Slow group velocity propagation of sound via defect coupling in a one-dimensional acoustic band gap array

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A simple experimental system is presented in which the group velocity of acoustic wave packets traveling in an air-filled waveguide can be slowed to values much smaller than the speed of sound in air. The experiment is an acoustic analog of the much-studied optical phenomenon of slow light propagation. Slow (or even stopped) light propagation has been observed in atomic vapors in the vicinity of strong dispersion, typically associated with electromagnetically induced transparency. In the acoustic experiment described here, strong dispersion is produced by the introduction of a defect in an otherwise perfectly periodic one-dimensional acoustic band gap array. The defect produces a narrow transmission band within the forbidden acoustic band gap region resulting in strong dispersion. By tuning the carrier frequency of the acoustic wave packet to the peak transmission of the defect, the group velocity can be slowed to $0.24v_s$, where v_s is the speed of sound in air. These results are shown to be consistent with theoretical calculations.

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There has been much recent interest in the phenomenon of slow, or even stopped optical wave packet propagation in atomic vapors.[1–5] The slow optical group velocity in these atomic vapor systems is due to the high dispersion typically created by the use of electromagnetically induced transparency (EIT). EIT creates a narrow frequency transmission band within an otherwise opaque medium. The rapid change in transmission leads to strong normal dispersion giving rise to a slow group velocity for optical wave packets whose frequency is centered on the narrow transmission band. In this paper we demonstrate the acoustic analog of the slow light phenomenon by reducing the group velocity of audio wave packets by over a factor of 4. In the acoustic case considered here, an acoustic band gap waveguide filter provides the acoustically ''opaque'' medium. The narrow transmission region necessary to produce strong dispersion is introduced by the use of a defect in the otherwise periodic acoustic band gap array. An audio wave packet whose center frequency is tuned to the defect transmission band travels with significantly reduced group velocity. We show that the experimental results are consistent with a straightforward theoretical model.

Audio frequency transmission measurements on acoustic band gap waveguides were accomplished using the experimental configuration shown schematically in Fig. 1. An enclosed speaker, driven by the output of the computer's sound card, coupled audio wave packets into a long (17 m) cylindrical waveguide of diameter 1.9 cm. The audio signal transmitted through the waveguide was transduced by a microphone, digitized, and recorded. At the center of the long waveguide that ran from the speaker to microphone was a segment that could be changed between uniform 1.9-cm-diam waveguide or an equal length of diameter modulated waveguide that forms an acoustic band gap filter. The arrangement permitted the comparison of audio wave packet transmission through a normal waveguide and through the acoustic band gap filter. To ensure that the acoustic signals consisted of single transverse-mode, plane waves, the frequency range of the experiments was chosen so that the long wavelength condition was satisfied for the diameter of the waveguides used.

The acoustic band gap waveguide filter array was created by joining alternating segments of cylindrical pipes of two different diameters of 1.9 and 3.2 cm. For the results presented here, the diameter-modulated waveguide array consisted of seven segments of pipe, six of which were 17.5 cm long and the center section, which was 35 cm long. Because of the acoustic impedance mismatch between different diameter pipes, sound waves traveling down the waveguide experience a reflection at each interface. This effect, combined with the overall 17.5 cm periodicity, gives rise to acoustic band gaps—frequency intervals in which transmission is strongly attenuated. The periodic system acts as a quarter-wave interference filter with a fundamental band gap at about 485 Hz (using $v_s = 340$ m/s and $\lambda/4 = 0.175$ m) and higher order band gaps at odd integer multiples of 485 Hz. The presence of the central 35 cm defect disrupts the periodic interference and leads to a narrow transmission band exactly in the center of the gap. A more complete description of the theoretical and experimental details of similar acoustic filters and defect modes is given in Ref. 6.

The experimental procedure consisted of two distinct measurements. The first determined the frequency-dependent transmission characteristics of the acoustic band gap waveguide, in particular the center frequency and transmission bandwidth of the defect mode. The second measurement determined the different transmission times of a narrow bandwidth wave packet through a uniform-diameter waveguide segment versus the acoustic band gap filter with a defect mode.

An impulse response method was used to characterize the band gaps and defect modes in the diameter-modulated waveguide array. A very short pulse with a correspondingly broad frequency spectrum was created numerically and saved as a computer audio file. The pulse was played repetitively through the stereo output of the computer's sound card. One channel of the stereo signal was fed to the trigger

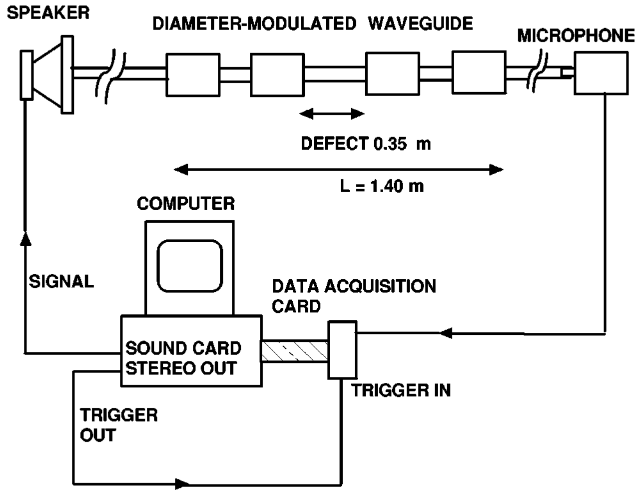

Fig. 1. Schematic of experimental configuration.

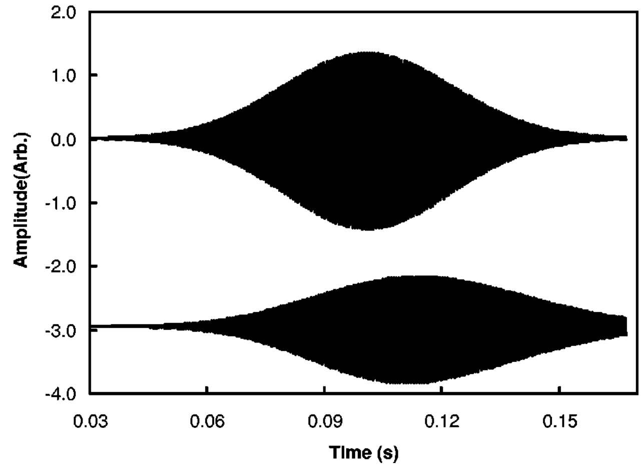

Fig. 3. Time wave packets for transmission through a uniform diameter waveguide (upper curve) and through the diameter modulated waveguide with a defect (lower curve).

input of a data acquisition card. The other channel was sent to a speaker coupled to the end of the cylindrical waveguide with the acoustic band gap filter in place in the center between the speaker and microphone. The acoustic impulse signal traveled through the waveguide and the resulting microphone signal was routed to a data acquisition card and digitized. The triggered data acquisition allowed an add-and-average technique to be used to achieve a time-domain signal with very high signal to noise ratio. The 17-m-long waveguide provided a large time-window, free from back reflections between the speaker and microphone, so that only the interference effect of the filter alone was measured in the time-domain experiments.

To provide a reference signal, an impulse measurement was performed on a piece of uniform-diameter waveguide of the same length as the diameter-modulated acoustic band gap waveguide. The transmission function of the diameter-modulated waveguide was determined by dividing the Fourier transform of the diameter-modulated waveguide time-domain impulse signal by the Fourier transform of the uniform waveguide reference impulse signal. The magnitude of the experimentally measured transmission function is displayed in Fig. 2. There are a series of acoustic band gaps at the expected frequencies and there are sharp narrow transmission peaks at the center of each band gap produced by the defect. The subsequently described experiments on wave packet transmission used the second band gap which has a defect mode at 1472 Hz near the band center, as indicated by the arrow in Fig. 2.

To determine the group velocity for propagation through the diameter-modulated waveguide, the arrival time of wave packets was measured again using the configuration of Fig. 1. The wave packets were pure sine waves modulated by a Gaussian envelope. The time extent of the Gaussian envelope was long enough that the spectra of the wave packets were narrower than the frequency width of the defect mode determined in the impulse response measurement. The wave packet had a carrier frequency of 1472 Hz, exactly coincident with the peak frequency of the defect mode. The wave packets were created numerically and stored as one channel of a stereo computer audio file. The second stereo channel consisted of a smooth sharp pulse that was used to trigger the data acquisition so that the add-and-average technique could again be used.

Using this narrow bandwidth Gaussian envelope wave packet, two separate measurements were made. The first measurement was made using a uniform-diameter waveguide segment to confirm that the wave packet travels in the normal waveguide at the speed of sound. The top trace of Fig. 3 shows the recorded trace of the wave packet that traveled through the uniform-diameter waveguide segment. Next, an equal length of the diameter-modulated acoustic band gap waveguide replaced the uniform-diameter waveguide and the arrival of the wave packet was again recorded. The lower trace of Fig. 3 shows the propagation of the Gaussian wave packet in this case. This wave packet is clearly delayed with respect to the wave packet that traveled through the straight waveguide, indicating a group velocity below that of the speed of sound. This result is the acoustic analog to the optical "slow light" experiments.[1–5] The wave packet is also slightly spread out and reduced in amplitude. The amplitude reduction results because of the imperfect transmission of the defect mode of the filter. The spread in the wave packet results from the fact that the dispersion varies with frequency over the narrow transmission band of the defect mode. Selecting a longer time envelope function with a correspondingly narrower frequency spread can mitigate this effect.

The slow group velocity can be quantified by measuring the difference in arrival time, Δt, between the wave packets

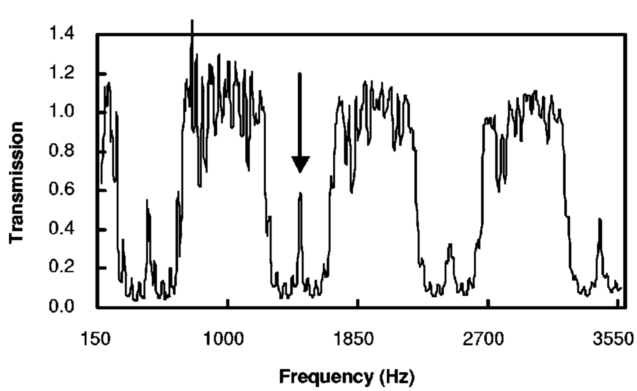

Fig. 2. Measured transmission through a diameter modulated waveguide with a defect. The defect transmission mode used in the experiment is indicated with an arrow.

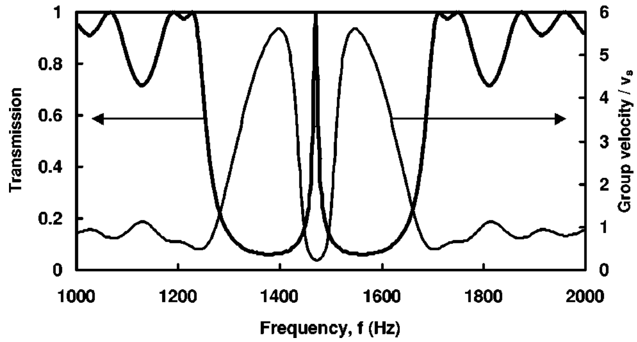

Fig. 4. Theoretically calculated transmission (with respect to the left-hand axis) and group velocity (right-hand axis) as a function of frequency.

traveling through the diameter-modulated waveguide and through the same length of uniform-diameter waveguide. The altered velocity occurs only over the length, L, of the diameter-modulated waveguide. The relation that permits the group velocity of the slow wave packets to be extracted is given by

$$\frac{L}{v_s} - \frac{L}{v_p} = \Delta t,$$

where v_s is the speed of sound in air, and v_p is the group velocity of slow wave packets. The appropriate value of the time delay for the data shown in Fig. 3 is $\Delta t = -0.014$ s. For the results shown in Fig. 3 the length $L = 1.45$ m. Using these values the group velocity of the defect mediated wave packet is $80\,\mathrm{m/s} = 0.24\,v_s$, significantly less than the speed of sound.

The variation of group velocity as a function of frequency tuning through the band gap and defect modes can be modeled theoretically. As described in Ref. 6, it is possible to accurately calculate the complex transmission function of diameter-modulated waveguides in the long wavelength limit. By forming the product of the Fourier transform of an incident wave packet with the theoretically calculated transmission function of the diameter-modulated waveguide, the spectrum of the corresponding transmitted wave packet is determined. The temporal form of the transmitted wave packet is obtained from an inverse Fourier transform of the transmitted wave packet spectrum. Comparison of the time difference between the peaks of the incident and transmitted wave packets permits the group velocity of the wave packet to be calculated. Figure 4 is a composite of the theoretically calculated frequency dependence of the transmission function of the acoustic band gap waveguide (left-hand axis) and the group velocity (right-hand axis). As expected, the group velocity is close to the speed of sound, v_s, in the allowed transmission zones outside of the band gap. Within the forbidden transmission regions the group velocity increases to a value well in excess of v_s. This phenomenon is due to the fact that the wave packets in these frequency regions tunnel through the forbidden band gap. As has been recently described,[7] in the process of tunneling the group velocity can exceed the speed of sound. This acoustic effect is the analog of the much-studied optical phenomena of superluminal wave packet tunneling.[8–13] Finally, of most relevance to the process studied here, in the vicinity of the defect mode the group velocity sinks to a value well below v_s. The experimentally measured group velocity of the defect-mediated wave packet ($0.24\,v_s$) agrees reasonably well with the theoretical value of $0.2\,v_s$.

In conclusion, we have demonstrated what we believe to be the first observation of slow group velocity of sound associated with the high dispersion in the vicinity of a narrow transmission band defect mode in an acoustic band gap system. Potential application of this phenomenon includes the development of novel acoustic filters particularly in acoustic band gap materials with periodicities in two or three dimensions. Furthermore, the strong confinement of sound in the vicinity of the defect mode could be of use in developing strong acoustic fields necessary for macrosonics applications.

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