Measuring air resistance in a computerized laboratory

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By dropping spherical party balloons onto a sonic motion sensor we show that the force associated with the air resistance is proportional to both the square of the velocity and to the cross-sectional area of the balloon. These results are in agreement with those expected for the value of the Reynolds number used, $R \approx 10^4$.

I. INTRODUCTION

A common introductory exercise in computational physics is to include the effects of air resistance in the equations of motion for a projectile. However, we have seen presented side by side in a conference[1] one set of calculations using a force due to the air resistance that is proportional to the velocity of the projectile

$$F \propto v \tag{1}$$

and another using a force proportional to the square of the velocity

$$F \propto v^2. \tag{2}$$

Both cannot, simultaneously, be correct.

The authors using Eq. (1) were obviously thinking of Stokes' law

$$F = 6\pi\eta r v, \tag{3}$$

which gives the viscous force on a sphere of radius r moving with velocity v in a fluid of viscosity η. This equation can be found in almost all introductory physics texts,[2] although the derivation is not trivial.[3] In fact, Stokes' law comes from an approximate solution to the equation of motion for a sphere moving in a fluid of infinite extent which is at rest at infinity. The approximation that is made is that the Reynolds number

$$R = \frac{\rho l v}{\eta}, \tag{4}$$

where ρ is the density of the fluid and l is a typical length scale, is much less than one. This means that we are dealing with a viscous fluid.

An air resistance proportional to v^2 occurs in the opposite limit of $R \gg 1$, and, in spite of not always being mentioned in introductory texts,[4] is quite easy to derive. An object of cross section A moving with velocity v sweeps out a volume $V = Av\Delta t$ in the time interval Δt. Therefore, it collides with $N = V\rho/m$ fluid particles, where m is the mass of the fluid particles. On average, each collision will result in a momentum change of the order of $\Delta p \approx mv$. The result is a force that is commonly parametrized by[5]

$$F = \tfrac{1}{2}\rho C_D A v^2, \tag{5}$$

where the drag coefficient C_D depends on the exact form of the object and on the medium but it is approximately independent of the velocity for large Reynolds numbers. For a sphere of radius r the drag force in this case is

$$F \propto r^2 v^2. \tag{6}$$

As can be seen, the viscosity does not appear as we are in the limit where inertial forces dominate. For objects falling in air, the viscosity is sufficiently small that Stokes' law is not valid for any but the lowest velocities.

In this paper we describe an experiment using party balloons that allows undergraduate students to distinguish between Eqs. (3) and (6). The diameter of the balloons is approximately 15 cm and the terminal velocity that they reach is 3 m s^{-1}. Combined with the density and viscosity of air, $\rho = 1.28\,\mathrm{kg\,m^{-3}}$ and $\eta = 1.83\times 10^{-5}\,\mathrm{N\,s\,m^{-2}}$, we find a Reynolds number $R \approx 10^4$. Obviously, we have $R \gg 1$ and we will show that Eq. (6) really is a much better description of air resistance than Eq. (3). Life can even be much more complicated, as discussed in Ref. 6, where it is considered that the missing constant of proportionality in Eq. (6) can be a function of the Reynolds number and the spin of the moving object.

II. EXPERIMENTAL METHOD

The equation of motion for an object of mass M falling in a fluid that produces a resistive force $F(v)$ is

$$M\frac{dv}{dt} = M^* g - F(v), \tag{7}$$

where M^* is the effective mass of the object taking into account the Archimedes upthrust due to the fluid displaced. As the resistive force increases with velocity, the object eventually reaches a terminal velocity v_∞ determined by

$$F(v_\infty) = M^* g. \tag{8}$$

If we work with objects of the same radius but different masses, distinguishing between Eq. (1) and Eq. (2) requires us to determine if the relation between the effective mass and the terminal velocity is

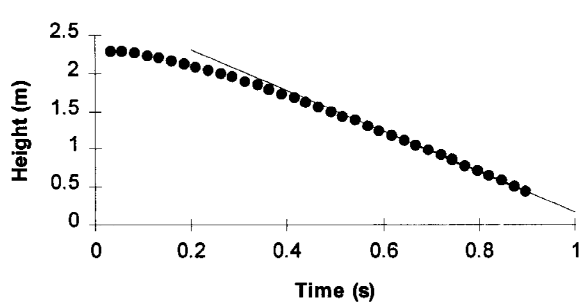

Fig. 1. The variation of the height of a balloon falling in air. A least-squares fit of a straight line to the last ten points is shown.

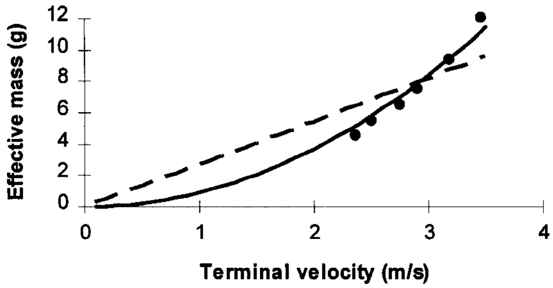

Fig. 2. The effective masses of the balloon vs the measured terminal velocity. The lines are least-squares fits to a straight line and to a parabola, in both cases the fit is constrained to pass through the origin.

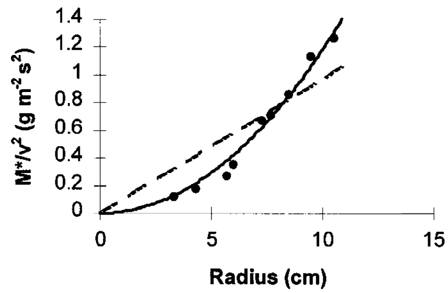

Fig. 4. Determination of the dependence of the resistive force on the radius of the balloons. The lines are least-squares fits to a straight line and to a parabola, in both cases the fit is constrained to pass through the origin.

$$M^* \propto v_\infty \tag{9}$$

or

$$M^* \propto v_\infty^2. \tag{10}$$

If we wish to show that Stokes' law is not always appropriate, then the experimental challenge is to find a combination of an object and a fluid such that the object reaches its terminal velocity in a distance that is of a laboratory scale, but where the inertial forces dominate the viscous ones. Our solution is to use party balloons. The advantage that balloons have over the coffee filters used by Derby, Fuller, and Gronseth[7] is that we can also vary the radii to check the r dependence of the resistive force. An alternative approach is to drive the object such that it executes uniform circular motion.[8]

A series of experiments was performed using a single balloon of fixed radius. The effective mass of the balloon, whose diameter was approximately 15 cm, was varied from 4.5 to 12 g by sticking coins to it. The effective mass was obtained by simply putting the balloon on a standard laboratory balance, because even there it experiences the Archimedes upthrust. The balloon was then dropped from a height of around 2.5 m onto a sonic motion sensor.[9] In Fig. 1 we show the evolution of the height of the balloon. The value of the terminal velocity was obtained by fitting a straight line to the last ten points in the graph of the position versus time. For each mass the terminal velocity was determined ten times and the standard deviation was less than 3% of the mean value. The results are shown in Fig. 2.

III. DATA ANALYSIS

The lines shown in Fig. 2 are least-squares fits to

$$M^* = k_1 v \tag{11}$$

and to

$$M^* = k_2 v^2, \tag{12}$$

where k_1 and k_2 are the fitting parameters. It can be seen immediately that the parabola is a far better fit than the straight line. The values of χ^2 are 13 for the straight line, and 1.6 for the parabola. Therefore, we can conclude that the better description of the air resistance to the movement of a balloon is not Stokes' law, but rather Eq. (2).

An alternative way of analyzing the data is to note that we are looking for a power law relation of the type $M^* \propto v^n$. The exponent n, which is equal to one in the case of Stokes' law and two in the case of Eq. (2), can be obtained as the slope of a graph of $\ln M^*$ vs $\ln v$. The data of Fig. 2 are replotted in this form in Fig. 3. The value obtained for the exponent is

$$n = 2.5 \pm 0.2. \tag{13}$$

This value is a little higher than two, but clearly inconsistent with the prediction of Stokes' law.

A second experiment that can be done with balloons is to check the dependence of the terminal velocity on the radius of the balloons. It is obvious that balloons of different radii have different masses, so the problem is to separate the contributions of the velocity and the radius. However, once we have established that the resistive force is proportional to the square of the terminal velocity we can write

$$\frac{M^*}{v_\infty^2} \propto r^n, \tag{14}$$

where we wish to distinguish between $n=1$ and $n=2$. The analysis is similar to that carried out to establish the velocity dependence. In Fig. 4 we show how M^*/v_∞^2 and r are related. Once again, the lines are least-squares fits to a straight line and to a parabola, both passing through the origin, and it

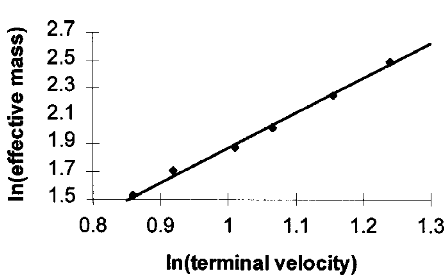

Fig. 3. A log–log plot of the data of Fig. 2. The straight line is the least-squares fit to the data.

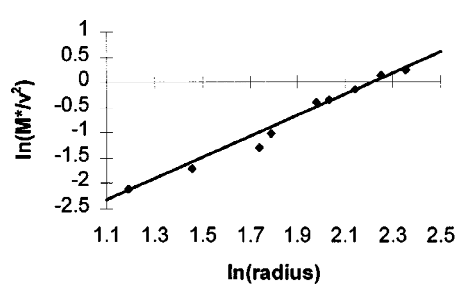

Fig. 5. A log–log plot of the data of Fig. 4. The straight line is the least-squares fit to the data.

is clear that $n = 2$ is the better of the two fits. The values of χ^2 are 0.33 for the straight line and 0.027 for the parabola. Figure 5 shows the same data as a log–log plot, which gives an exponent

$$n = 2.1 \pm 0.4. \tag{15}$$

IV. CONCLUSIONS

We have shown that party balloons are well suited to studying air resistance. They rapidly reach their terminal velocity and so their movement can easily be studied in a laboratory with a sonic motion sensor. As they can be prepared with different masses and radii they can be used to check both the velocity dependence and the size dependence of the resistive force. We have shown that the resistive force is proportional to both the square of the velocity and the square of the radius of the balloon. This is consistent with the motion being dominated by inertial forces, as is to be expected for the Reynolds numbers observed.

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STATISTICS

Outside psychology [philosophy] plays almost no part in the functions of the research machine. Brain scientists—Steven Pinker included—are defensive about their flirtation with the mystics. They know that they cannot afford a relationship with their subject as austere as that of the physicist Lord Rutherford with his; he claimed that ''if your experiment needs statistics, you should have done a better experiment.'' Even biologists see that as unfair; in the messy world of real life, statistics reveal the general through the mists of the particular. Psychologists, with minds of their own to deal with, may need yet another level of explanation. The cynical view that if their science needs philosophy they should do better science is less than reasonable. It may mean, though, that large parts of their enterprise are for the time being beyond the limits of science altogether.

Steve Jones, ''The Set Within the Skull'' (a review of *How the Mind Works*, by Steven Pinker, Norton), New York Review of Books, November 6, 1997.