NOTE

# Heating Water: Rate Correction Due to Newtonian Cooling

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Newton's law of cooling is one of those empirical statements about natural phenomena that shouldn't work, but does. Objects change their temperature because of the often simultaneous processes of heat conduction, convection, and radiation. Each of these processes—for small temperature differences from their surroundings—has a rate of heat transfer proportional to the ambient temperature difference that leads to an exponential decrease of temperature with time. This is Newton's law of proportional cooling.[1] The cooling constant of proportionality depends on details of geometry and materials.

Most introductory textbooks neglect this cooling and show a linear graph of temperature rise versus time when a container of liquid water is heated from 0°C to 100°C. However, in a laboratory exercise the rising temperature curve is not linear (see Fig. 1). The deviation from linearity can be quantitatively explained by an application of Newton's law of cooling. Students can find Newton's cooling constant by measuring the cooling curve of temperature versus time after boiling when the heat source is removed. The constant is used in a correction term in the heating equation to calculate a realistic heating curve.

## Equations for Temperature Versus Time in Heating and Cooling

The rate of temperature rise, $\dot{T}$, for a water mass, $m$, heated at a constant rate, $\dot{Q}$, is

**Table I. Constants used or measured in the experiment.**

| | | |
|---|---|---|
| Mass of water | $m$(kg) | 1.00 |
| Heating rate | $\dot{Q}$(J/s) | 192 |
| Specific heat | $c$(J/kg°C) | 4180 |
| Newtonian constant | $\alpha$(J/°C s) | 1.74 |
| Cooling time constant | $mc/\alpha$(s) | 2400 |
| Room temperature | $T_{RT}$(°C) | 22 |
| Initial heating slope | $\dot{T}_h$ (°C/s) | 0.046 |
| Initial cooling slope | $\dot{T}_c$ (°C/s) | -0.030 |

$$\dot{T} = \dot{Q}/(mc) \qquad (1)$$

where $c$ is the specific heat of water. The dot over a symbol indicates a time derivative, for example, $\dot{T} = \frac{dT}{dt}$. When Newtonian cooling is neglected, the solution to Eq. (1) is a linear relation up to the boiling point

$$T - T_{RT} = \left(\frac{\dot{Q}}{mc}\right)t \qquad (2)$$

where $T_{RT}$ is the room temperature. When Newtonian cooling is taken into account, Eq. (1) is modified to

$$\dot{T} = \left(\frac{\dot{Q}}{mc}\right) - \left(\frac{\alpha}{mc}\right)(T - T_{RT}) \qquad (3)$$

with $\alpha$ the Newtonian constant for the specific setup. The solution[2] to Eq. (3) is

$$T - T_{RT} = \left(\frac{\dot{Q}}{\alpha}\right)\left[1 - e^{-\frac{\alpha t}{mc}}\right] \qquad (4)$$

for $T < 100^{\circ}$C. If we assume cooling is much less than the heating rate, Eq. (4) can be approximated as

$$T - T_{RT} = \left(\frac{\dot{Q}}{mc}\right)\left[t - \frac{1}{2}\left(\frac{\alpha}{mc}\right)t^2 + \cdots\right] \qquad (5)$$

The first term shows the linear heating rise, Eq. (2); the second term shows a quadratic cooling correction.

## Analysis

The constants $\dot{Q}$ and $\alpha$ can be determined from the experimental heating and cooling data. The value of $\dot{Q}$ is found by measuring the slope of the heating curve near room temperature where Newtonian cooling is negligible. The value of $\alpha$ is found by measuring the initial slope of the cooling curve

$$T = \left(T_{max} - T_{RT}\right)e^{-\frac{\alpha t}{mc}} \qquad (6)$$

where $T_{max}$ is the temperature where cooling began, e.g., slightly below 100°C, to extract $mc/\alpha$, the 1/e cooling time constant.

Once the two constants are determined from the data, the full heating curve can be calculated from Eq. (4)

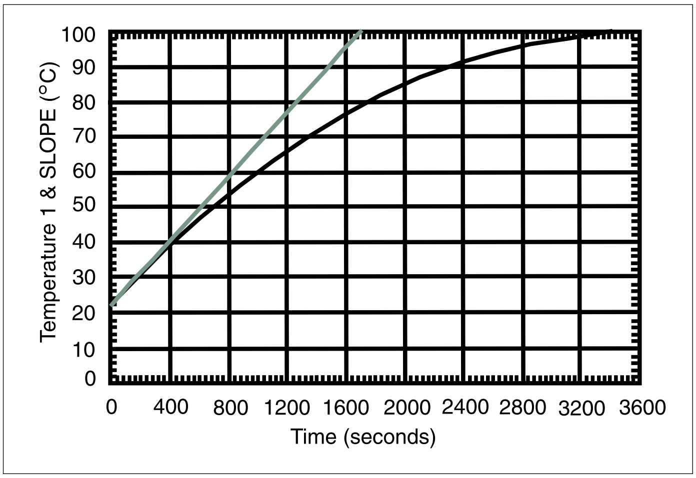


**Fig. 1. Temperature-vs-time graph for water heating from room temperature to boiling. Straight line is the initial slope, $\dot{T}_h$, used to measure the heating rate, $\dot{Q}$.**

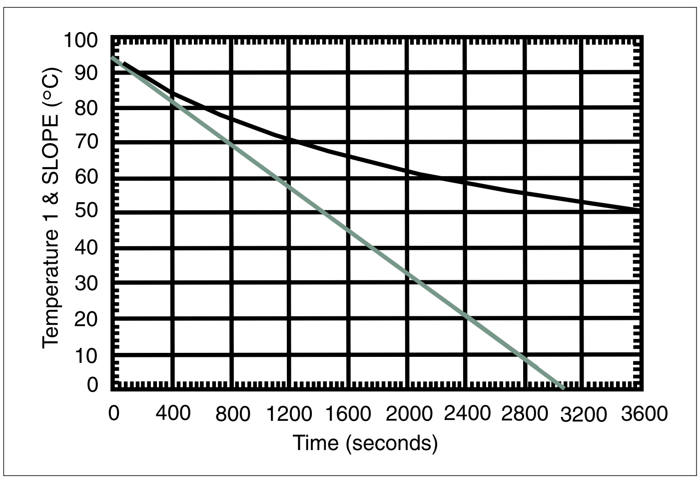


**Fig. 2. Temperature-vs-time graph for water cooling from near boiling to room temperature. Straight line is the initial slope, $\dot{T}_c$, used to measure the heating rate, $\alpha$.**

and compared with the data to test the assumption that the temperature nonlinearity is due to Newtonian cooling. For the constants given in Table I, the deviation of Eq. (4) from the data is less than 2°C over the heating range except near the boiling point where latent-heat loss adds to the cooling correction.

If the heating rate is too low, the boiling temperature is never reached. In that case, Eq. (4) can be used to find the equilibrium temperature when cooling balances heating

$$T_{\text{equil}} - T_{\text{RT}} = \left(\frac{\dot{Q}}{\alpha}\right) \qquad (7)$$

The data in Figs. 1 and 2 were obtained by placing a one-liter beaker of room-temperature water on an electrically heated hot plate operating at a moderate setting. Rapid heating does not allow time for the development of Newtonian cooling. After the data were collected, showing the change from room-temperature to boiling (Fig. 1), the beaker was removed to an insulated surface while the cooling data was collected (Fig. 2). The initial slopes of both curves were measured to determine $\dot{Q}$ and $\alpha$, and then Eq. (4) was compared with the heating data.

## Discussion

This heat transfer experiment and its analysis can be done with high or low technology. The data can be taken with a thermometer and wall clock or with a computer connected to an electric temperature probe. The analysis can be done using calculus starting with the differential equation, Eq. (3), or with the algebraic solutions, Eqs. (4) and (6). The heating project is a straightforward student-lab experiment about a common phenomenon to gain a basic insight into heat transfer mechanisms.

## References

1. Colm T. O'Sullivan, "Newton's law of cooling: A critical assessment," *Am. J. Phys.* **58**, 956 (Oct. 1990). See also: Craig F. Bohren, for a clarifying "Comment" on this paper, *Am. J. Phys.* **59**, 1044 (Nov. 1991).
2. Note the similarity of the heating equation to the voltage equation for charging a capacitor through a resistor, $V = V_o(1 - e^{-t/RC})$. For water, the "heat charging" is interrupted at the boiling-phase transition.
3. The data in Figs. 1 and 2 were taken using a Vernier Direct-Connect Temperature Probe connected to a Serial Box Interface and plotted—but not fitted—with the Macintosh version of Data Logger.