Velocity dependence of friction on an air track

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An experiment to measure the velocity dependence of friction on an air track using a photogate timer is described. This experiment is suitable as a laboratory exercise or a demonstration experiment in an upper division level course in mechanics.

The experiment described in this paper has been used as a student laboratory exercise in an upper division course in mechanics[1] at Lycoming College. It serves the dual purpose of providing a concrete example of velocity dependent friction and of providing an opportunity for the students to use their calculus skills in analyzing the results of an experiment.

The friction on an air car is calculated from the following experimental data. One end of the air track is raised slightly (Fig. 1). The air car is given a velocity uphill and passes a photogate timer, where the initial velocity v_i is measured. The air car undergoes gravitational deceleration and comes to rest at a measured distance L from the photogate timer. Gravity then accelerates the air car back to the photogate timer, where the final velocity v_f is measured. The difference between the initial and final kinetic energy of the air car is attributed to friction.

The student is asked to assume that the friction is given by cv^n, where c and n are unknown constants to be calculated from the experimental data. The change in kinetic energy ΔK is then approximately given by

$$\Delta K = mv_i^2/2 - mv_f^2/2 = 4cL\,(2aL)^{n/2}/(n+2), \tag{1}$$

where $a = g \sin\theta$ is the acceleration due to gravity and higher order terms in $cv_i^2/(ma)$ are neglected. The derivation is given in the Appendix.

Figure 2 shows a graph of the natural log of L versus the natural log of ΔK. Ten data points were taken using a 5-m air track, which was at an angle of 0.61° from horizontal. A photogate timer accurate to 0.1 ms was used. The slope is theoretically predicted to be $n/2 + 1$. The fit slope of 1.466 ± 0.045 gives $n = 0.932 \pm 0.090$, consistent with a linear dependence of friction on velocity.

Although this experiment is best suited for a laboratory taken in conjunction with an intermediate course in mechanics, it can also be used as a lecture demonstration with the theoretical derivation and data analysis given as a homework assignment. It takes very little time to obtain enough data to make a good estimate of n.

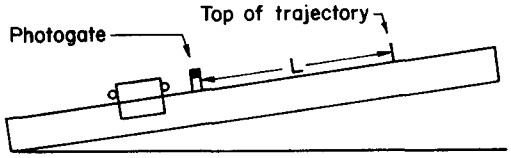

Fig. 1. Experimental setup.

APPENDIX

There are two derivations of Eq. (1) which are discoverable by students with a background of one year of calculus, provided suitable hints are given. The first method invokes the principle that the change in total energy is equal to the nonconservative work. The total nonconservative work is approximately equal to

$$2\int_0^L cv^n\,dx, \tag{A1}$$

where v is the average between the velocity going uphill and the velocity going downhill at a distance x from the top of the air car's path. Substituting

$$v = (2ax)^{1/2} \tag{A2}$$

into Eq. (A1), one gets Eq. (1). A more rigorous method is to use the work energy theorem

$$\frac{d\,(mv^2/2)}{dx} = F \tag{A3}$$

to derive the integral

$$\int dx = L = \int_{v_i}^{0} mv \frac{dv}{(-ma - cv^n)} = \int_0^{v_f} mv \frac{dv}{(ma - cv^n)}. \tag{A4}$$

Now, using the approximation

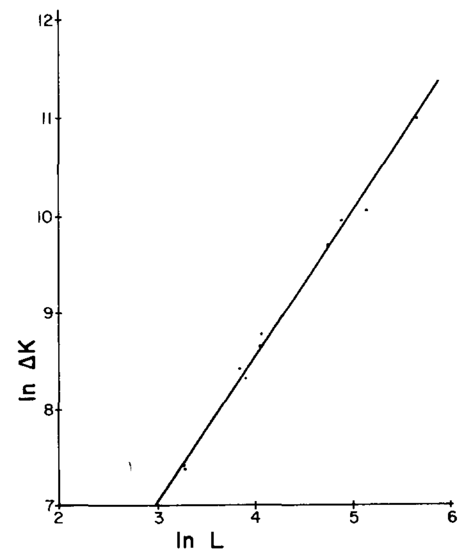

Fig. 2. A graph of the natural log of L in cm versus the natural log of K in ergs.

$$[1 + cv^n/(ma)]^{-1} = 1 - cv^n/(ma), \quad \text{(A5)}$$

one obtains

$$L = (v_i^2/2a)\{1 - 2cv_i^n/[ma(n+2)]\}$$
$$= (v_f^2/2a)\{1 + 2cv_f^n/[ma(n+2)]\}. \quad \text{(A6)}$$

Since this is a first-order approximation, one can substitute

$$cv_i^n = cv_f^n = (2aL)^{1/2} \quad \text{(A7)}$$

into the first-order terms in Eqs. (A6). This gives

$$mv_i^2/2 = maL/\{1 - 2c(2aL)^{n/2}/[ma(n+2)]\},$$
$$mv_f^2/2 = maL/\{1 + 2c(2aL)^{n/2}/[ma(n+2)]\}. \quad \text{(A8)}$$

The difference between Eqs. (A8) is Eq. (1) to first order. The more rigorous proof shows that the approximation is valid provided that cv_i^n is much less than ma. This is equivalent to demanding that ΔK be much less than $mv_i^2/2$.

[1]The text used is G. R. Fowles, *Analytical Mechanics* (Holt, Rinehart and Winston, New York, 1977).

Young's experiment with polarized light: Properties and applications

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We discuss in a simpler manner, several properties of the fringes obtained if Young's experiment is performed with polarized light. These properties are applied to implement three contemporary experiments of optical image processing: weighted addition and subtraction of two images, contrast reversal, and pseudocolor encoding of gray level information.

I. INTRODUCTION

The double slit, or Young's experiment, is the classical evidence of the wave characteristics of light[1] or that associated with electrons.[2] With polarized light, the experiment is particularly interesting since it proved that light can be described as a transversal wave rather than a longitudinal one.[3,4] Besides its historical interest, the double-slit experiment can be employed as a simple, but powerful device, to implement contemporary image processing techniques, such as addition and subtraction of two images,[5–9] contrast reversal of an image,[10–13] and the codification of gray levels into pseudocolors.[14–18] For all these reasons, the double-slit experiment, under polarized illumination, can have a strong appeal for presenting and exploring classical and contemporary experiments in optics; it can be a lasting experience in the curricula of an optics student.

Our aim is to discuss, in a simple fashion, the general properties of the fringes obtained in the double-slit experiment under polarized light. Next, we show how these properties can be applied to implement, in an easy way, three modern techniques of optical image processing.

In Sec. II we discuss in a simple, although general way, several properties associated with the interference fringes obtained in the double-slit experiment if polarized illumination is used. In Sec. III we discuss an equivalent experimental setup that makes Young's experiment more versatile. Finally, in Sec. IV, the above results are applied to implement the following experiments: the weighted addition and subtraction of two images, a method for obtaining a contrast reversal version of an image, and the codification of gray level information into pseudocolors.

II. BASIC THEORY

Let us consider the classical, double-slit setup as shown in Fig. 1. S represents a slit source with monochromatic light, which is linearly polarized after passing through the polarizer P. In front of each of the two slits, S_1 and S_2, there is a linear polarizer (P_1 and P_2, respectively). The transmission axes of P_1 and P_2 are mutually perpendicular. The plane (x,y) of observation is located at the plane 0 behind the analyzer A. For our discussion, we employ the coordinate system that is shown in Fig. 2. The axes $0X$ and $0Y$ coincide with the transmission axes of P_1 and P_2, respectively. The angles formed by the transmission axes of P and A are denoted by Ψ and α.

In these terms the resultant complex amplitude, from S_1 and S_2, at a point Q over 0, is

$$A(x,y) = A_1 + A_2 \exp[i\phi(x,y)],$$

where

$$A_1 \propto A_0 \cos\alpha \cos\Psi, \quad \text{(1a)}$$

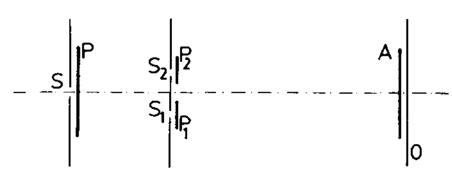

Fig. 1. Young's experiment with polarized light.