Collision cross sections and the size of a coin

A C F Santos and A Fröhlich

Department of Physics, University of Missouri–Rolla, Rolla, MO 65401, USA

Abstract

An important question in education nowadays is how to convey some concepts of Modern Physics to students in high school, secondary education or the sixth form. This article presents an approach to atomic collisions suitable for intermediate level students. We describe an alternative way of measuring the area of a coin using a macroscopic analogy of the scattering of radiation by an atom or molecule—the collision cross section. We hope this article will help educators to illustrate such an important procedure and related concepts of Modern Physics.

Introduction

When a beam of particles or, more generally, radiation passes through a material many distinct processes can take place. Some particles pass directly through the material without any kind of interaction (see figure 1) while others are deflected due to their interaction or collision with the atoms of the material.

The elastic scattering of light in the atmosphere produces the blue colour of the sky. Similar examples of light scattering are common everyday experience: a rainbow, for instance, is formed by the preferential dispersion of light at a given angle when it collides with water drops. In these cases the effects of the collision are observed in the scattered particles (radiation). As a result of the interaction the target material also suffers a transformation. These transformations give rise to a large number of applications of scattering, including the use of radiation to treat cancer in patients, the use of ionizing radiation to preserve food, and the study of the composition of materials.

The interaction or collision between two particles is usually described in terms of the *cross section* [1], which is defined as: *'A measure of the probability that an encounter between particles will result in the occurrence of a particular atomic or nuclear reaction. Also called collision cross section'* [2]. This parameter is basically a measure of the effectiveness of the projectile–target interaction. The larger the cross section, the more likely it is that the projectile particles are deflected. Cross section has the dimension of area (or length squared) and represents the effective area of the region of the collision [3–5]. A cross section depends on the types of particle involved

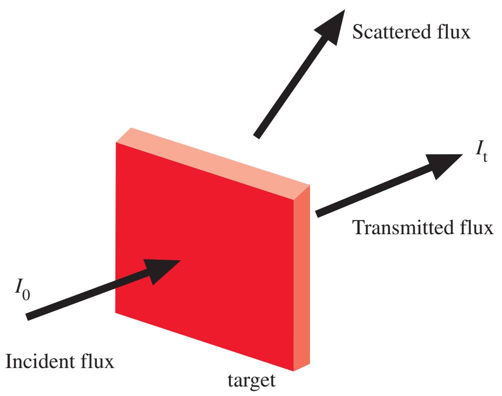

Figure 1. A beam of radiation of intensity I_0 moving through a target. A fraction of the incident beam is scattered while the remaining particles are transmitted without colliding with the target particles.

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and usually depends on the velocity or energy of the particles in the beam.

The formal definition of microscopic cross section is given as follows. Consider a beam of particles with intensity I_0, incident upon a target of density ρ, i.e. particles per unit area, as shown in figure 1. Now look at the number of particles absorbed or scattered by the target, I_s. Because of the randomness of the impact parameter, which is the distance between the projectile and a target particle, the number of scattered projectiles will fluctuate over different measurements. Even so, if one averages many measurements, this number will tend towards a fixed amount. The cross section, σ, is then defined as

$$\sigma = \frac{I_s}{\rho I_0}. \qquad (1)$$

That is, σ is the average fraction of scattered projectiles divided by the number of target particles per unit area. Equation (1) tells us that, if the number of particles present in the target, ρ, increases, so does the number of collisions, I_s, given that the cross section is a fixed number. One must divide by this factor if the cross section is to represent the effective area of interaction between a *single* beam particle and a target.

In atomic and nuclear physics, the cross section for an interaction to occur does not have to depend on the geometric area of a particle. Two particles may have the same geometric areas (sometimes known as geometric cross section) and yet have very different interaction cross sections or probabilities for interacting with a projectile particle.

We now describe a method using equation (1), which is based on the measurement of the growth of the number of scattered projectiles as a function of the target density ρ. Equation (1) can be rewritten as

$$\frac{I_s}{I_0} = \sigma\rho. \qquad (2)$$

The above equation says that the fraction of scattered projectiles, I_s/I_0, depends linearly on the target density. Therefore, the measurement of the cross sections can be reduced to the study of the dependence of the growth rate of the scattered projectiles on the target density. Consequently, the cross section can be determined from the linear part of the growth rate of the scattered projectiles, i.e. from the slope in the plot of I_s/I_0 versus ρ. In a real situation, however, the target will always contain impurities, such as residual gases from background. Since the impurities are independent of the target density and are roughly constant in a given system, they do not influence the slope of the growth rate of the main collision products, from which the cross sections are determined. In order to take into account the contribution of the residual impurities, a constant term, f_{BG}, is added to equation (2):

$$\frac{I_s}{I_0} = f_{BG} + \sigma\rho. \qquad (3)$$

Experiment

The experiment that we shall describe is a macroscopic analogy of the scattering cross sections. In the present case, the target is a sheet of paper of area S, containing many scattering centres, the coins T, and it is desired to know the size of the target coins T. A certain number of coins were uniformly distributed on the sheet of paper. In order to simulate the residual impurities present in the real situation, coins R, of a different size, were added.

Thus, the procedure for computing the cross section, or effective area, of a coin in the present experiment for which a projectile (a pen or pencil) is directed at a target is as follows.

1. Attach the paper to the floor using self-adhesive tape.
2. Measure the area of the sheet of paper.
3. Spread the background coins R uniformly on the sheet of paper[1].
4. Take roughly 100 shots and measure the number of collisions I_s of the projectile with the residual coins, R. Do not take into account any shots that miss the paper. At this point the target density ρ is still zero.
5. Add target coins T.
6. Determine the number of target particles per unit area ρ. Basically it is the number of coins T divided by the area of the sheet of paper.

[1] Here the teacher may not want to use any background coins at all. The analysis becomes easier if one uses solely target coins T. In that case, the term f_{BG} is zero.

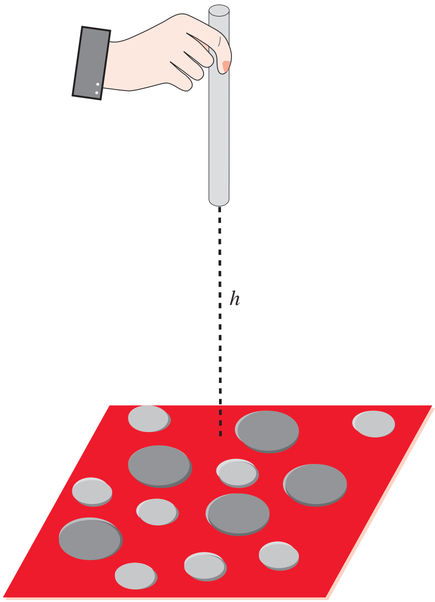

Figure 2. The projectile (grey cylinder) falls vertically from a height h onto a sheet of paper covered with coins.

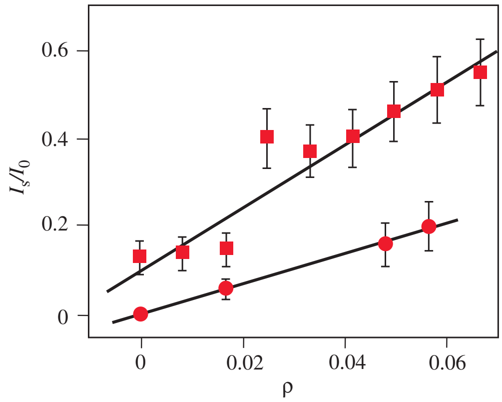

Figure 3. The growth method for pennies. Circles: the projectile is a sharp pencil and no background impurities were added. Squares: the projectile used was a pencil with dimensions comparable to the target dimensions. In order to simulate the impurities present in the background, six American quarters were used.

7. Let the projectile falls from a height h onto the sheet of paper I_0 times, as shown in figure 2. Note that h must be sufficiently high so that statistically the projectile could interact with all target coins: 1 or 1.5 m should be enough.
8. Take roughly 100 shots and measure the number of collisions I_s of the projectile with any target or residual coin, T or R. Do not take into account any shots that miss the paper. Make yourself a table showing the number of collisions I_s as a function of the number of target coins ρ per unit area.
9. Repeat items 5 to 8 until your table has a minimum of four or five rows.
10. In order to determine the interaction of the projectile with the target particles T present in the target, use the basic cross section formula given above. Take the quantity I_s and divide it by I_0. Plot I_s/I_0 as a function of ρ.

This is called the growth method for determining cross sections.

Figure 3 shows two case studies using American pennies as target coins. Case study 1 (circles): The set of data was taken without adding 'impurities' and using a sharp pencil. Case study 2 (squares): In this case, six quarters (23 mm diameter) were added as background or residual impurity and a pencil of 8 mm diameter was used. The error bars represent the statistical uncertainties for a given number of collisions, $\sqrt{I_s}/I_0$. Using the least-squares method the slopes of the plot of figure 3 are calculated as 3.45 ± 0.13 cm^2 (circles) and 7.01 ± 0.85 cm^2 (squares).

Analysis: the size of the projectile and the effect of the background

We now describe how to take into account the size of the projectile in the measurement of the size of the coins. For this calculation we shall assume that the coins are circles of radius r_1 and the cross-sectional area of the projectile is also a circle of radius r_2. We see from figure 4 that the collision will not take place unless the distance between the two centres is less than the sum $r_1 + r_2$ as they approach each other. In this case our target has an effective area given by $\pi(r_1 + r_2)^2$. In the present experiment the projectile was either a sharp pencil (diameter negligible compared with the target dimensions) or a pencil of diameter 8 mm (measured using a caliper); a penny has a diameter of 19.1 mm. In the first case (case study 1), $r_2 \ll r_1$ and the effective area is roughly the target area (2.86 cm^2). On the other hand, in the second example given above (case study 2), the dimensions of the projectile have to be taken into account. In that case, the effective area is

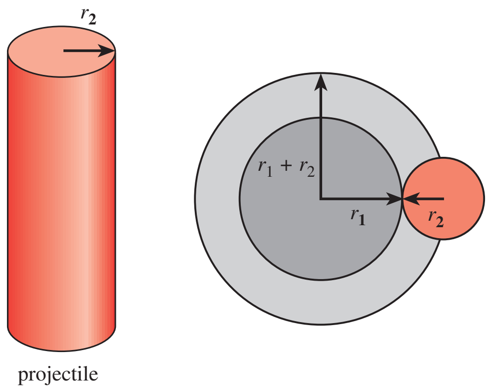

Figure 4. Left: the projectile is represented by a cylinder of diameter $2r_2$. Right: the projectile of radius r_2 and the target of radius r_1 collide if their centres are within a distance $r_1 + r_2$ of each other.

5.76 cm^2. Comparing those areas to the measured slopes from figure 3, the percentage errors are roughly 21%.

One should point out that, since the cross section for the penny is small compared with the quarters (23 mm diameter), a small percentage of impurities has a strong effect on the measured cross section. The larger the cross sections of the impurities, the stronger the effect. On the other hand, if one use quarters as target and pennies as background impurities, the effect would be minimized. In addition, using a sharp pencil makes the analysis easier since we do not have to take into account the projectile size.

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Antonio (Toni) Santos is a Visiting Scholar at the University of Missouri-Rolla. He received his PhD in physics from the Catholic University of Rio de Janeiro. He is an experimentalist who specializes in inelastic interactions between atomic and molecular particles.

Alexandre Frölich has taught physics for secondary students in Brazil for 11 years and he is now working towards a Master's degree at the University of Missouri-Rolla.