A simple method for estimating the thickness of the atmosphere by light scattering

Michael Vollmer[a)]
Physikalische Ingenieurwissenschaften, University of Applied Sciences Brandenburg, 14770 Brandenburg, Germany

(Received 20 September 2002; accepted 18 April 2003)

A simple experiment on light scattering in the atmosphere is presented, which allows us to estimate the thickness of the atmosphere. The experiment is based on an idea by Wood and uses only tubes, diaphragms, and a partially reflecting mirror as optical elements.
[DOI: 10.1119/1.1582188]

I. INTRODUCTION

This paper discusses a light scattering experiment based on an idea by Wood[1] that can be performed with the naked eye and that allows us to estimate the thickness of the atmosphere by comparing the sunlight scattered along a path through the atmosphere with that scattered along a path (of adjustable length) parallel to the ground. The setup is very simple and can easily be built in the classroom.

An assumption inherent in the comparison is that both scattering paths contain the same scattering particles. Therefore, to yield good results, the experiment must be done when the air is pure with few aerosol particles present, because aerosol particles are not distributed uniformly throughout the atmosphere. If the atmosphere contains only the gases that constitute normal air, the light scattered by such an atmosphere is described by Rayleigh theory. The light scattered from larger liquid or solid particles is significantly more complicated and is better described by Mie theory. The characteristics of Rayleigh and Mie scattering have been discussed extensively in the literature (see, for example, Refs. 2–8).

In Sec. II some characteristics of Rayleigh scattering in the atmosphere as well as scattering in general are briefly reviewed as background for the understanding of the experiment. The experimental arrangement as well as a quantitative analysis is presented in Sec. III, a comparison to related work is given in Sec. IV, and finally some possible extensions for student projects are discussed in Sec. V.

II. SCATTERING OF LIGHT IN THE ATMOSPHERE

The earth's atmosphere includes atoms, molecules, raindrops, ice crystals, and aerosols. These constituents can scatter light described by either Rayleigh or Mie scattering theory depending on the size of the particles compared with the wavelength of the incoming radiation. Here, we review the general characteristics of Rayleigh scattering that are relevant for the understanding of the suggested experiment. The scattering from larger particles, such as aerosols, whose size is comparable to or larger than the wavelength of light, is not treated here. Our experiment may be done only at times or places where the atmosphere is nearly free of these larger particles.

A. Rayleigh scattering

Rayleigh scattering describes the elastic scattering of electromagnetic waves whose wavelength λ is much larger than the size of the scatterer. This is the case for the scattering of sunlight from the electrons of atoms or molecules or from very small particles in the atmosphere. The angular distribution, polarization, and wavelength dependency of Rayleigh scattering has characteristic features.

The polarization of Rayleigh-scattered light is easily understood in terms of the classical Drude model. The electric field of the wave leads to forced oscillations of harmonically bound electrons (the resonant frequencies of electronic transitions of the air molecules are far in the ultraviolet). Consequently, the periodic acceleration of the electrons results in the emission of electromagnetic waves with the angular characteristics of a dipole antenna (see Fig. 1). In particular, no radiation is emitted in the direction of the oscillation. This behavior helps to explain the polarization of the scattered light.[9] If unpolarized light is incident on ideal Rayleigh scatterers, the scattered light should be perfectly polarized at a scattering angle of 90° (see Fig. 2).

In reality, even if very pure air is used, the degree of polarization is never 100% for several reasons. First, the air molecules are not spherical, but rather spheroidal. This shape factor results in a maximum degree of polarization of about 94%.[3,6] Second, the atmosphere usually contains an appreciable number of larger particles, which show Mie scattering with a different angular polarization dependence. Third, there are contributions of multiply scattered light as well as backscattering from the surface of the earth. Overall, the polarization typically reaches values of around 80% and depends on the concentration of particles. It is thus related to the transmission of the atmosphere (see Rozenberg in Ref. 5).

The wavelength dependence of Rayleigh scattering can be easily derived from the characteristics of dipole scattering.[10] The total intensity of the scattered light varies as a function of wavelength according to $I(\lambda) \propto 1/\lambda^4$, indicating that blue light is scattered much more efficiently than red light. Due to dispersion effects, the exponent is even slightly larger and has a value of about 4.08.[3,6]

B. Geometry and air mass factor

The geometry of the scattering of sunlight in the atmosphere is described in Fig. 3. The sunlight comes from an angle ϕ_s above the horizon. Rather than using ϕ_s, one often uses the zenith angle ϕ_z. To view sunlight scattered by 90°, we look in a direction with a zenith angle equal to the sun elevation angle ϕ_s (broken line in Fig. 3).

The zenith angle defines the air-mass factor. If an observer

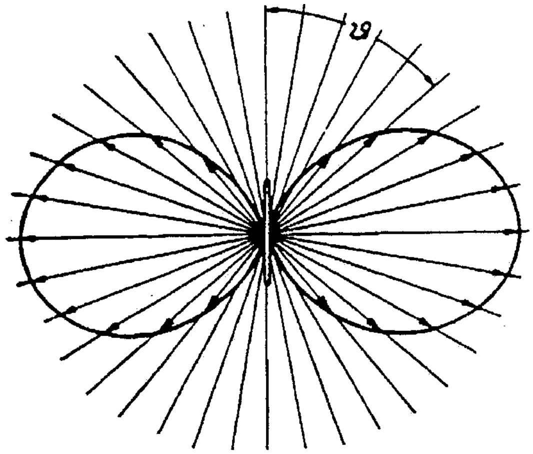

Fig. 1. Emission of electromagnetic waves (bold solid line) by a Hertzian dipole as a function of angle (the lines are in intervals of 10°). The intensity of the scattered light in a particular direction is indicated by the length of the arrows in this direction; no light is emitted along the (vertical) axis of oscillation.

looks straight up, he/she is looking through an air mass defined to be equal to unity (AM=1.0). If the observer looks up at a zenith angle of ϕ_z, the line of sight through the atmosphere is longer by about the factor $1/\cos\phi_z$ for a planar earth. For example, for a zenith angle of 30°, AM=1.15. In the context of the present experiment, AM is a measure of the number of light scatterers along a given line of sight.

C. Single versus multiple scattering

The amount of matter along a sight line determines whether single or multiple scattering dominates (see, for example, Ref. 6). The relevant parameter is the optical thickness τ, which is defined as the thickness in units of the optical mean free path. For $\tau<1$, single scattering is the dominant process. This is the case for light incident near $\phi_z=0°$. For larger ϕ_z, the path of light through the atmosphere and therefore also τ increases. For $\tau>1$, a photon has an increasing chance of being scattered many times as its path becomes longer than the optical mean free path. If the sight line is nearly horizontal, the very long path through the atmosphere gives rise to multiple scattering, which accounts for the whiter color of the sky near the horizon compared with the bluer sky near the zenith (see for example, Refs. 11 and 12).

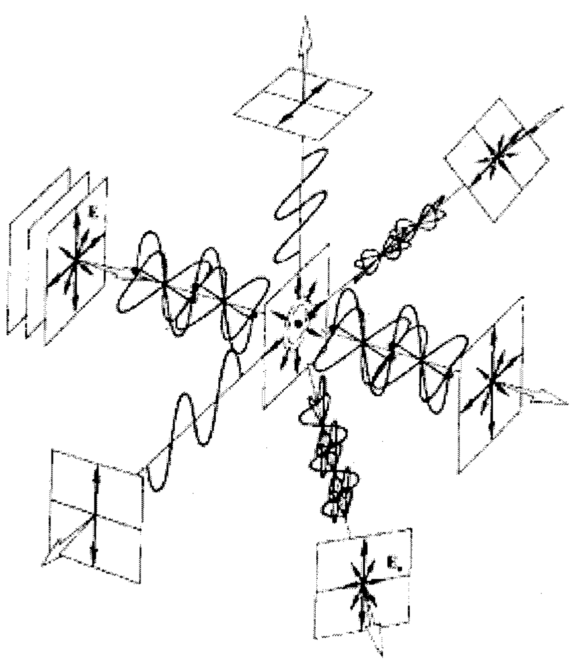

Fig. 2. Unpolarized light, indicated by the respective E-field vectors, is scattered by electrons of air molecules. At a scattering angle of 90°, the scattered light is linearly polarized (after Ref. 9).

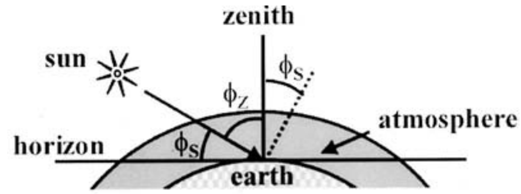

Fig. 3. The geometry for light scattering in the atmosphere with respect to an earthbound observer is usually described by the sun elevation angle ϕ_S and the zenith angle ϕ_Z. For simplicity, atmospheric refraction effects, which become important for low sun elevations, are neglected here. The curvature of the earth is strongly exaggerated compared to the thickness of the atmosphere.

III. ESTIMATING THE THICKNESS OF THE ATMOSPHERE

Several demonstrations and lab experiments of light scattering phenomena have been described in the literature (see, for example, Refs. 1, 5, and 13–17). This paper describes an old experiment that was originally suggested by Wood[1] and later discussed by Minnaert.[18]

A. Experimental arrangement and qualitative explanation

The principal idea is to compare the intensities of the scattered sunlight along two lines of sight. Both lines are at 90° to the direction of the sun. One is in the vertical plane passing through the sun, and the other is parallel to the ground. This horizontal path is limited in length by the experiment. The light scattered along the horizontal path will be denoted as air light, the light scattered along the vertical path will be denoted as sky light in the following. The basic geometry of the sun, observer, sky light, and air light is depicted in Fig. 4.

To see only the scattered light, both scattered light sources should have black backgrounds. For sky light this back-

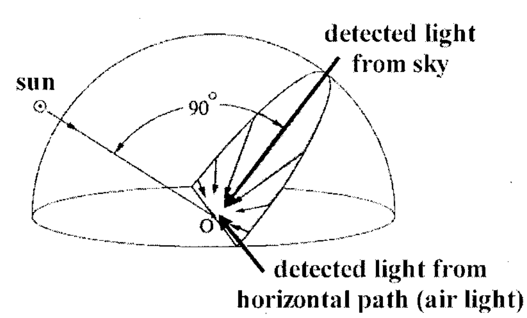

Fig. 4. Light scattering geometry for an experiment to estimate the thickness of the atmosphere by light scattering.

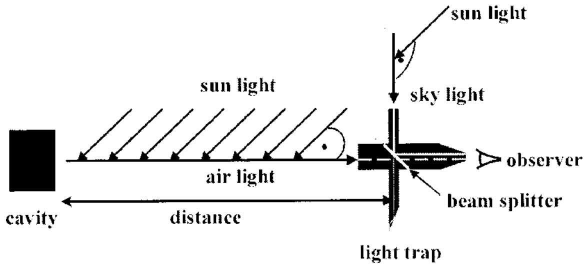

Fig. 5. Experimental apparatus for estimating the thickness of the atmosphere. For clarity, the sky and air light beams within the tube (solid and broken white lines) are shown vertically displaced in the instrument; in practice the displacement is horizontal.

ground is obviously the blackness of the universe. For the limited horizontal scattering path a black background can be achieved by looking into the direction of the opening hole of a large cavity, for example, a single window in a big room. We used an opening of about 1.2 by 1.8 m^2 in an otherwise dark room. The opening resembles a blackbody (see for example Ref. 19), absorbing nearly 100% of the incoming radiation.

To observe both sources of scattered light simultaneously, a horizontal tube (diameter equal to 10 cm and length equal to 1 m) is combined with another tube (diameter equal to 6 cm and length equal to 0.6 m) that points in the vertical plane, 90° from the direction of the sun. They both are mounted on a tripod. A beam splitter, consisting of a normal glass plate at an angle of 45° which covers half of the field of view, is inserted (see Fig. 5). A number of black apertures (not shown in Fig. 5) are inserted into the horizontal tube to trap any stray light effects of light entering the tube at an oblique angle and to prevent any air light from hitting the beam splitter. In addition, the sky light that is transmitted by the beam splitter is caught in a light trap.

In the experiment, the sun elevation was $\phi_s \approx 30°$ yielding a zenith angle of $\phi_z \approx 30°$ for the sky light. This angle corresponds to an air mass factor of AM$\approx$1.15.

The experiment consists of varying the distance of the tube from the terrestrial cavity until the brightness of the light scattered along the two paths is equal (Fig. 6). If the observer is close to the cavity, the air light will appear dimmer than the sky light. At some distance d, the brightness of the scattered air light will be the same.

The qualitative analysis of the experiment is as follows. If we assume a purely Rayleigh scattering atmosphere (deviations will be discussed below), sky light that travels along

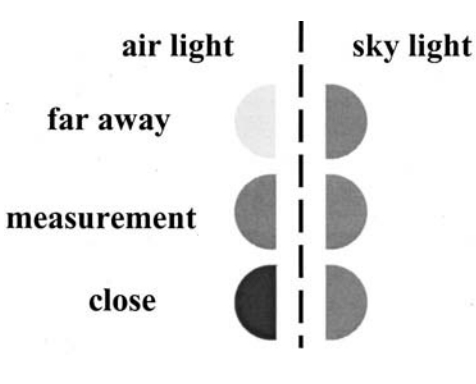

Fig. 6. The air light has less, equal, or more brightness than the sky light, depending on the length of the horizontal scattering path.

the line of sight to the eye of the observer traverses an atmosphere thickness with AM=1.15 for $\phi_z \approx 30°$. This thickness is too short for multiple scattering to be significant. Because of the low reflectivity of the partially reflecting mirror, the air mass in the horizontal beam at the brightness match is less that that in the sky light beam. Light scattered toward the eye of the observer along either line of sight is therefore of bluish color as predicted by Rayleigh scattering theory for single scattering events. The measurement itself is quite accurate, because the human eye is very sensitive to the comparative brightness of neighboring areas. However, some necessary assumptions for a simple quantitative analysis introduce greater uncertainties.

B. Quantitative analysis

To obtain a measure of the thickness of the atmosphere, we assume an isothermal atmosphere with the pressure decreasing with height h above sea level, according to the barometric formula. In a Rayleigh scattering atmosphere, the number of scatterers per unit length along the path of a light wave, N, follows an exponential, that is,

$$N(h)=N(0)\exp\left\{-\frac{h}{H}\right\}. \tag{1}$$

$N(0)$ is the number of scatterers at sea level and $H \approx 8000$ m is a measure of the equivalent thickness of the atmosphere.

If we assume only single scattering events and that all scatterers are subject to the same incident light intensity (see below), it follows that the scattered light is proportional to the total number of scatterers along the light path of length d. Under these assumptions, when the brightness of the scattered light along the two lines of sight is equal, the number of scatterers in each path is the same. If we consider the effect of a mirror with a reflectivity x in one of the beams, this statement implies that,

$$N(0)d=xAM\int_0^\infty N(0)\exp\left\{-\frac{h}{H}\right\}.\,dh, \tag{2}$$

which yields

$$d=x\mathrm{AM}\ \ \mathrm{H}. \tag{3}$$

Hence, the equivalent thickness of the atmosphere can be calculated by measuring the distance d between the tube and the cavity if AM and x are known. The air mass factor AM is easily estimated from sun's elevation. The reflectivity of the beam splitter x can either be measured or estimated theoretically. If we know the index of refraction of the beam splitter, x can be computed according to the following argument. Figure 5 shows a plane that passes through the observer and is perpendicular to the sun-observer direction. In an ideal Rayleigh-scattering atmosphere, the observer looking anywhere in this plane will see light that is 100% polarized. During a measurement, one of the tubes is oriented in a horizontal direction to view the air light, and the other also points in the plane to view the sky light. Both of the tubes lie in this plane, which coincides with the plane of incidence of the mirror. By reference to Fig. 2, we can see that sky light will be polarized parallel to the plane of incidence.

For glass with $n=1.5$, the reflectivities for perpendicular ($\perp$) and parallel ($\parallel$) polarization and an angle of incidence of 45° are $R_\perp \approx 9\%$ and $R_\parallel = 0.8\%$. If the sky light has a degree of polarization of 100% at the observation angle of 90° toward the sun, the light is polarized parallel to the plane of incidence of the beam splitter. Therefore, the front and back surfaces of the beam splitter give rise to an overall reflectivity of about 1.6%. In this case, reflections from perpendicularly polarized light (usually about 17% from the front and back surface) would not be possible. It is, however, well known that the sky light for clear skies has a polarization of only about 80%. In this case, the total sky light signal consists of $(0.8\times 1.6\%)+(0.2\times 17\%)$, i.e., the overall reflectivity of the beam splitter for the sky light amounts to $x=4.7\%$. If we assume that the polarization is within the range $80\%\pm 5\%$, we obtain $x\approx 4.7\pm 0.8\%$. The relative error of about 17% dominates the total error.

In the experiment, a distance of $d=270\pm 15$ m was found to give equal brightness for sky light and air light. From Eq. (3) with $x=0.047\pm 0.008$ and $\mathrm{AM}=1.15\pm 0.03$, we obtain $H=5000\pm 900$ m.

C. Discussion

If we compare $H=5000$ m to the expected value of $H=8000$ m, the deviation of about 37% is appreciable, but still satisfactory, particularly when considering the simplicity of the method and the fact that we assumed a purely Rayleigh scattering atmosphere. The deviations are due to a number of reasons: (1) The assumption of an isothermal atmosphere is an approximation. However, more realistic models show similar dependencies of pressure on the altitude.[18] (2) The degree of polarization is an adjustable parameter. It would be possible to obtain an estimate for H close to 8000 m if the polarization of the sky light were 91%. However, this value seems unreasonably high (compare to Rozenberg in Ref. 5). Obviously, it would be better to measure the degree of polarization independently. If this is not possible, in particular for simple experiments, an estimate of 80% seems reasonable. (3) The assumption that all scatterers are subject to the same incident light intensity is only a rough approximation. It is well known that the incident light intensity changes with height, the attenuation being due to Rayleigh scattering, as well as absorption and scattering by haze and absorption in the Chappuis absorption bands of ozone. [The Chappuis absorption bands of ozone are between 500 and 700 nm (see Fig. 3 of Hulbert in Ref. 5).] As a consequence, the light intensity at ground level is typically only about 50% of the intensity of light incident on the upper atmosphere. Therefore, the left-hand side of Eq. (2) should be corrected by a factor of 0.5. On the other hand, the light intensity also changes with height, that is, the light reaching a certain point along the line of sight of the sky light in Fig. 5 has already passed a certain path in the atmosphere. The lower the position along the line of sight, the lower is the light intensity at this position. Due to the Chappuis bands of ozone, the spectrum of the solar radiation at different heights of the line of sight of the sky light also changes, that is, the color of the light from various heights may differ slightly. As an example, Hulbert[5] has calculated the attenuation of sun light at sunset while passing through the atmosphere before reaching the zenith line of sight at various heights. Unfortunately, there are no analytical formulas for the attenuation, which due to the Chappuis bands strongly depend on wavelength.

Estimates of the effect of attenuation on the above analysis of the experiment leads to slightly worse estimates for the thickness of the atmosphere. There is, however, no simple derivation for a correction factor of this effect.

Finally, the most severe objection to the above analysis is that scattering near the ground does not only come from pure air but also from haze. In particular, there often is a layer of haze in the lower atmosphere that will affect the air light much more strongly than the sky light beam. Because the density of the haze does not follow an exponential law and because the wavelength dependence of the scattering is different, we would not expect to obtain good approximate results for the vertical sky light in a hazy atmosphere.

Obviously, the experiment only works well for a clear atmosphere; for example, at high elevations and/or very far from industrial regions. In urban areas, a qualitative criterion for good measurement conditions is a clear atmosphere. The above measurement was performed after several days of rain, giving a very clear atmosphere with an extraordinary blue sky.

IV. COMPARISON TO THE EXPERIMENTS BY WOOD AND MINNAERT

Wood compared the scattering power of dust-free air as filtered by cotton with the power of the whole atmosphere by comparing the respective scattering intensities. In this context, he described a setup similar to the present experiment.[1] Minnaert[17] used a similar setup to estimate the thickness of the atmosphere. Both authors mention that in very clear air, an intensity match of sky light with air light was found for distances of about 330 m whereas on slightly hazy days, only 130 m were found (this numerical coincidence suggests that Minnaert may have just reproduced Wood's numbers). By using a rough estimate of the reflectivity of 5% and not taking into account the air mass factor, Minnaert came close to 6.6 km. Wood made a much more thorough analysis than Minnaert and took into account the air mass factor, a rough estimate of the change of light intensity as a function of height, and the polarization of the light. Both Wood and Minnaert attribute the deviations from the theoretical expectations to the scattering of larger haze particles in the lower atmosphere as was done in the present work.

V. SOME IDEAS OF POSSIBLE EXTENSIONS FOR STUDENT PROJECTS

The method for estimating the equivalent thickness of the atmosphere by light scattering can be used for further discussions and extended analysis. Two ideas will be briefly discussed here, estimating the mass of air molecules and monitoring air pollution over extended periods of time. It is well known that in an isothermal atmosphere, the equivalent thickness of the atmosphere H is related to the mass m of a gas molecule by[20]

$$H=kT/mg, \tag{4}$$

where T is the temperature, k is Boltzmann's constant, and g is the acceleration of gravity. Equation (4) easily follows from the barometric formula, because H is related to the pressure and density at ground level, $H=P_0/(\rho_0 g)$. If we use the ideal gas law, the mass density $\rho=Nm/V$ can be substituted to find H from Eq. (4). For $T=290$ K and $H=8000$ m, the mass m in Eq. (4) is found to be 5.2×10^{-26} kg. This is reasonably close to the expectation of about $29\times 1.67\times 10^{-27}\approx 4.8\times 10^{-26}$ kg, which resembles the mass of 29 nucleons each with a mass of 1.67 $\times 10^{-27}$ kg. The number of 29 is chosen to approximately account for the respective concentrations of N_2 (28 nucleons) and O_2 (32 nucleons) in the air. The result of the experiment, $H = 5000$ m, gives $m = 8.2 \times 10^{-26}$ kg. The measurement of the equivalent thickness of the atmosphere also can be used to roughly estimate the mass of an air molecule, which is close to that of a nitrogen molecule.

The analysis of H referred to in Eq. (3) assumes that the reflectivity of the beam splitter depends on the index of refraction n of the glass as well as on the degree of polarization P of the sky light. A rough estimate of H was given by assuming typical values for n and P. The analysis can be improved by independently measuring both quantities. The index of refraction is easily determined, for example, by measuring Brewster's angle using polarized light or by measuring the transmission and reflectance for near normal incidence.

Although n stays constant, the polarization of the sky light can vary appreciably from day to day. Our analysis used typical values for clear skies as a rough estimate. If the polarization were measured accurately each day, the analysis according to Eq. (3) would be more precise. After finding a value for H for clear sky conditions, measurements with the present apparatus would also allow us to obtain some quantitative estimates for the degree of atmospheric pollution under certain conditions. Regarding the analysis, optimum conditions would be if the aerosol particles were located only in a thin horizontal layer close to the ground. In this case, the vertical signal would remain unchanged, and the decrease of d would directly reflect the number density of the aerosols.

Although this idea is very attractive, a quantitative analysis in general usually is difficult, because the vertical distribution of the aerosol particles in the atmosphere also effects the vertical signal. And, as discussed in Sec. III C, as long as the vertical distribution of the light scattering particles and the wavelength dependence of the scattering are not known, the simple theory of this paper cannot be applied.

VI. CONCLUSIONS

Historically, experiments have been of major importance for understanding light scattering in the atmosphere. The theoretical approaches of Rayleigh and Mie are well known and are used to explain many of the fascinating colorful phenomena in nature. However, both qualitative and quantitative experiments on the fundamentals of the scattering processes can help improve our understanding of these phenomena. The experiment described here is surprisingly simple to set up and allows a rough estimate of the equivalent thickness of the atmosphere. The deviations from a purely Rayleigh scattering atmosphere reveal the importance of scattering by atmospheric haze.

ACKNOWLEDGMENTS

I want to thank A. Young for stimulating discussions and R. Greenler for very helpful suggestions during the preparation of this manuscript.

[a]Electronic mail: vollmer@fh-brandenburg.de

[1]R. W. Wood, ''Light scattering by air and the blue colour of the sky,'' Philos. Mag. **39** (232), 423–433 (1920); see also R. W. Wood, *Physical Optics*, 3rd ed. (Macmillan, New York, 1934).

[2]H. C. van de Hulst, *Light Scattering by Small Particles* (Dover, New York, 1981).

[3]A. T. Young, ''Rayleigh scattering,'' Phys. Today **35** (1), 2–8 (1982).

[4]C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles* (Wiley, New York, 1983).

[5]*Selected Papers on Scattering in the Atmosphere*, edited by C. F. Bohren (SPIE Optical Engineering, Bellingham, WA, 1989).

[6]C. F. Bohren, ''Atmospheric optics,'' in *Encyclopedia of Applied Physics* (VCH, Weinheim, 1995), Vol. 12, pp. 405–434.

[7]U. Kreibig and M. Vollmer, *Optical Properties of Metal Clusters* (Springer, Berlin, 1995).

[8]''On Minnaert's shoulders: Twenty years of the Light and Color conferences,'' edited by C. L. Adler, available from ⟨www.blueskyassociates.com/CDROMS.html⟩.

[9]E. Hecht, *Optics*, 3rd ed. (Addison Wesley, Reading, MA, 1998).

[10]F. S. Crawford, Jr., *Berkeley Physics Course, Vol. 3* (McGraw–Hill, New York, 1965).

[11]D. K. Lynch and W. Livingston, *Color and Light in Nature* (Cambridge U.P., Cambridge, 1995).

[12]R. Greenler, *Rainbows, Halos and Glories* (Cambridge U.P., Cambridge, 1980).

[13]J. Tyndall, ''On the blue colour of the sky, the polarization of skylight and on the polarization of light by cloudy matter generally,'' Philos. Mag. **37** (250), 384–394 (1969).

[14]C. F. Bohren, *Clouds in a Glass of Beer* (Wiley, New York, 1987).

[15]Video by R. Greenler, ''Red sunsets, black clouds and the blue moon: light scattering in the atmosphere,'' available from www.blueskyassociates.com

[16]M. Vollmer and R. Tammer, ''Laboratory experiments in atmospheric optics,'' Appl. Opt. **37**, 1557–1568 (1998).

[17]G. Hoeppe, *Blau: Die Farbe des Himmels* (Spektrum Akademischer Verlag, Heidelberg, 1999).

[18]M. G. J. Minnaert, *Light and Color in the Outdoors* (Springer, Berlin, 1993).

[19]*Handbook of Optics, Vol. 1*, edited by M. Bass (McGraw-Hill, New York, 1995).

[20]C. Kittel, *Thermal Physics* (Wiley, New York, 1969).