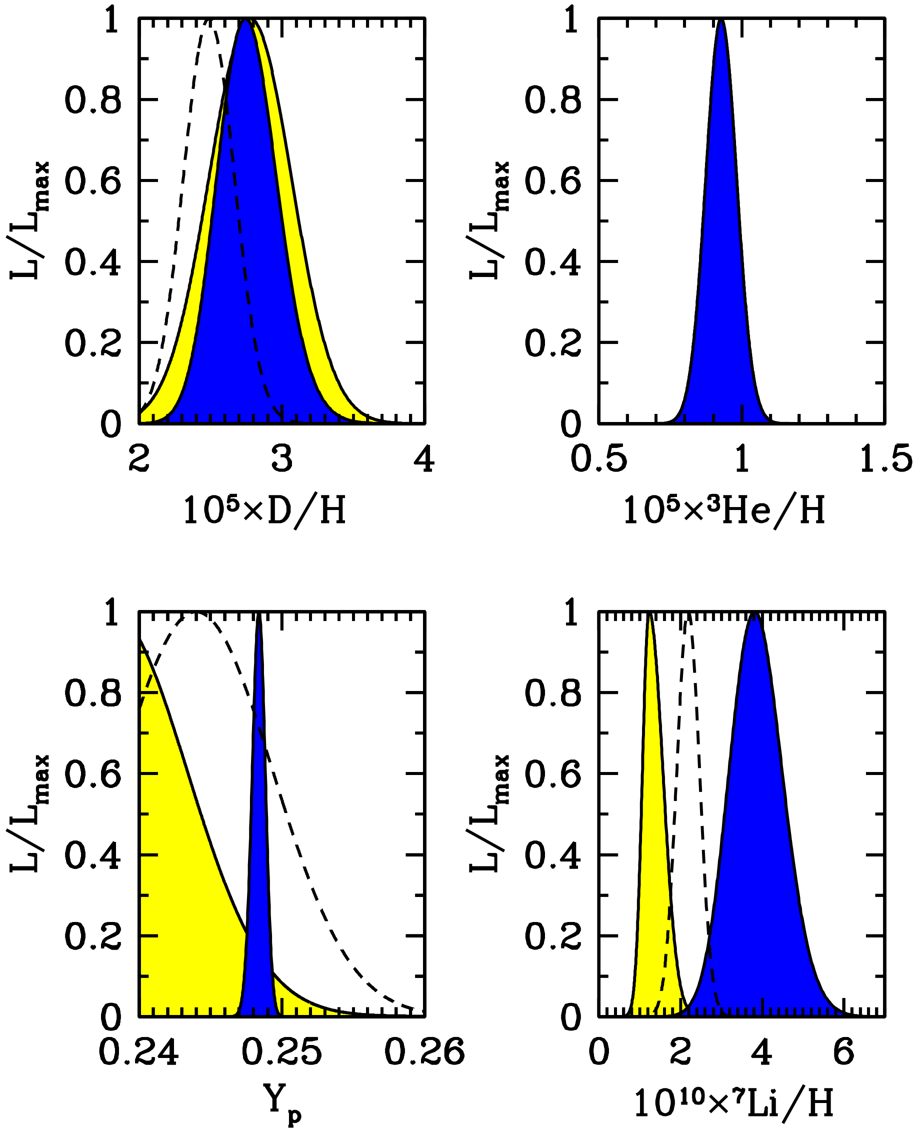

Figure: Primordial light element abundances as predicted by BBN and WMAP (dark shaded regions). Different observational assessments of primordial abundances are plotted as follows: **(a)** the light shaded region shows $D/H = (2.78 \pm 0.29) \times 10^{-5}$ [20]-[23], while the dashed curve shows $D/H = (2.49 \pm 0.18) \times 10^{-5}$ [21, 22]; **(b)** no observations plotted **(c)** the light shaded region shows $Y_p = 0.238 \pm 0.002 \pm 0.005$ [25], while the dashed curve shows $Y_p = 0.244 \pm 0.002 \pm 0.005$ [26]; **(d)** the light shaded region shows $^7Li/H = 1.23^{+0.34}_{-0.16} \times 10^{-10}$ [27], while the dashed curve shows $^7Li/H = (2.19 \pm 0.28) \times 10^{-10}$ [28].

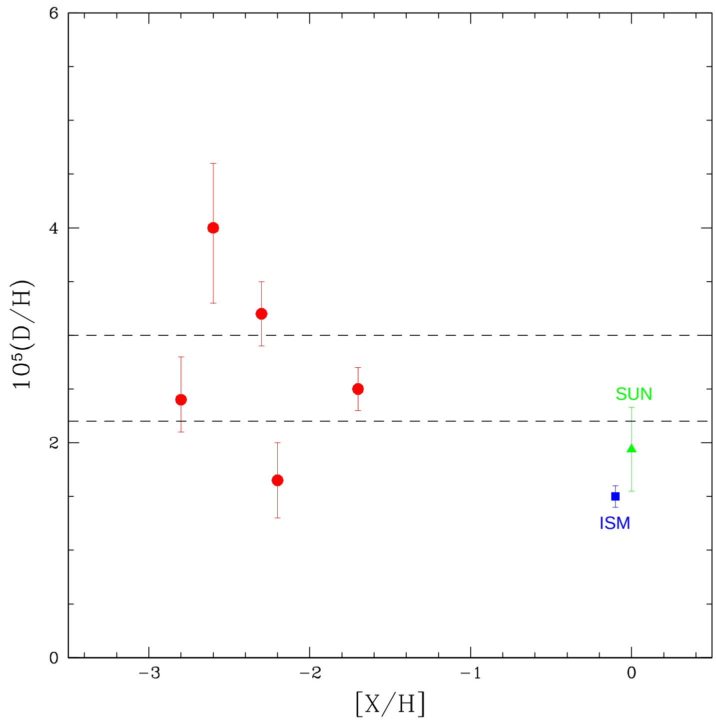

Figure: The deuterium abundance by number with respect to hydrogen versus the metallicity (relative to solar on a log scale), from observations (as of early 2003) of QSOALS (filled circles). Also shown for comparison are the D abundances for the local ISM (filled square) and the solar system ("Sun"; filled triangle). The dashed horizontal lines represent the range of the $\pm1\sigma$ estimate for the primordial deuterium abundance ($y_{\rm D} = 2.6 \pm 0.4$) based on the QSOALS data.

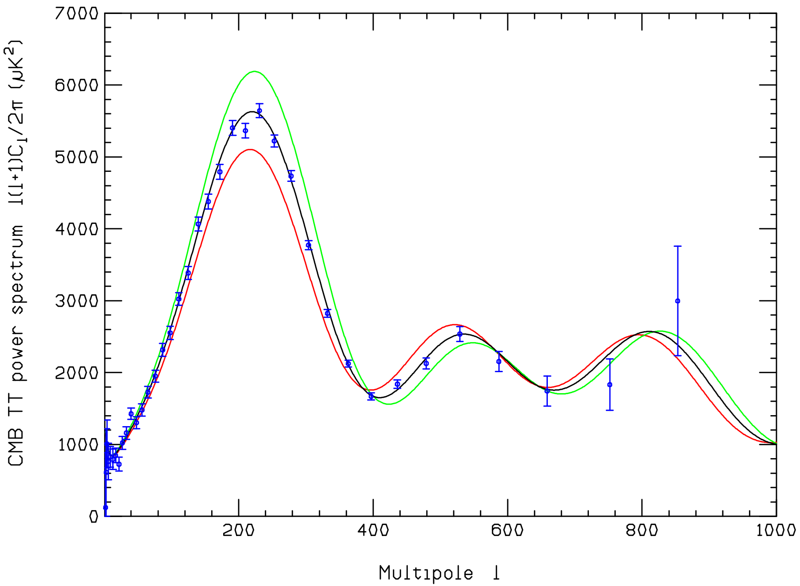

Figure: The CBR temperature fluctuation anisotropy spectra for three choices of the baryon density parameter $\omega_{\rm B} = 0.018,\ 0.023,\ 0.028$, in order of increasing height of the first peak. The WMAP data points [2] are shown.

The rest of the story involves the subsequent reaction network. One can look at the various steps:

• $n+p \rightarrow d + \gamma \qquad n/p \sim 1/7$

• $d(n,\gamma)^3H$ and $d(d,p)^3H$

$d(p,\gamma)^3He$ and $d(d,n)^3He$

$^3He(n,p)^3H$ (at long times ^3H β decays to ^3He)

• $^3H(p,\gamma)^4He$ and $^3H(d,n)^4He$

$^3He(n,\gamma)^4He$ and $^3He(d,p)^4He$ and $^3He(^3He,2p)^4He$

Note: center of mass energies 100 kev $\Rightarrow$

Coulomb suppression effects

• trace amounts of ^7Li,^7Be

$^4He(^3H,\gamma)^7Li$ and $^4He(^3He,\gamma)^7Be$

(^7Be later decays to ^7Li, but only when it becomes an atom

that is, after recombination: interesting story here)

• the usual explanation for the termination of the reaction network with the species mentioned above is that there are no stable nuclei with A=5 and A=8. Thus the obvious potential reactions for going further, $^4He+^4He$ and ^4He+p, are ineffective. Actually, this common explanation is not quite true. If one cheats and makes up stable isotopes at these mass numbers with modest binding energies, the chain still largely terminates as above. The reason is that the Coulomb barriers at these low temperatures (100 keV) become increasingly hard to penetrate as Z increases.

The repulsive Coulomb barrier near a nucleus is

$$\alpha \frac{Z_1 Z_2}{r} \sim 1.44 Z_1 Z_2 \frac{1fm}{r}$$

So if $Z_1 = Z_2 = 2$ and $r \sim 2fm$, this is about 3 MeV. So it is very hard for charged particles with kinetic energies of $\sim$ 100 keV to tunnel through the Coulomb barrier to the regions where the strong nuclear force can bind the reacting particles to form a new nucleus.

•Once the bottleneck to forming deuterium is broken, the rest of the network described above proceeds quickly to produce ^{4}He.

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Now some comments about systematics:

1) The larger η, we showed the larger $T_{nucleosynthesis}$, and thus the larger n/p ratio. Therefore larger η lead to larger ^{4}He abundance.

2) One can think of d, ^{3}He as "catalysts" in the network: they are produced and then consumed, and thus reach an equilibrium value that depends on the competition between production and consumption. What happens is $T_{nucleosynthesis}$ is increased? The production channel is effectively n+p, where there is no Coulomb barrier. Destruction channels include reactions like d+d,d+p, etc, which are Coulomb inhibited. So increasing the T effects the destruction channels more, since Coulomb barrier penetration is exponential, enhancing the destruction. The conclusion is that higher $T_{nucleosynthesis}$ should produce lower d, ^{3}He.

3) For low η (and therefore low T) ^{7}Li is made and destroyed by:

$$^4He(^3H,\gamma)^7Li \qquad ^7Li(p,\alpha)^4He$$

The second reaction has the more effective Coulomb barriers (effecting both initial and final states). Thus a lower T will more effectively turn off the destruction of ^{7}Li than its production. Thus a lower T (with low η) means more ^{7}Li.

But it turns out that for high η, the primary way to make ^{7}Li changes. High η means more ^{3}He, as we have noted, so

$$^4He(^3He,\gamma)^7Be$$

becomes important. This production clearly benefits by high T because of the Coulomb barrier. Furthermore there are very few neutrons around to kill this isotope by (n,α). Thus, ^{7}Li produced as ^{7}Be begins to turn up again at high T and high η. This physics is discussed in some detail in Kolb and Turner.

Another interesting issue is the dependence on number of relativistic species: adding another neutrino species increases the energy density and therefore the Hubble rate. Therefore weak interactions fall out of equilibrium earlier, when the n/p ratio is higher. It follows that this forces ^{4}He production upward. And conversely...