AAPT UNITED STATES PHYSICS TEAM
AIP 2002

2002 Semi-Final Exam

INSTRUCTIONS
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems.
- After you have completed Part A, you may take a break.
- Then work Part B. You have 90 minutes to complete both problems.
- Show all your work. Partial credit will be given.
- Start each question on a new sheet of paper. Be sure to put your name in the upper right-hand corner of each page, along with the question number and the page number/total pages for this problem. For example,

  Doe, Jamie
  A1 – 1/3
- A hand-held calculator may be used. Its memory must be cleared of data and programs. It may not be used in graphing mode. Calculators may not be shared. You may not use any tables, books, or collections of formulas. You may use a ruler or straight edge.
- Questions with the same point value are not necessarily of the same difficulty.
- Good luck!

Possibly Useful Information

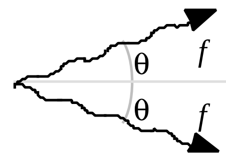
Gravitational field at the Earth's surface	$g = 9.8$ N/kg
Newton's gravitational constant	$G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Coulomb's constant	$k = 1/4\pi\varepsilon_o = 9.0 \times 10^{9}$ N·m^2/C^2
Biot-Savart constant	$k_m = \mu_o/4\pi = 10^{-7}$ T·m/A
Speed of light in a vacuum	$c = 3.0 \times 10^{8}$ m/s
Boltzmann's constant	$k_B = 1.38 \times 10^{-23}$ J/K
Avogadro's number	$N_A = 6.02 \times 10^{23}$ (mol)$^{-1}$
Ideal gas constant	$R = N_A k_B = 8.31$ J/(mol·K)
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}$ J/(s·m^2·K^4)
Elementary charge	$e = 1.6 \times 10^{-19}$ C
1 electron volt	1 eV = 1.6×10^{-19} J
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s = 4.14×10^{-15} eV·s
Electron mass	$m = 9.1 \times 10^{-31}$ kg = 0.51MeV/c^2
Binomial expansion	$(1 + x)^n \approx 1 + nx$ for $\lvert x \rvert << 1$

2002 Semi-Final Exam
Part A

A1. A wire of length l, radius r_1, and resistivity ρ is tightly wound in a single layer into the shape of a solenoid with circular cross section of radius r_2. Assume it is an ideal solenoid. A DC voltage V is placed across the ends of the solenoid.

(10) a. What is the magnitude of the magnetic field inside the solenoid?
(10) b. What is the self inductance of the solenoid?
(5) c. If the DC voltage source is replaced with an AC source with rms voltage V_{rms} and frequency f, what is the rms current I_{rms} through the solenoid?

A2. A particle decays into two photons which are both observed to have frequency f, one moving at an angle θ above the positive x-axis and the other at an angle of θ below the positive x-axis. See the accompanying diagram. The energy and momentum magnitude of a relativistic particle of mass m and speed v are:



$$E = \frac{mc^2}{\sqrt{1-(v/c)^2}} \qquad p = \frac{mv}{\sqrt{1-(v/c)^2}}$$

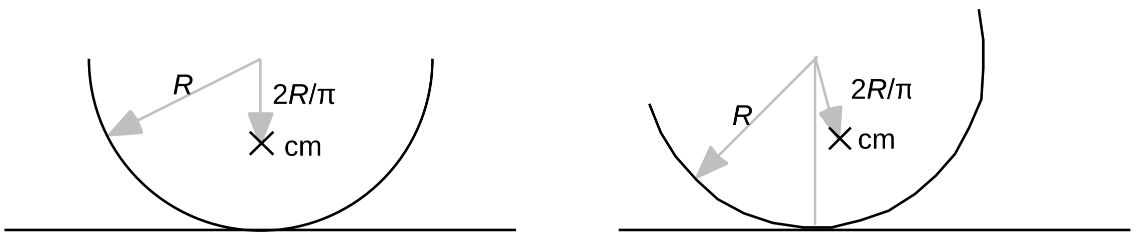
(15) a. Find the velocity of the original particle.
(5) b. Find the rest mass of the original particle.
(5) c. What are the frequencies of the decay photons in the rest frame of the particle? What are their directions of travel in this rest frame?

(25) **A3**. Fundamental quanta of length, time, and mass are the Planck length λ_P, the Planck time t_P, and the Planck mass m_P. Use the fact that they depend only on the Newton's gravitational constant G, Planck's constant h, and speed of light in a vacuum c and no other constant to derive expressions for them and then calculate their numerical values.

A4. A 0.75-m rod has a uniform linear mass density of λ. A small mass m with negligible volume is attached to one end of the rod. The rod with the attached mass is placed in a container of unknown fluid and after oscillating briefly comes to rest in its equilibrium position. At equilibrium it floats vertically (mass m end down) with 2/3 of its length submerged. If the rod were fully submerged it would displace 7.5×10^{-4} kg of fluid.

(10) a. What is the maximum value that the mass m can have?
(10) b. What is the minimum value that the mass m can have?
(5) c. Sketch a graph that shows the values of λ as a function of m.

2002 Semi-Final Exam
Part A



B1. A thin semicircular hoop of radius R and mass m rocks back and forth without slipping on a rough surface. The center-of-mass of this hoop lies a distance $2R/\pi$ below its center of curvature when it is in its equilibrium position. See diagrams above.

(10) a. What is the semicircular hoop's moment of inertia about its center of mass?

(20) b. Find $T_{\text{no slip}}$ the period of oscillation for small amplitude oscillations. Ignore all terms higher than first order in the oscillation angle θ.

(20) c. The hoop now undergoes oscillations on a frictionless surface with the period of oscillation T_{slip}. Ignore all terms higher than first order in the oscillation angle θ. Find the ratio of T_{slip} to $T_{\text{no slip}}$.

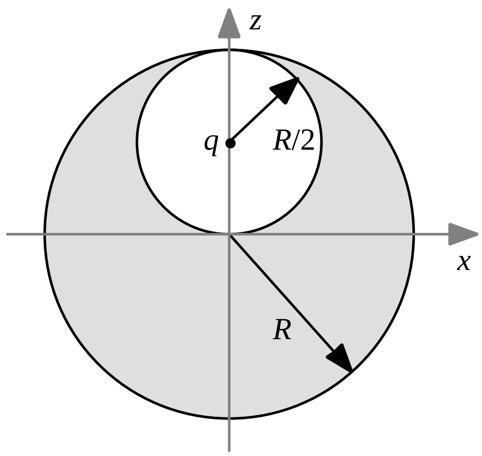


Figure B2-1

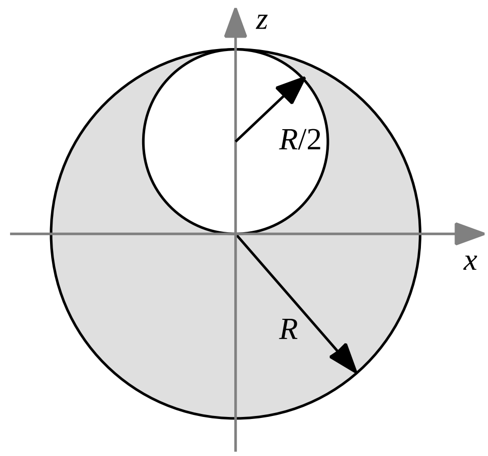


Figure B2-2

B2. A total charge $+Q$ is uniformly distributed throughout a sphere of radius R. Charge is removed from a spherical hole of radius $R/2$ centered on the $z = R/2$ point of the z-axis and concentrated into a point charge q which is placed at the center of the hole. See Fig. B1-1 above.

(5) a. Find the charge q in terms of Q.

(10) b. Find the electric field at all points along the x-axis.

(10) c. The charge q is now removed, leaving the charge distribution seen in Fig. B2-2 above. Find the electric field at all points along the z-axis.

(10) d. Find the electrostatic potential at an arbitrary point x, y, z outside the sphere of radius R as shown in Fig. B2-2 above. Assume the potential vanishes at an infinite distance from the charges.

(10) e. Very far from the charge distribution, the electrostatic potential may be written in the following form

$$V = k\left[\frac{a}{r} + \frac{\vec{b}\cdot\vec{r}}{r^3} + \dots\right]$$

where r is the distance from the origin at the center of the large sphere. Using your result from part d) above, determine the constant a and the constant vector $\vec{b}$. ($\vec{b}$ is called the electric dipole moment.)

(5) f. Interpret a and $\vec{b}$ in terms of a set of discrete charges and their locations.

The 2002 Semi-Final Examination questions were written by the coaches of the United States Physics Team. The coaches are: Academic Director, Mary Mogge – Professor of Physics at California State Polytechnic University, Pomona, CA; Senior Coach, Leaf Turner – Physicist in the Theoretical Division of Los Alamos National Laboratory, Los Alamos, NM; Warren Turner – Physics Teacher at Massachusetts Academy of Math and Science, Worcester, MA; Robert Shurtz – Physics Teacher at the Hawken School, Gates Mills, OH; Boris Zbarsky –undergraduate student at MIT, Cambridge, MA; Andrew Lin – undergraduate student at Yale University, New Haven, CT. Questions A2, A4, B1, and B2 were suggested by Leaf Turner and Questions A1 and A3 by Mary Mogge.