Experiments with electrical resistive networks

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Theoretical problems involving equivalent resistances of large or infinite networks of resistors have received substantial attention. We consider two *actual* networks. In the first, the resistance is measured across one end of a ladder whose number of loops is incremented until the precision of the multimeter is exceeded. In the second, resistances are measured across nodes near the center of a 12 by 12 square grid of resistors. These experiments are useful in the introductory physics laboratory as interesting examples of equivalent resistance, and can be added to a standard Ohm's law experiment. The square grid apparatus also can be employed for lecture demonstrations. In addition, this apparatus offers approximate experimental confirmation of complicated theoretical calculations for the equivalent resistance between two nonadjacent nodes of an infinite square grid. These experimental results are verified numerically.

I. INTRODUCTION

Problems involving large or infinite electrical resistive networks are interesting and educational.[1,2] Applications include geophysical exploration with electrical currents, petroleum flow in oil wells, and random walks.[3] Although there is a substantial amount of theoretical work in the educational literature,[1,2,4–9] we have not found any experimental investigations. Our main purpose in this article is to show that interesting resistive networks can be incorporated into Ohm's law experiments in the educational laboratory.

Two resistive networks are considered. The first is the ladder shown in Fig. 1(a), where we investigate the convergence of the equivalent resistance across the terminals to the limiting value corresponding to the infinite ladder. We show that this limit can be experimentally probed to a high degree of precision as a result of an approximate exponential convergence, which eliminates both the need for a large number of resistors and the effect of variations of the individual resistances.

The second resistive network we consider is the two-dimensional square grid shown in Fig. 2. The problem of the equivalent resistance across two *adjacent* nodes in an *infinite* square grid of identical resistors is well-known.[1,6] The solution can be simply obtained by superposing two current distributions: one in which current i is injected at one node and removed at infinity, and the other in which current i is injected at infinity and removed at a node adjacent to the original node. In the first case, the symmetry implies that current $i/4$ flows in each of the four segments emanating from the injection point. Similarly, in the second case, current $i/4$ flows in each of the four segments terminating at the removal node. The superposition of these two distributions yields net current $i/2$ through the segment between the two nodes. The equivalent resistance, which is the potential difference divided by the injected or removed current, is thus $R_0/2$, where R_0 is the resistance of each resistor.

This argument cannot be extended beyond the equivalent resistance across two adjacent points: When current is injected at one node and removed at infinity, the branching ratio of the current at a nearest-neighbor node cannot be determined by symmetry. The problem was solved by van der Pol and Bremmer,[7] who analytically performed difficult integrations to find definite numbers, although these authors do not explain how these integrations were accomplished. Lavatelli[8] generalized the results to finite grids, although the results are in terms of integrals. Venezian,[9] who was evidently unaware of any previous work, employed a different approach to the infinite grid problem, and arrived at integrals that do not appear to be equivalent to those of van der Pol and Bremmer. However, a comparison shows that Venezian's numerical integrations agree with van der Pol and Bremmer's analytical results. Venezian's method was recently illuminated and generalized to other lattices by Atkinson and van Steenwijk,[10] who performed algebraic integrations with MATHEMATICA. The results are equivalent to those of van der Pol and Bremmer. One of our motivations is to experimentally and computationally confirm the theoretical results. Although these results are supported by independent calculations, such confirmations are appropriate due to the difficulty of the problem and the usefulness in the educational laboratory.

II. LADDER THEORY

Figure 1(a) shows an electrical resistive ladder composed of n loops of resistors each of resistance R_0. The equivalent resistance R_{n+1} across the terminals of the ladder with $n+1$ loops can be determined[1,2] in terms of the equivalent resistance R_n of the ladder with n loops by considering Fig. 1(b). The result is $R_{n+1}=R_0+R_0R_n/(R_0+R_n)$, or

$$R_{n+1}=\frac{R_0+2R_n}{R_0+R_n}R_0, \tag{1}$$

for $n=1,2,3,...,$ where $R_1=2R_0$. The equivalent resistance of the infinite ladder network is then determined by setting $R_{n+1}=R_n=R_\infty$ in Eq. (1), and solving for R_∞. The result is

$$\frac{R_\infty}{R_0}=\frac{1+\sqrt{5}}{2}. \tag{2}$$

The resistive ladder is useful as an educational laboratory experiment to verify the progression of $R_1,R_2,R_3,\ldots,$ to the limiting value R_∞. As listed in the third column of Table I,

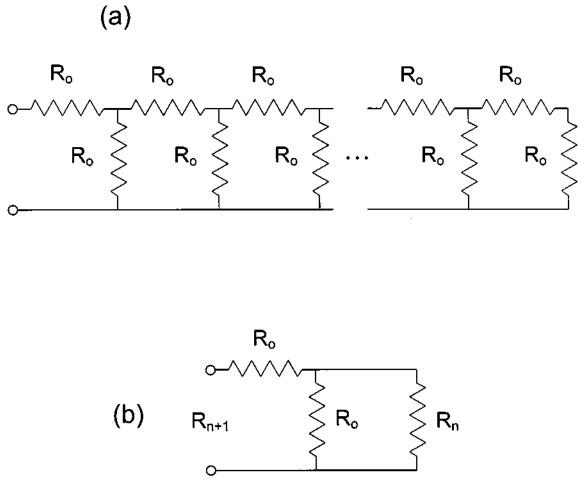

Fig. 1. (a) Resistive ladder network consisting of n loops of pairs of identical resistors of resistance R_0. (b) Diagram for determining the equivalent resistance R_{n+1} of $n+1$ loops in terms of the equivalent resistance R_n of n loops.

calculations of the recursion relation (1) reveal that the convergence is very fast. To analytically investigate this convergence, we substitute $R_n = R_\infty + \delta_n$ into Eq. (1), and assume that n is sufficiently large such that $\delta_n \ll R_\infty$. This leads to the recursion relation $\delta_{n+1} = (2 - R_\infty/R_0)(1 + R_\infty/R_0)^{-1}\delta_n$ which, by Eq. (2), is $\delta_{n+1} = \alpha^{-2}\delta_n$ where $\alpha = (3+\sqrt{5})/2$. The decaying solution obeys $\delta_n \propto \alpha^{-2n}$, which shows that δ_n exponentially (or geometrically) approaches zero, and so R_n exponentially approaches R_∞. Specifically, the theory shows that $\log[(R_n - R_\infty)/R_0]$ vs n asymptotically approaches a negative slope of magnitude $2\log(\alpha)$. By plotting the theoretical values in Table I, one can observe that the asymptote is essentially reached by the $n=2$ point, and that even the $n=1$ point falls nearly on the asymptote.

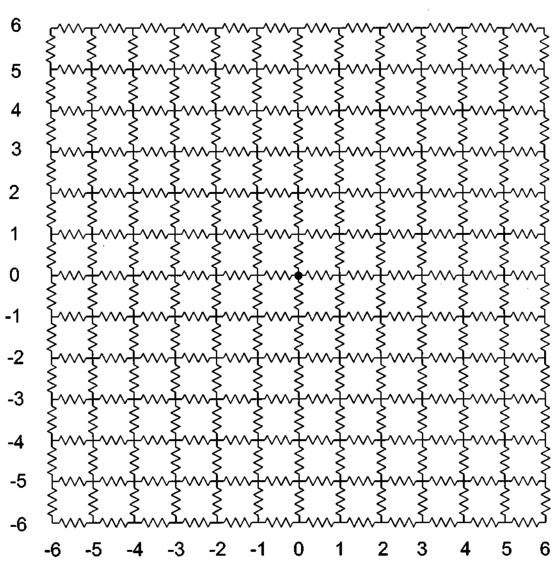

Fig. 2. A 12 by 12 grid of identical resistors of resistance R_0. The dot labels the center node (0,0) of the grid. Measurements of equivalent resistance were made with the center always as one terminal, and were across a side, diagonal, double side, knight's move, and double diagonal.

Table I. Theoretical and experimental equivalent resistances R_n for a ladder of n loops of resistors shown in Fig. 1(a). In the experiment, $R_o = 1.00\ \mathrm{k}\Omega$.

Number n of loops in ladder	Theoretical value of R_n/R_0	Relative theoretical deviation $(R_n - R_\infty)/R_0$	Measured value of R_n/R_0
1	2	3.82×10^{-1}	1.990 01
2	5/3	4.86×10^{-2}	1.657 72
3	13/8	6.97×10^{-3}	1.616 30
4	34/21	1.01×10^{-3}	1.610 38
5	89/55	1.48×10^{-4}	1.609 53
6	233/144	2.16×10^{-5}	1.609 41
7	610/377	3.15×10^{-6}	1.609 40
8	1597/987	4.59×10^{-7}	1.609 40
∞	$(1+\sqrt{5})/2$	0	

The exponential approach of R_n to R_∞ suggests that when a potential difference is applied to the terminals in Fig. 1(a), the currents in successive loops of the ladder decrease exponentially. To analytically investigate this, it is convenient to define ''mesh'' currents i_k circulating in the same sense in the loops, where $k = 1,2,...,n$. Kirchhoff's loop rule for the kth loop yields the recursion law $3i_k = i_{k+1} + i_{k-1}$, for $k = 2,3,...,n-1$. For the infinite ladder ($n = \infty$), the solution which vanishes as $k \to \infty$ is $i_k \propto \alpha^{-k}$, where $\alpha = (3+\sqrt{5})/2$ as above. Hence, the currents in the infinite ladder indeed decrease exponentially for successive loops. That is, for each successive loop the branching ratio of the current is a constant, which is a plausible result. The recursion law is also satisfied by the diverging sequence $i_k \propto \alpha^k$, which must be included for finite ladders, so $i_k = A\alpha^{-k} + B\alpha^k$ in these cases. The ratio B/A is determined by Kirchhoff's loop rule for the final loop, $3i_n = i_{n-1}$, which yields $B/A = -\alpha^{-2n}(3-\alpha)/(3-1/\alpha)$. Although the currents do not strictly decrease exponentially in successive loops of a finite ladder, the effect of the exponentially increasing term decreases as n increases. Moreover, plotting the current distributions reveals that the effect is only noticeable for $k = n$ for any value of n, and that even this deviation is small.

III. LADDER EXPERIMENT

We performed an experiment with the resistive ladder shown in Fig. 1(a) with $R_0 = 1.00\ \mathrm{k}\Omega$ resistors of $\pm1\%$ accuracy mounted on a small electronics breadboard. Such an experiment is suitable for the introductory laboratory because no soldering is required, and because the resistors are inexpensive, especially when purchased in large amounts.[11] To precisely measure the equivalent resistances, we employed a six-digit multimeter (Hewlett-Packard model 34401A). Successive loops of the ladder were naturally added to the end *opposite* to that where the multimeter was connected. (The importance of this is explained below.) Listed in the final column of Table I are the data normalized by the individual resistance $R_0 = 1.00\ \mathrm{k}\Omega$.

The convergence of the experimental data to R_∞, can be exhibited by a plot of $\log[(R_n - R_\infty)/R_0]$ vs n. However, the predicted value (2) is unsuitable for R_∞/R_0 here because the actual value will differ on the order of 1% due to variations in the resistance of the resistors, and because the plot is very sensitive to the value of R_∞/R_0. If the predicted value is used, the data agree with the theory for typically only one

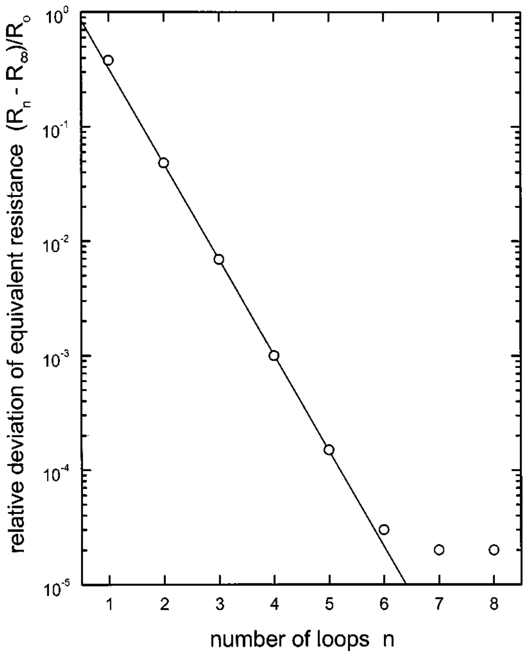

Fig. 3. Equivalent resistance of the ladder in Fig. 1(a) as a function of the number n of loops. The points are experimental; the line is a fit to the points corresponding to $n=2$, 3, 4, and 5. The leveling-off of the points is due to the precision of the multimeter being exceeded.

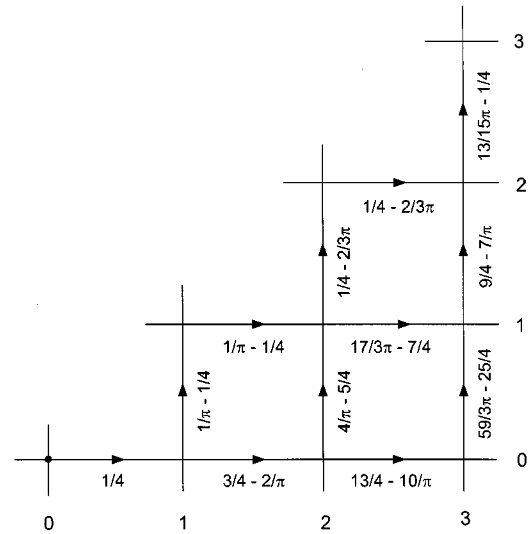

Fig. 4. Theoretical current distribution in an infinite two-dimensional square grid of identical resistors, according to Ref. 7 or 10. Unit current is injected at the lower left node (0,0) and removed at infinity. The distribution has eightfold symmetry, and so only an octant is shown.

decade, and then strongly diverge. We thus treat R_∞/R_0 as a fit parameter. The fit can be achieved simply and quickly by manually adjusting the value until the plot is linear. The results are shown in Fig. 3. The data extend beyond four decades, which is remarkable for any experiment. Seldom do introductory physics students have the opportunity of dealing with measured values where six figures are significant in testing a theory. The expense of a multimeter with such precision can be mitigated by having groups share one or several multimeters, which is suitable here because the data can be quickly gathered while other groups are performing a standard Ohm's law experiment.

The measured equivalent resistances R_6, R_7, and R_8 approach a constant value in Fig. 3 because the precision of the multimeter is exceeded. The line in Fig. 3 was obtained by adjusting the value of R_∞/R_0 such that only the R_2 through R_5 data are fitted, because the asymptote is not predicted to be reached by R_1 (Sec. II), and because the R_6 through R_8 data are imprecise. The value of the fit parameter is $R_\infty/R_0 = 1.609\,380$, with an upper bound of 1.609 392 where the R_6 datum exhibits negative deviation from the linear relationship in the plot, and a lower bound of 1.609 370 where the R_5 datum begins to exhibit positive deviation from the linear relationship. The experimental value of R_∞/R_0 is within the 1% resistance tolerance of the predicted value 1.618 034 of Eq. (2). The slope of the line in Fig. 3 has magnitude 0.835, which closely agrees with the predicted value of $2\log(\alpha) = 0.835\,951$ in Sec. II. The experimental results are thus consistent with the theory.

It is surprising that the data in Fig. 3 have negligible deviations even though the values of the resistors vary on the order of 1%. Indeed, how can experimental values with a precision of one part in 10^5 be meaningful when random variations of 1% exist in a system? The high precision in this experiment results from the manner in which the loops are added, and the approximate exponential decrease in the current in successive loops (Sec. II). This rapid decrease in current causes the equivalent resistance across the terminals in Fig. 1(a) to be insensitive to variations in the added individual resistances as loops are added to the opposite end. This is *not* the case, for example, if the order of adding loops is reversed so that loops are added at the terminals as in Fig. 1(b), which is necessary for the theoretical calculation. The values of the equivalent resistance continue to deviate on the order of 1% as loops are added (although the final value is the same regardless of the order), which does not allow a precise asymptotic analysis of the data as above. We confirmed this experimentally for the ladder that yielded the data in Fig. 3. If the above fit value of R_∞/R_0 is used in the plotting of the data, the deviations are dramatic in that many of the ordinate values are negative, so they cannot appear on the semilogarithm plot. (The data are thus not shown.)

IV. SQUARE GRID THEORY

Figure 4 shows the local current distribution according to the theory of van der Pol and Bremmer[7] for the case of an infinite square grid of identical resistors where a unit current is injected at one node (the origin) and removed at infinity. It is remarkable that π enters into the branching values of the current at the nodes. We confine our interest to the equivalent resistances across a ''side,'' ''diagonal,'' ''double side,'' ''knight's move,'' and ''double diagonal.'' If the nodes are labeled by a unit lattice as in Figs. 2 and 4, examples of these are segments with one endpoint at the origin (0,0) and the other at (1,0), (1,1), (2,0), (2,1), and (2,2), respectively. By superposing the results in Fig. 4 such that a unit current is injected at the origin and removed at one of the desired nodes, one can calculate the local current distribution that would occur when an ohmmeter is connected across the nodes. Once these currents are known, the equivalent resistance can be determined as the potential difference divided by the input (or output) current. The values of the

Table II. Theoretical and experimental equivalent resistances R_{eq} for a two-dimensional square grid of resistors with resistance R_0. The theoretical values are from Ref. 7 or 10 for an infinite grid, and the experimental and numerical values are for a 12 by 12 grid (Fig. 2). In the experiment, the individual resistance is $R_0 = 0.995\ \text{k}\Omega$.

Segment	Value of R_{eq}/R_0			Deviation from infinite grid value	
	Infinite grid	Experimental	Numerical	Experimental	Numerical
side	1/2	0.503 09±0.000 09	0.503 280	0.62%	0.66%
diagonal	$2/\pi$	0.643 21±0.000 12	0.643 364	1.04%	1.06%
double side	$2-4/\pi$	0.739 99±0.000 30	0.740 090	1.82%	1.83%
knight's move	$4/\pi-1/2$	0.790 59±0.000 37	0.790 587	2.24%	2.24%
double diagonal	$8/3\pi$	0.878 50±0.000 18	0.878 448	3.50%	3.49%

equivalent resistances, normalized by the resistance R_0 of the individual resistors, are displayed as the infinite grid values in Table II.

These calculations are very useful for introductory physics students because the superposition principle is required in an interesting manner. The calculations also strongly reinforce the notion of equivalent resistance because the situation is more challenging than usually encountered. The students are expected to completely understand the equivalent resistance of a side, but are asked to otherwise accept the theoretical values of the currents in Fig. 4 as results of calculations beyond the level of introductory physics. This naturally motivates an experimental confirmation of the values.

The equivalent resistances for the infinite square grid serve as approximations to equivalent resistances near the center of the 12 by 12 grid in the experiment (Sec. V). For an accurate comparison with the experimental values, one could numerically perform integrals of Lavatelli.[8] It is easier and more interesting to use the *relaxation method*[12] to numerically determine the values of the potential at the nodes. The potential values are held fixed at the two particular nodes where the current is injected and removed. Once the potential values are known, the currents can be calculated from Ohm's law, and the equivalent resistances across the input and output nodes then follow as above.

We accomplished the numerical determination of the potentials as follows. First, note that the existence of definite potential values at the nodes automatically ensures that the sum of the potential differences around any loop vanishes (Kirchhoff's loop rule). Next, it is readily shown with Ohm's law that the vanishing of the net current into a node (Kirchhoff's junction rule) is equivalent to the value of the potential at that node equaling the average of the values at the four nearest-neighbor nodes. In the relaxation method for a resistive grid, the values of the potentials at the input and output nodes are fixed, say, dimensionlessly at 1 and -1, and the values at all other nodes can be initially set to zero. This latter set of values is then continually updated by requiring that each value equals the average of the four nearest-neighbor values, while the values at the input and output nodes remain fixed. We impose the boundary conditions of only three nearest neighbors along the edges, and only two nearest neighbors at the corners, by implementing ''ghost'' nodes at adjacent points just outside the grid. The potential at each ghost node is constrained to equal the potential at the actual node adjacent to it, so that no current flows. In this way, all of the actual nodes are conveniently treated in the same manner; i.e., with each value of the potential being continually replaced by the average of the four nearest-neighbor values of the potential. Listed in Table II are the results for the equivalent resistances after roughly 3000 iterations of the potential values. We confirmed these results by creating a virtual 12 by 12 square grid of identical resistors with the ELECTRONICS WORKBENCH™ software,[13] which employs general numerical methods to solve arbitrary circuits.

V. SQUARE GRID EXPERIMENT

To experimentally test the theoretical values of the equivalent resistances in Sec. IV, we constructed a grid of $2\times 12 \times 13 = 312$ resistors shown in Fig. 2. For accuracy, we first measured the individual resistances of 400 resistors rated at $1.00\ \text{k}\Omega \pm 1\%$,[11] and plotted a histogram of the results. We found that the distribution was roughly Gaussian with mean value 0.995 kΩ, and that we could obtain a worst-case uncertainty of $\pm 1/2\%$ by removing only a few percent of the resistors. Hence, the value of the individual resistance for our apparatus is $0.995 \pm 0.005\ \text{k}\Omega$.

A plywood board 2 ft by 2 ft with 3/4 in. thickness was used in the construction of the grid. A white sheet of Formica was glued to the top so that the resistor grid would be clearly visible. We then drilled $13^2 = 169$ holes 1.25 in. apart for #8-size brass wood screws. For convenience in quickly identifying the center node, a small brass disk was concentrically inlaid in the board. The screws were flat-head Phillips type. Using a Dremel tool, we extended the channels in each screw head completely across the face of the head. The screws were then partially screwed into the board. Resistors were randomly selected from our pool of $0.995 \pm 0.005\ \text{k}\Omega$ resistors, and the resistor leads were placed in the channels of the screws and soldered with an iron capable of high temperature, because the brass screws acted as heat sinks. Rubber feet were attached to the bottom of the board for use in the laboratory. Finally, wire was attached to the back of the board so that it could be conveniently and safely stored by hanging on a wall. Such a display of an apparatus also serves to initiate physics discussions when curious students and professors first observe it.

An alternative apparatus could be constructed with nichrome wire that is looped around posts on a frame such that a square grid is created. The wire would then be silver-soldered at the overlapping points. We chose the resistor grid described above due to our desire for an accurate experimental verification of the theory. However, the nichrome wire grid may be more interesting and appropriate for lecture demonstrations as well as the educational laboratory.

Resistance measurements were made with a high-precision multimeter (Hewlett-Packard 34401A), with the center node (labeled by the dot in Fig. 2) always as one terminal. We made the four possible measurements of each of the side, diagonal, double side, and double diagonal segments, and the eight possible measurements of the knight's move. Listed in Table II are the averages and deviations (one-half the difference of the maximum and minimum values), normalized by the individual resistance $R_0 = 0.995\,\mathrm{k}\Omega$. As part of an Ohm's law experiment in the educational laboratory, a single resistive grid board can be easily passed among groups of students during a laboratory period, because the measurements are quickly performed. Students can also numerically investigate the equivalent resistances with existing code or software (Sec. IV).

The near identity of the two columns of percentage differences in Table II strongly suggests that the experimental and numerical results agree. The strict test, however, is whether the numerical values lie within the experimental means and uncertainties of the means due to the random variations of the individual resistances. These uncertainties are *not* the deviations in Table II, which are due to the different possible measurements of the segments in our particular apparatus. In fact, only the final two segments in Table II have equivalent resistance means and deviations that contain the corresponding numerical values. The desired uncertainties can be determined experimentally or computationally by an ensemble of apparatus, or analytically by a propagation-of-error formula. An experimental ensemble is impractical here, and the derivation of a propagation-of-error formula presents an interesting but apparently difficult problem for a two-dimensional grid of resistors whose values randomly deviate about a common value. We have left this for possible future work.

Table II shows that the finiteness of the grid causes the equivalent resistances to be greater than the values for an infinite grid, which is expected because the current has fewer paths. However, the deviations from the values of the infinite grid are surprisingly large if the current or potential distributions decrease exponentially from the terminals as in the case of the ladder (Sec. II). This suggests that the decrease is *algebraic* rather than exponential, in contrast to the resistor ladder. We confirmed this with our numerical relaxation program (Sec. IV). The algebraic decrease is reasonable because in the continuum limit (sufficiently far from the terminals) the potential distribution should be that of a dipole. However, this argument is not rigorous because the assumption of a continuum limit may preclude the possibility of an exponentially decreasing distribution. A related argument follows from a connection between the rule used in the relaxation method (Sec. IV) and Laplace potential theory. The rule is that the value of the potential at a node is the average of the values of the nearest-neighbor potentials. Consider a two-dimensional continuum in which there is a potential obeying Laplace's equation, and an imaginary square grid that covers the continuum. In the limit that the grid spacing is small compared to the change in potential, it is readily shown[14] that the value of the potential at a node of the grid is the average of the values of the nearest-neighbor potentials, which is identical to the rule for a grid of resistors. Hence, the solution for a grid of resistors is expected to be similar to a continuum solution. The argument is not rigorous, however, because the grid problem has the condition that the dimensionless potential values are ± 1 at two nodes separated by one or several grid spacings, which violates the continuum assumption of slow variation.

An indication that the decrease in current with distance from the terminals of the multimeter is indeed algebraic rather than exponential can be dramatically demonstrated with the grid. The multimeter leads are connected across a side, with the center node as one terminal. The resistance reading is noted, and a wire is then used to short a single resistor adjacent to a corner of the grid. Remarkably, the resistance reading is observed to change, decreasing roughly one part in 10^5.

Employing a short to observe changes in equivalent resistance offers a useful technique for students to probe the grid. For example, consider any two nodes whose perpendicular bisector coincides with the line that passes through the terminals of a side or diagonal segment of any length, where the origin lies on the line. When these nodes are shorted, there is *no* change in the equivalent resistance. The two nodes have the same potential as a result of the infinite lattice symmetry, which is preserved by the boundaries of the actual lattice. When the terminals correspond to a knight's move, the situation is more complicated for several reasons. First, the infinite lattice symmetry is not as obvious as in the other cases, although the same condition holds. Second, the boundaries of the actual lattice break the symmetry in this case. Hence, the lack of change of the equivalent resistance of the knight's move occurs only if the shorted nodes are not near the boundary of the lattice. Whereas the side and diagonal cases offer students an interesting but straightforward exercise in symmetry, the knight's move case is more challenging.

Finally, it should be noted that the grid of resistors can be employed as a lecture demonstration apparatus. Examples of demonstrations are as follows. First, the apparatus serves as an example of a network whose equivalent resistance across any two nodes cannot be obtained by a sequence of series and parallel reductions. In fact, a 3 by 3 grid of identical resistors is not completely reducible in this manner, and a 2 by 2 grid of nonidentical resistors is not in general completely reducible. Second, the resistance across any two nodes can be measured with an ohmmeter, where it should be pointed out to students that the ohmmeter injects a small amount of current and displays the ratio of the potential difference to the current. A physical apparatus helps students visualize this process. Approximate theoretical equivalent resistances such as $R_0/2$ across a side and $2R_0/\pi$ across a diagonal can be verified by measurement. Third, that the current distribution of the ohmmeter probes the entire grid can be dramatically demonstrated by shorting a single resistor far from the terminals, as described above. The effect of shorting other nodes can be observed, and students can be asked if there exist pairs of shorted nodes such that the equivalent resistance does not change, as described above.

VI. CONCLUSIONS

We have considered the equivalent resistance across a ladder network whose number of loops is incremented, and across nodes near the center of a large two-dimensional square grid. Experiments with these resistor networks offer interesting additions to a standard introductory laboratory experiment in Ohm's law, and can be readily performed. The square grid can also serve as a lecture demonstration apparatus.

As loops are added to the ladder network, the approximate exponential convergence of the equivalent resistance values to the limiting value is experimentally convenient because a large number of loops is not required. The approximate exponential decrease in current in successive loops has a consequence that 1% variations of the individual resistance values are negligible if successive loops are added to the end of the ladder opposite to the end with the terminals. This allows a precise analysis of the equivalent resistances extending over four decades, where the limitation is due to a six-digit multimeter.

For ohmmeter terminals near the center of the two-dimensional square grid, the current distribution decreases algebraically with distance from the terminals. This leads to small deviations from the theory for equivalent resistances across nodes of an infinite square grid. The algebraic decrease can be dramatically demonstrated by shorting a single resistor near a corner of the grid, which causes an observable decrease in equivalent resistance across two adjacent nodes near the center. Shorting any two nodes of the grid offers a means of probing the current distribution due to the ohmmeter. In particular, if the nodes have the same potential as dictated by symmetry, there is no change in the equivalent resistance.

The experimental square grid results were confirmed numerically by the relaxation method, and alternatively with commercial software. The results are consistent with the theory. A remaining problem is to derive a propagation-of-error formula for the equivalent resistance across two nodes of a large or infinite square grid of resistors whose values are randomly distributed about a common value. Another remaining problem is to build a square resistive grid with nichrome wire.

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[a]Undergraduate students.

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BLOCK THAT METAPHOR! PLEASE!

While the student of physics faces a vast body of accepted wisdom to be mastered before the occasional genius can push the frontier forward, sociology is all frontier. Physics is a string of nourishing sausages to sociology's hunk of bleeding flesh, still quivering from the slaughterhouse.

The best thing to be in life is one of that tiny minority who can eat the sausages and grind up an extra bit of meat for the Nobel prize. But perhaps the second-best thing to be, if you have the temperament, is someone trying to get your arms round a big chunk of quivering flesh.

Harry Collins, in a review of *The Social Animal*, by Gary Runciman, New Scientist, 14 March 1998.