Strong shielding due to an electromagnetically thin metal sheet

P. Rochon and N. Gauthier
Department of Physics, Royal Military College of Canada, Kingston, Ontario K7K 5L0, Canada

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Under certain conditions, "electromagnetically thin" metal shields may, somewhat paradoxically, be used to prevent the penetration of alternating fields inside a Faraday cage. By definition, a metal sheet of thickness d is said to be electromagnetically thin if, for the frequency of interest, the skin depth[1] $\delta = (2/\sigma\mu_0\omega)^{1/2}$ is much larger than d (SI units are used throughout), where σ = conductivity of the metal, ω = angular frequency of the ac fields, and μ_0 = vacuum permeability. This surprising effect is not very well known,[2,3] even though, as pointed out recently by Fahy *et al.*,[1] it is easily derived from Faraday's and Ampere's laws. These authors considered the case of a thin cylindrical shell of thickness d and inner radius R, placed coaxially in the field of a long solenoid. They showed that "the interior of the tube is effectively screened from the applied field when $d > d_0$, where $d_0 = \delta^2/R$...," provided that the following conditions are met: $d \ll \delta \ll R$. As stated by Fahy *et al.*,[2] general expressions for the fields in this situation were originally worked out by King in 1933. However, King did not consider the limit of these expressions in the case where $d \ll \delta \ll R$.

Stimulated by the analysis of Fahy *et al.*[1] we decided to test the effect in the laboratory. We also decided to extend these authors' considerations to the simpler case of two parallel flat sheets. The present note reports observations of the Fahy-Kittel-Louie (FKL) effect in the cylindrical and flat geometries. The experiment is easy to perform and could be of interest at the intermediate level of a physics laboratory.

Our results for the cylindrical case are presented in curves (a) and (b) of Fig. 1 (log-log plot). The frequency range 1-100 kHz was investigated; the vertical axis gives the absolute value of the ratio of the magnetic fields inside (i) and outside (o) the metal cylinder. The solenoid used to produce the magnetic field was constructed by winding N = 100 turns of 24-gauge wire on a hollow bakelite cylindrical core of outer radius R = 8.5 cm; the solenoid length was L = 23 cm. The current through the solenoid was supplied by a Wavetek function generator. The field near the center of the solenoid was measured by a small 300-turn 40-gauge wire pickup coil mounted on the tip of a long wooden needle. The twisted leads from the pickup coil were shielded to prevent unwanted signals. The signals were viewed on an oscilloscope and measured using a digital voltmeter.

The field on the solenoid axis was measured without the shield first, then with the shield inserted. The interior field probe could also be easily moved from the region inside to that outside the shield to obtain the field ratio. The sample characteristics were $R = 0.850 \pm 0.005$ cm for the aluminum shield and $R = 1.060 \pm 0.002$ cm for the copper shield; both samples had thickness $d = 0.080 \pm 0.002$ cm. According to the FKL theory, the logarithm of the field ratio is simply given by

$$\ln|H_i/H_o| = \ln(2/\sigma\mu_0 Rd) - \ln\omega. \qquad (1)$$

This result is only useful if the conditions $2Rd \gg \delta^2$ and $\sqrt{2}d \ll \delta$ are valid. Otherwise, the full theory[1] must be used. Curves (a) and (b) show systematic deviations away from the prediction of Eq. (1) when $\delta \lesssim d$, at higher frequencies. This deviation starts to be important at about 20 kHz in both cases. Similarly, at very low frequencies, δ becomes comparable to or larger than R, $\delta \gtrsim R$, and Eq. (1) again breaks down. A linear least-squares fit of our data in the frequency range 4-20 kHz gives a slope of -1.02 ± 0.03 for both cases, in excellent agreement with Eq. (1). From the results, we estimate the sample conductivies to be

$$\sigma(\mathrm{Al}) = (3.25 \pm 0.06) \times 10^7\ \mathrm{S \cdot m^{-1}}$$

and

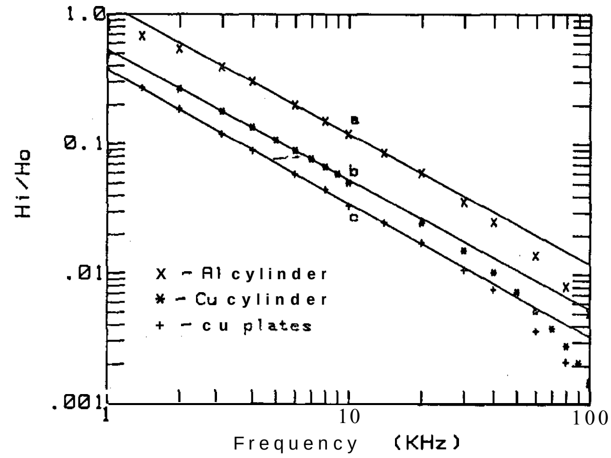

Fig. 1. Log-log plot of the magnitude of the field ratio versus the frequency (kHz). The points represent measured values: (x) for the Al cylinder; (*) for the Cu cylinder; (+) for the Cu plates. The solid lines represent the best straight-line lit to the data, which in each case is based on the frequency range from 4 to 20 kHz.

$\sigma(\text{Cu}) = (5.58 \pm 0.08) \times 10^7\ \text{S}\cdot\text{m}^{-1}$.

2)

An extension of the FKL analysis to the case of two flat parallel sheets, each of thickness d, conductivity σ, and separated by a distance $2R$, is easily done. Let the x axis be perpendicular to the plane of the plates, with the alternating magnetic field pointing parallel to the plates along the y axis. Then the electric field is parallel to the z axis of coordinates and the laws of Faraday, Ampère, and Ohm give the equations

$$i\omega\mu_0 H_l = -\frac{\partial}{\partial x} E_l; \quad \sigma_l E_l = \frac{\partial}{\partial x} H_l, \tag{2}$$

subject to the usual boundary conditions at $x = \pm R$ and $\pm (R+d)$. In Eq. (2), $l=1$ for the inside region $(-R<x<+R)$, $l=2$ for the metal regions $(-R-d<x<-R;\ R<x<R+d)$, and $l=3$ for the outside regions $(|x|>R+d)$. The displacement term is neglected for the small frequencies of interest here, and so, $\sigma_1 = \sigma_3 = 0$, $\sigma_2 = \sigma$. By symmetry, $E_l(-x) = -E_l(x)$ and $H_l(-x) = H_l(x)$ so that, with $K \equiv \sqrt{2i}/\delta$ and A, B, H_o, and H_i constants, we have

$$H_1 = H_i; \quad E_1 = i\omega\mu_0 x H_i; \tag{3}$$

$$H_2 = Ae^{Kx} + Be^{-Kx}; \quad E_2 = (K/\sigma)(Ae^{Kx} - Be^{-Kx}); \tag{4}$$

$$H_3 = H_o; \quad E_3 = E_2(R+d) + i\mu_0\omega(R+d-x)H_o. \tag{5}$$

By definition, H_o is the field at the sample surface and H_i is the interior field. The three independent boundary conditions give, with $X = R + d$,

$$\begin{aligned} &-H_i + Ae^{KR} + Be^{-KR} = 0, \\ &Ae^{KX} + Be^{-KX} = H_o, \\ &-H_i + A(e^{KR}/KR) - B(e^{-KR}/KR) = 0; \end{aligned} \tag{6}$$

this system is readily solved for the required field ratio,

$$H_i/H_o = [\cosh(Kd) + KR \sinh(Kd)]^{-1}. \tag{7}$$

In the limits $|Kd| \ll 1$, $|K|^2 Rd \gg 1$, we then obtain the desired ratio:

$$\ln|H_i/H_o| \simeq -\ln(\sigma\mu_0 Rd) - \ln\omega. \tag{8}$$

Curve (c) of Fig. 1 shows our results for a copper-plate system ($d = 0.051 \pm 0.001$ cm, $R = 1.215 \pm 0.005$ cm; plate width = 6 cm; plate length = 8 cm). The same solenoid was used for this sample and for the earlier cylindrical ones. The results are linear over a wider range of frequencies than for the cylindrical cases because R is larger and d is smaller in the present situation. (Good electrical contact must be provided at the edges of the plates to ensure a closed circuit for the current flow. The contact resistance should be much smaller than the resistance of the sheet.) The conductivity was estimated to be $(5.49 \pm 0.08) \times 10^7\ \text{S}\cdot\text{m}^{-1}$ for this case. Again, deviations away from linearity are seen to occur because Eq. (8) is only valid for intermediate frequencies. Outside this range, Eq. (7) should be used to describe the results.

[1]P. Lorrain, D. R. Corson, and F. Lorrain, *Electromagnetic Fields and Waves* (Freeman, New York, 1988), 3rd ed.

[2]S. Fahy, C. Kittel, and S. G. Louie, "Electromagnetic screening by metals," Am. J. Phys. **56**, 989–992 (1988).

[3]G. Warner and R. Anderson, "The shielding effect of a conducting tube," Int. J. Elect. Eng. Educ. **18**, 231–245 (1981).