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Building a copper pipe 'xylophone'

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Abstract

Music is central to the life of many students. This article describes using the equation for frequency of vibration of a transversely oscillating bar or pipe (with both ends free to vibrate) to build a simple and inexpensive xylophone-like musical instrument or set of chimes from a 3 m section of copper pipe. The instrument produces a full major scale and can also be used to investigate various musical intervals.

With even a quick look around most school campuses, it is easy to see that students enjoy music. Ears are sometimes hard to find, being covered or plugged by headphones that are connected to radios or portable CD players. The music emerging from these headphones inspires, entertains and provides emotional release. A closer look reveals that much of the life of a student either revolves around music or is at least strongly influenced by it. The radio is the first thing to go on in the morning and the last to go off at night (if it goes off at all). T-shirts with the logos and tour schedules of popular bands are the artifacts of the most coveted teenage activity—the concert. The teacher of physics has the unique ability and opportunity to bridge the gap between the sometimes-perceived irrelevant curriculum offered within the school and the life of the student outside the school. This article describes using the equation for frequency of vibration of a transversely oscillating bar or pipe (with both ends free to vibrate) to build a simple and inexpensive xylophone-like musical instrument or set of chimes from a 3 m section of copper pipe (the type used by plumbers for water service).

In a bar or pipe of length L with both ends free

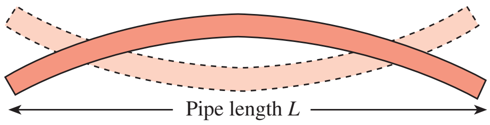

Figure 1. A bar or pipe vibrating transversely in its first mode.

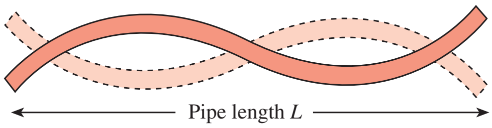

Figure 2. A bar or pipe vibrating transversely in its second mode.

to vibrate, a transverse standing wave condition is created when it is struck on its side, as in the case of the marimba or glockenspiel. The constraint for this type of vibration is that both ends of the bar or pipe must be antinodes. The simplest way for a bar or pipe to vibrate with this constraint is to have one antinode at its centre, in addition to the antinodes at each end. The nodes occur at $0.224L$ and $0.776L$ [1]. This first mode of vibration produces the fundamental frequency, f_1 (see figure 1). The second mode of vibration, f_2, producing the next higher frequency, is the one

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Table 1. Speed of sound within various metals.

Material	Speed of sound ($m\,s^{-1}$)
Aluminium	5150
Brass	3500
Copper	3700
Steel	5050
Glass	5200

with a total of four antinodes, including the ones at the ends. This mode has a node at the centre of the pipe and two other nodes at $0.132L$ and $0.868L$ (see figure 2). There are many higher modes of vibration. The frequency of the nth transverse mode of vibration for a bar or pipe is given by [1]:

$$f_n = \frac{\pi v K}{8L^2} m^2 \qquad (1)$$

where v is the speed of sound in the material of the bar or pipe (see table 1 for the speed of sound in common materials); L is the length of the bar or pipe; $m = 3.0112$ when $n = 1$, $m = 5$ when $n = 2$, $m = 7$ when $n = 3, \ldots (2n + 1)$; K is the radius of gyration and is given by

$$K = \frac{\text{thickness of bar}}{3.46}$$

for rectangular bars, or

$$K = \tfrac{1}{2}\sqrt{(\text{inner radius})^2 + (\text{outer radius})^2}$$

for pipes.

The values for m indicate that, unlike the vibration modes of plucked and bowed strings or of air columns within pipes, the transverse vibration modes of bars and pipes are anharmonic ($f_2 \neq 2f_1$, $f_3 \neq 3f_1$). Instead

$$\frac{f_2}{f_1} = \frac{5^2}{3.0112^2} = 2.76$$

and

$$\frac{f_3}{f_1} = \frac{7^2}{3.0112^2} = 5.40.$$

Given equation (1), students can easily calculate the lengths required for a bar or pipe to produce various frequencies of vibration within the audible range. If several of these frequencies are chosen to match certain frequencies within an accepted musical scale, then a simple musical instrument can be built which will produce recognizable tones. I recently built such an instrument from a 3 m length of copper pipe of 15 mm diameter (see figure 3). Desiring to produce the fundamental frequency, I decided to support the cut pipes at the nodes for the first mode of vibration. This desired mode also required giving m a value of 3.0112. The inside and outside radii of the pipe were measured to be 7 mm and 8 mm respectively, giving K a value of

$$K = \tfrac{1}{2}\sqrt{(7\text{ mm})^2 + (8\text{ mm})^2} = 5.32\text{ mm}.$$

Wanting to be able to cut eight lengths of pipe representing a C major scale (C_x, D, E, F, G, A, B, C_{x+1}), I first calculated the length for a pipe vibrating transversely in the frequency of the note C_5. The frequency of C_5 in the Scale of Equal Temperament [2] is 523.25 Hz. Rearranging the equation for frequency of vibration and calculating the necessary length for the first mode to produce the frequency of the C_5 note gives:

$$f_1 = \frac{\pi v_L K}{8L^2} m^2 \Rightarrow$$

$$L = \sqrt{\frac{\pi v_L K m^2}{8 f_1}}$$

$$= \sqrt{\frac{\pi(3700\text{ m s}^{-1})(0.00532\text{ m})(3.0112)^2}{8(523.25\text{ Hz})}}$$

$$= 0.366\text{ m}.$$

If all eight pipes were the same length, then the total length of pipe needed would be 2.93 m. However, if C_5 is defined as the lowest note produced by the instrument, then all the other pipes will be smaller. Thus a 3 m length is quite sufficient. Since the only variable in the equation for determining the length for the same copper pipe is the desired frequency of vibration, the other elements of the equation can be combined into one constant, simplifying the calculation for the remaining lengths:

$$L = \sqrt{\frac{\pi v_L K m^2}{8f}} = \sqrt{\frac{70}{f/\text{Hz}}}\text{ m}.$$

Table 2 provides the notes in the C_5 major scale, their frequencies, and the corresponding lengths for the copper pipe sections that will vibrate transversely at those frequencies.

Figure 3. This simple xylophone-like musical instrument, built from a single 3 m length of copper pipe, produces all the notes in the C_5 major scale.

Table 2. Lengths for 15 mm diameter copper pipes to vibrate at the frequencies of the notes in the C_5 major scale.

Notes	Frequencies (Hz)	Pipe lengths (m)
C_5	523.25	0.366
D_5	587.33	0.345
E_5	659.26	0.326
F_5	698.46	0.317
G_5	783.99	0.299
A_5	880.00	0.282
B_5	987.77	0.266
C_6	1046.5	0.259

At this point the bars, if they were to be used as chimes, could be drilled through with a small hole at one of the nodes. After filing off any sharp areas around the holes, fishing line or other suitable string could be strung through each pipe. The pipes could then be hung from the fishing line and would produce a high quality, first-mode-dominated tone if struck at their centres. However, I chose to make a simple xylophone-like instrument. This was accomplished by attaching 25 mm sections of 'self-stick rubber foam weatherseal' at both nodes of each pipe. A 3/8″ wide and 7/16″ thick (9.5×11.1 mm) variety commonly available at most home improvement stores was used. There are many other options for mounting the pipes depending on the quality and permanence desired [3]. Simple mallets for striking the pipes can be made from leftover pieces of the 3 m section of copper pipe. A 150 mm piece of copper pipe wrapped at the end with several layers of office tape produces tones that are satisfactory. Wooden spheres about 25 mm in diameter or hard rubber balls mounted on wooden dowels or thin metal rods will also produce satisfactory, but different, tones. Reference [3] provides detailed instructions for constructing a variety of types of mallets.

Given only one significant figure of accuracy in the measurement of the radius of the pipe, I was prepared for its vibration frequency to deviate significantly from that used to calculate the length, but testing the C_5 pipe's frequency against the 523.25 Hz tone from an audio frequency generator produced a beat frequency of only 2 Hz, an unusually good level of accuracy. Yet even if the frequency of the cut pipe had deviated considerably from the desired frequency, the simple xylophone is still a useful endeavour—the instrument would still sound as though it plays the full scale perfectly. As long as the lengths are cut to the nearest millimetre, three significant figures of precision are possible in the frequencies of the pipes. The result is a set of pipes whose vibrations are consistent with each other. The first and last still differ in frequency by an octave (a frequency ratio of 2:1) and the pipes, when played from longest to shortest, still go up in frequency with Equal Temperament half-step or whole-step increments in the same pattern as the C major scale. '*Do–Re–Me–Fa–So–La–T–Do*' is instantly familiar. Critics might counter that the whole scale is either too flat or too sharp, but the frequency of any standard note (Concert A = 440 Hz, for example) is arbitrary. And unless the xylophone were to be played along with some other instrument, it's only the intervals between the notes that are important. The copper pipe xylophone produces these very nicely. A musician can also easily use the xylophone to play good-sounding tunes as long as all notes of the tune are within the major scale (no flats or sharps).

For those who are familiar with the physical basis for the musical scale and wish to use the xylophone to explore the physics of the musical scale, it is easy to demonstrate the musical importance of the octave by striking the two 'C' pipes simultaneously or one right after the other. They are clearly different frequencies, but at the same time have a similar sound, hence the foundation of the musical scale being around the phenomenon of the octave. One can also appreciate the particularly consonant intervals of the perfect fifth (a frequency ratio of 1.5 : 1) and the perfect fourth (a frequency ratio of 1.33 : 1) by striking the C_5 and G_5 or C_5 and F_5 pipes simultaneously. The relative consonance of many other musical intervals can also be explored. In addition, chords of three or four notes can be produced if two people work together to strike three or four pipes simultaneously. For those with limited or no knowledge of the physics of the musical scale, there are good introductory sources of information available [4–6].

Building the copper pipe xylophone provides the student with an engaging and meaningful way to apply the equations describing the transverse vibrations of bars and pipes. The end product brings together the disciplines of physics, music and art in a coherent and authentic way. Finally, it provides a means to further explore the ideas of consonance and musical temperament.

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David Lapp has taught physics at the high school and introductory college levels since 1984. In 2003 he is working as a Research Fellow at the Wright Center for Innovative Science Education at Tufts University in Medford, Massachusetts. His research is primarily in the area of the physics of music and musical instruments.