Inward "Centrifugal" Force on a Helium-Filled Balloon: An Illustrative Experiment

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Uniform circular motion is a topic covered in all introductory physics textbooks. The description using a nonaccelerated reference frame, in which Newton's second law is valid, shows that the resultant must be a real force directed toward the center of curvature, which is responsible for the centripetal acceleration of the circular motion. When using an accelerated reference frame moving with the body, it is necessary to introduce a purely fictitious force in the outward direction (*centrifugal* force) that enables one to apply Newton's equation. Although both descriptions lead to the same results, the centrifugal force is frequently a source of trouble to elementary physics students when attempting to distinguish between real and fictitious forces.[1,2]

The notion of centrifugal force appears when one tries to explain the common experience of

Fig. 1. A helium-filled balloon inside a moving car. The car is turning toward the left, and the balloon is deflected toward the center of curvature.

"being pulled outward" in a circular motion. This leads to the erroneous conclusion that this force, as its own (Latin) name suggests, must be always directed outward. We propose here to analyze the motion of a helium balloon placed inside a car that is traveling around a curve. The balloon is fixed to the rear seat of the car by means of a lightweight thread in such a way that the balloon does not touch the car ceiling. Thus, we can easily see the direction of the net force acting on the balloon. This simple illustrative experiment can be used as an "outdoor classroom demonstration" and shows that the centrifugal force can be, in certain cases, directed *toward* the center of the circular trajectory.

When students are asked to guess about the motion of the balloon, most of them agree in saying that it will move toward the external part of the bend, as do the passengers of the vehicle, because of the effect of the *centrifugal* force. However, what is actually observed is that the balloon moves toward the inner part of the curve, thus giving the impression that a "special" *centrifugal* force, of opposite direction to the usual one, is exerted on the balloon (see Fig. 1).

This erroneous prediction of most students is obviously a consequence of an inaccurate application of Newton's second law using fictitious forces. However, the analysis of the problem in an inertial frame of reference fixed somewhere outside the car leads to the right answer and avoids these kinds of misunderstandings.

The explanation for the observed behavior of the balloon is as follows. The circular motion requires a net force directed toward the center of curvature. Since the tension exerted by the thread is directed outward, the force responsible

for the circular motion can be exerted only by the fluid in which the balloon is immersed. The air contained inside the car is not at rest in the inertial frame of reference, but rotates together with the car. This movement causes the local air pressure to increase with distance from the rotation axis. Consequently, the force exerted on the fluid nearer the external part of the car is greater than the force acting on the fluid that is closer to the rotation axis, and a net force directed toward the center of curvature acts on the balloon.

This reasoning can be mathematically expressed as follows. The force over each infinitesimal volume of rotating fluid of density ρ_F is

$$d\vec{F} = -\rho_{\mathrm{F}}\, \omega^2\, r\, dV\, \vec{u}_{\mathrm{r}},$$

where r denotes the perpendicular distance to the rotation axis, ω is the angular velocity, and $\vec{u}_{\mathrm{r}}$ is the unit vector in the radial direction. This force changes with radial distance and is related to the pressure change as:[3]

$$d\vec{F} = -\frac{dp}{dr}\, dV\, \vec{u}_{\mathrm{r}}.$$

Then, the net force exerted by the fluid on the helium balloon with density ρ and volume V is

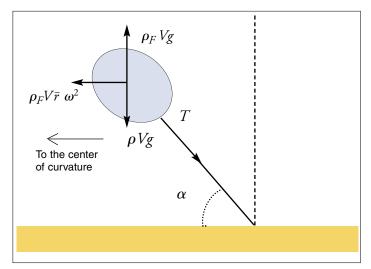**Fig. 2. Free-body diagram of the helium balloon when fixed to the rear seat of the car.**

$$\vec{F} = -\int_v \frac{dp}{dr} dV\, \vec{u}_{\mathrm{r}} = -\rho_{\mathrm{F}}\omega^2 \int_v r\, dV\, \vec{u}_r = -\rho_{\mathrm{F}} V\bar{r}\omega^2 \vec{u}_{\mathrm{r}},$$

where $\bar{r}$ is the distance from the body-mass center to the rotation axis. If we consider now the motion of the balloon, two factors must be taken into account. The first is the above force on the balloon exerted by the air contained in the vehicle, and the second is the pressure differences inside the balloon as a consequence of

the motion of the helium contained in it. As a result, there appears a radial acceleration of the balloon with respect to the rotating air given by

$$\vec{a} = \left(1 - \frac{\rho_F}{\rho}\right)\bar{r}\,\omega^2\,\vec{u}_r\,.$$

Thus, when the motion of the balloon is described using the accelerated reference frame of the rotating air, one needs an additional force to account for the motion of the balloon. If the air is denser than the gas inside the balloon ($\rho_F > \rho$, as is the case in a helium balloon), this fictitious force is directed to the center of curvature, giving rise to a paradoxical inward *centrifugal* force.

Let us come back to the nonaccelerated reference frame located at the center of curvature. The angle of the thread with the horizontal di-

rection may be used to estimate the above acceleration. Newton's second law equations for the net force in the radial and vertical directions give, respectively,

$$\rho_F\,V\bar{r}\,\omega^2 - T\cos\alpha = \rho V\bar{r}\,\omega^2$$
$$\rho_F\,Vg - T\sin\alpha - \rho Vg = 0\,.$$

This is a set of two equations for the unknowns T and α. After eliminating T, the tangent of the angle α yields:

$$\tan\alpha = \frac{g}{\bar{r}\omega^2}\,.$$

Thus,

$$\vec{a} = \left(1 - \frac{\rho_F}{\rho}\right)\frac{g}{\tan\alpha}\ .$$

For a car moving at 25 mph on a curve of radius 15 m, we get tan $\alpha \cong 0.87$, which corresponds to an angle of 50°, easily seen with the naked eye.

Comments

In summary, we propose here a simple demonstration illustrating a common misconception that arises when Newton's second law is applied in a noninertial frame of reference.[4] The analysis of the problem in an inertial frame allows for the right explanation of the phenomenon observed, and avoids the use of a paradoxical *centrifugal* force directed, in this case, to the center of curvature of the trajectory.

References

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Cartoon by Sidney Harris