Cool in the Kitchen: Radiation, Conduction, and the Newton "Hot Block" Experiment

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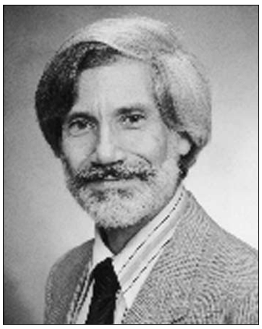
The rate at which an object cools down gives valuable information about the mechanisms of heat loss and the thermal properties of the material. In general, heat loss occurs by one or more of the following four processes: (a) conduction, (b) convection, (c) evaporation, and (d) radiation.

In *conduction*, heat is transferred through a medium by the collisional encounters of thermally excited molecules vibrating about their equilibrium positions, or, in the case of metals, by mobile, unbound electrons; only energy, not bulk matter itself, moves through the material. *Convection*, by contrast, refers to the transfer of heat through the action of a moving fluid; in free or natural convection, the motion is principally the result of gravity acting on density differences resulting from fluid expansion. *Evaporation* entails the loss of heat as a consequence of loss of mass, the faster-than-average molecules escaping from the free surface of a hot object, thereby removing kinetic energy from the system. Lastly, *radiation* involves the conversion of the kinetic and potential energy of oscillating charged particles (principally atomic electrons) into electromagnetic waves, ordinarily in the infrared portion of the spectrum. From the perspective of classical physics, charged particles moving periodically about their equilibrium positions (or indeed undergoing any kind of acceleration) radiate electromagnetic energy.

Although the physical principles behind the four mechanisms lead to different mathematical expressions, it is widely held that, if the temperature of a hot object is not too high, then the decrease in temperature in time follows a simple exponential law, an empirical result historically bearing Newton's name. As student (CRS) and teacher (MPS) we encountered Newton's law several times in our course of study together in both high-school math[1] and high-school physics.[2]



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Our curiosity was aroused. How good an approximation to reality *is* Newton's law—and what in any event determines whether the temperature of the hot object is too high? Furthermore, although Newton's name is readily associated with his laws of motion, law of gravity, and various optical phenomena and apparatus, it does not usually appear in discussions of thermal phenomena. Indeed, apart from this one instance, a search through a score or more of history of science books and thermal physics books at various levels of instruction produced but one other circumstance for noting Newton's name—and that was his failure to recognize the adiabatic nature of sound propagation in air.[3] This historical footnote accentuates, however, the circumstance that Newton pursued his interests at a time long before the concept of heat was understood. He died in 1727, but the beginning of a coherent system of thermal physics might arbitrarily be set at nearly a hundred years later when Sadi Carnot published (1824) his fundamental studies on "the motive power of fire" (*La puissance motrice du feu*). What, then, prompted Newton to study the rate at which hot objects cool, how did he go about it, and where did he record his work?

"The heat of a little kitchen fire..."

We address the historical questions first. In stark contrast to Newton's other eponymous achievements, for which a physics teacher or student desirous of knowing their origins could turn to such ageless sources as *Principia* or *Optiks*, the paper recording the law of cooling is decidedly obscure. After much searching, we discovered a reprinting of this elusive work in an old (and rather dusty) physics sourcebook.[4] According to the author, William Francis Magie, the paper, "A Scale of the Degrees of Heat," was published anonymously in the *Philosophical Transactions* in 1701 although Newton was known to have written it. (Such a paper is briefly cited—but with no mention of the law of cooling—in *Never At Rest*, Richard Westfall's definitive biography of Isaac Newton.[5])

Despite its obscurity, this is, like much of Newton's work, a fascinating paper. In contrast to what we might expect, Newton's principal concern was *not* to nail down the precise formulation of another physical law, but rather to establish a practical scale for measuring temperature. By 1701, Newton, then about 60 years old, had long since completed the fundamental studies of his youth—motion, gravity, the calculus, spectral decomposition of light, diffraction of light, and much else—to take up the job of a British functionary. In 1695 he had been appointed Warden of the Mint, and moved from Cambridge to London. It seems reasonable to us to speculate that Newton's concern with temperature and the melting points of metals was motivated by his responsibility for overseeing the purity of the national coinage.

All the same, the experiment was vintage Newton: clever use of the simplest materials at hand to carry out a measurement of broad significance.[6] Having selected linseed oil—which has a relatively high boiling point (289 °C) for an organic material—as his thermometric substance, Newton presumed that the expansion of the oil was linearly proportional to the change in temperature. With this thermometer and a chunk of iron heated by the "coals in a little kitchen fire," Newton proceeded to establish what quite possibly was the first temperature scale by which useful measurements were made. He set 0 on his scale to be "the heat of air in winter at which water begins to freeze" and defined 12 to be "the greatest heat which a thermometer takes up when in contact with the human body." On this fixed two-point scale the "heat of iron...which is shining as much as it can" registered the value 192.

Having established the above points, as well as other intermediate values (e.g., 17: "The greatest heat of a bath which one can endure for some time when the hand is dipped in it and is kept still"[7]), Newton sought an independent procedure for confirming their validity. To do this,

> ... I heated a large enough block of iron until it was glowing, and taking it from the fire with a forceps ... I placed it at once in a cold place ... and placing on it little pieces of various metals and other liquefiable bodies, I noted the times of cooling until all these bodies lost their fluidity and hardened, and until the heat of the iron became equal to the heat of the human body. Then by assuming that the excess of the heat of the iron and of the hardening bodies above the heat of the atmosphere, found by the thermometer, were in geometrical progression when the times were in arithmetical progression, all the heats were determined.... The heats so found had the same ratio to one another as those found by the thermometer.

And thus Newton's law of cooling first saw light of day.

In fact, that small section above is *all* that Newton had to say about "Newton's law." Note that not once in the entire article does Newton mention the word "temperature." At this time the concepts of heat and temperature were poorly understood and confounded; Newton refers to both as "heat" (*calor* in Latin). Note, too, that nowhere does Newton mention the word "exponential" or give the equation of exponential form

$$T - T_0 = (T_m - T_0)e^{-kt} \tag{1a}$$

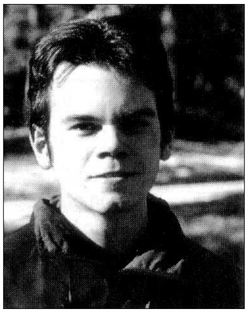
with rate constant k, ambient temperature T_0 and maximum temperature T_m) that explicitly shows the temporal variation synonymous with Newton's law. However, in verifying the points of his scale, Newton asserted that "the heat which the hot iron communicates in a given time to cold bodies...is proportional to the whole heat of the iron"—or, as we would express mathematically in current symbolism

$$dT / dt = -k(T - T_0) \tag{1b}$$

Equation (1a) is the solution to Eq. (1b), and from (1a) the reader will readily confirm that

$$\frac{T_1 - T_0}{T_2 - T_0} = \frac{T_2 - T_0}{T_3 - T_0} = \frac{T_3 - T_0}{T_4 - T_0} = \ldots = e^{k\Delta t} \tag{1c}$$

where the temperatures T_1, T_2, T_3, ... are all measured at equal intervals of time ($t_1 = \Delta t$, $t_2 = 2\Delta t$, $t_3 = 3\Delta t$...). This is the "geometrical progression" of temperatures (above the



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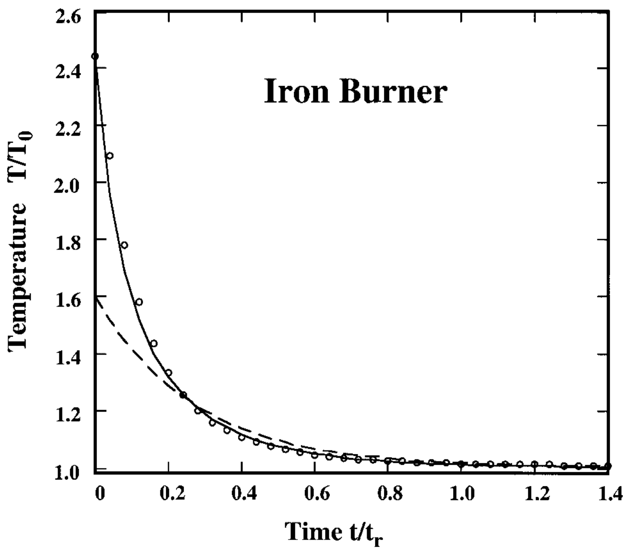

Fig. 1. Variation of reduced temperature with reduced time for the cooling of an electric burner. Circles mark experimental points; solid line represents Stefan's law (heat loss by radiation); dashed line is an exponential fit (Newton's law). Radiation time is t_r = 25 min.

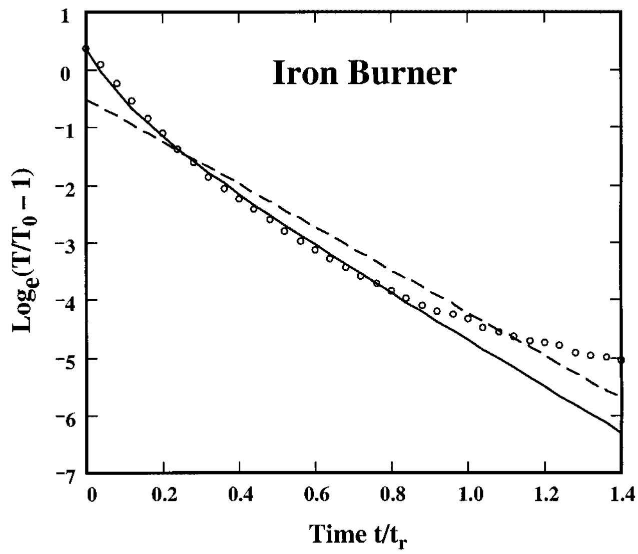

Fig. 2. Representation of Fig. 1 on a logarithmic scale.

ambient temperature) when the times are in "arithmetical progression," which Newton assumed.[8]

The law is simple and useful. But is it true?

Radiant Energy—The "Hot-Block Kitchen Experiment" (300 Years Later)

"It is certain," wrote Benjamin Thompson (Count Rumford) at the opening of his own seminal paper on the flow of heat,[9] "that there is nothing more dangerous in philosophical investigations than to take anything for granted, however unquestionable it may appear, till it has been proved by direct and decisive experiment." Thus inspired, we retired to our kitchen to test, as best we could, the law governing the cooling of a hot block of iron.

As a substitute for the block of iron and Newton's open kitchen fire (which surely would have invalidated our home insurance contract), we used, instead, an electric range and turned the right rear burner on HI so that it "was shining as much as it can." The ambient temperature was measured to be 25.5 °C with a mercury-in-glass thermometer, which we also used to calibrate a digital thermocouple thermometer[10] placed in contact with the burner. The glowing burner registered 456 °C, which would appear to be somewhat cooler than Newton's kitchen fire.[11] All the same, it was hot enough to test Newton's law.

Turning the range off, we simultaneously activated a stopwatch and recorded the temperature of the burner at intervals of one minute for a total of 35 minutes, at which time it approached ambient temperature closely enough to terminate the experiment. The temperatures, measured to a precision of 1 °C for $T \geq 200$ °C and 0.1 °C for $T < 200$ °C, are plotted with small circles in Fig. 1. It is convenient and instructive to plot the data as dimensionless quantities. The vertical axis gives the ratio of the instantaneous temperature to the ambient temperature, all temperatures being in degrees kelvin. The horizontal axis registers the time in units of a characteristic "radiation time" t_r, which in this experiment was found to be 25 minutes. The dashed line in the figure is the exponential curve (i.e., Newton's law) obtained as a least-squares fit to the data. The fitting procedure, performed with *Cricket Graph*, minimizes the sum of the square of the deviations of a straight line from the natural logarithm of $T - T_0$, which, according to Eq. (1a), should be a linear function with slope $-k$ and intercept $\log_e (T - T_m)$.

It is clear that Newton's law does not represent very well the mechanism of heat loss. If not Newton's, then what law governs the physics at work here?

We believe that under the conditions of our experiment—initially glowing solid iron in (for the most part) stationary air—the principal mode of heat loss is radiation until the reduced temperature (T/T_0) has fallen to about 1.2. The net rate (dQ/dt) at which a hot body immersed in an ambient medium of temperature T_0 loses energy by radiation is given by Stefan's law[12]

$$\left(\frac{dQ}{dt}\right)_{\rm rad} = -\epsilon\, \sigma A\, (T^4 - T_0^4) \qquad (4)$$

in which ϵ is the emissivity of the material, $\sigma = 5.67 \times 10^{-8}$ W/m$^2\cdot$K^4 is a universal constant, A is the effective radiating area, and T is the absolute temperature. The first term on the right-hand side is the radiant power lost to the environment; the second term is the radiant power received from the environment. A general thermodynamic argument can be given (although not here) that the material parameter ϵ must be the same for both radiant emission and absorption. Note that the rate of energy loss is proportional to the fourth power of T whereas in Newton's law [Eq. (1b)], it is proportional to the first power of T. [Under the present circumstances, Eq. (1b) corresponds to net cooling by conduction, as will be seen shortly.]

When an object radiates an amount of energy dQ, the drop in temperature dT depends linearly on the mass m and specific heat capacity c of the material

$$dQ = mc\,dT \tag{5}$$

Upon substituting (5) into (4) and dividing both sides of the equation by T_0, we cast Eq. (4) into the dimensionless form

$$\frac{d\mathcal{T}}{d\tau} = 1 - \mathcal{T}^4 \tag{6}$$

with reduced temperature $\mathcal{T} \equiv \frac{T}{T_0}$ and reduced time $\tau \equiv \frac{t}{t_r}$, the characteristic radiation time referred to earlier being

$$t_r = \frac{mc}{\epsilon\sigma A T_0^3} \tag{7}$$

Equation (6) no longer explicitly contains material properties or physical constants and can be solved readily by separating variables, decomposing the right-hand side into a sum of rational terms

$$d\tau = \frac{d\mathcal{T}}{1-\mathcal{T}^4} = \frac{1}{4}\left(\frac{d\mathcal{T}}{1-\mathcal{T}} + \frac{d\mathcal{T}}{1+\mathcal{T}} + \frac{2d\mathcal{T}}{1+\mathcal{T}^2}\right)$$

and applying the elementary integration formulas for the natural logarithm and inverse tangent. This leads to the implicit relation for $\mathcal{T}$

$$\left(\frac{\mathcal{T}-1}{\mathcal{T}+1}\right) = \left(\frac{\mathcal{T}_m - 1}{\mathcal{T}_m + 1}\right) e^{-2(\arctan\mathcal{T}_m - \arctan\mathcal{T})}\, e^{-4\tau} \tag{8}$$

with $\mathcal{T}_m = \frac{T_m}{T_0}$.

With a little additional effort, it is not difficult to reduce Eq. (8) to an approximate explicit relation for $\mathcal{T}(\tau)$. Combine the two phase terms into a single phase by using the trigonometric identity,[13]

$$\arctan x + \arctan y = \arctan\left(\frac{x+y}{1-xy}\right),$$

make the small-argument ($x < 1$) approximations $\arctan x \approx x$ and $e^x \approx 1 + x$, and carry through the algebraic manipulations to isolate $\mathcal{T}(\tau)$, obtaining

$$\mathcal{T} = 1 + \frac{2(\mathcal{T}_m - 1)(\mathcal{T}_m^2 - 2\mathcal{T}_m + 3)}{(\mathcal{T}_m^2 + 1)[(\mathcal{T}_m + 1)e^{4\tau} - \mathcal{T}_m + 1] - 4(\mathcal{T}_m - 1)} \tag{9}$$

Applied to Eq. (8), the small-argument approximation implies that $\frac{\mathcal{T}_m - \mathcal{T}}{\mathcal{T}_m^2 + 1} << 1$, which is best fulfilled when $\mathcal{T}$ is close to its maximum value. In other words, we would expect the reduction (9) to describe radiative heat loss well, and to become progressively poorer as the temperature approaches ambient temperature (in which case radiation becomes secondary to conduction). By contrast, it is to be noted that when $\mathcal{T}$ is close to the ambient temperature ($\mathcal{T} \sim 1$), the radiative cooling law takes the form of Newton's law, for Eq. (6) becomes approximately

$$\frac{d\mathcal{T}}{d\tau} = (1-\mathcal{T})(1+\mathcal{T})(1+\mathcal{T}^2) \approx -4(\mathcal{T} - 1) \tag{10}$$

Looking again at Fig. 1, we see that this expectation is indeed borne out. The solid curve, which closely matches the experimental points, is calculated from Eq. (9) with the radiation time $t_r \sim 25$ minutes the only adjustable parameter. Figure 2, in which $\log_e(\mathcal{T} - 1)$ is plotted against τ, shows the experimental results from another perspective. Clearly the locus of experimental points (small circles) is not linear. [Actually, the log of the log of $\mathcal{T}$ makes a nearly straight line.] The solid line, Eq. (9), follows the experimental points up to about 0.8 t_r units of time, after which conduction sets in and pure radiation theory is no longer adequate. Note, however, that the log function greatly exaggerates what are actually small discrepancies between theory and experiment since (for any base a) $\log_a(x) \rightarrow -\infty$ as $x \rightarrow 0$. The dashed line is the least-square linear fit leading to the exponential curve in Fig. 1.

From Eq. (7) and the empirical radiation time, we can estimate the emissivity of the burner. Substituting the physical quantities $m = 0.263$ kg, $A = 0.056$ m^2, $T_0 = 299$ K, $c = 448$ J/kg·K, and $t_r = 25$ min into (7), we obtain $\epsilon \sim 0.91$, which is quite reasonable for an object with blackened, oxidized surface. By comparison, the emissivity of soot is 0.95 and that of flat black paint is 0.94.[14]

How is it possible that Newton, who started out with an even higher temperature than we did, obtained "Newton's law," i.e., an exponential decrease in temperature? Actually, who can say with certainty that he did? His short paper contains no experimental record at all of the variation in temperature of the hot iron with time. He states, but does not demonstrate, that "the heat which the iron loses in a given time is proportional to the whole heat of the iron." Moreover, no information is given as to how Newton measured intervals of time, no mean task in an age when an inexpensive Casio wristwatch (our own chronometer) did not exist.

Last and conceivably most significant, Newton did something with his hot block that we did not do with our burner: He removed it from the fire and "placed it...where the wind was constantly blowing." Newton did this specifically so that "equal parts of the air are warmed in equal

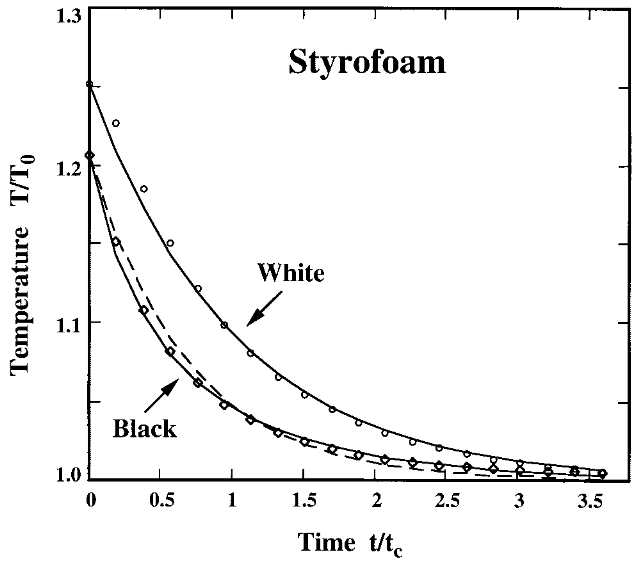

Fig. 3. Variation of reduced temperature with reduced time for cooling of a block of Styrofoam. Circles mark experimental points of white Styrofoam covered with reflective foil; diamonds mark experimental points of the same object painted black. Upper solid line is derived from Eq. (14) with conduction time t_c = 5.3 min; lower solid line is derived from Eq. (17) with radiation parameter $\gamma = \frac{t_c}{t_r} = 0.21$; dashed line is derived from Eq. (14) with conduction time t_c = 3.6 min.

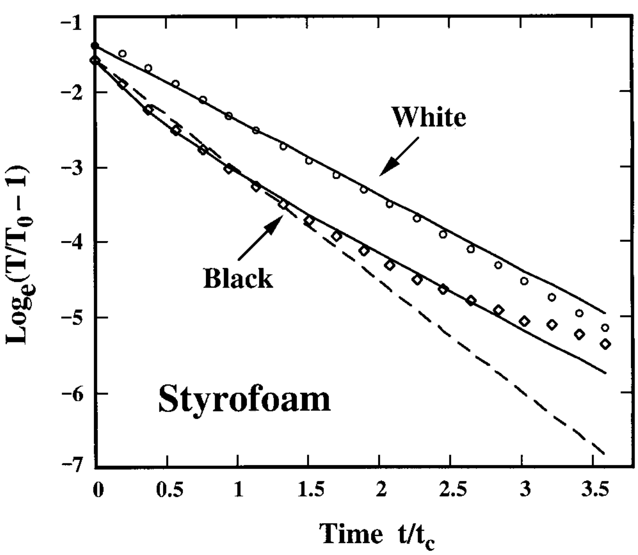

Fig. 4. Representation of Fig. 3 on a logarithmic scale.

times and carry away a heat proportional to the heat of the iron." Forced convection, which played no role in our own experiments, would have provided an additional cooling mechanism.

Study in Black and White

Having satisfied ourselves that our own hot-block experiment could be accounted for satisfactorily by Stefan's law rather than by Newton's law, we inquired next into the consequences of both conductive and radiative energy loss occurring together. It is of particular interest to ascertain whether the effects of radiation are perceptible over a temperature range sufficiently low that heat loss is dominated by conduction, and to determine whether, in fact, Newton's law provides a good model under these circumstances.

The rate at which a hot object initially at maximum temperature T_m loses heat by conduction across a region of thickness d bounded by a surface of area A is described adequately by the relation

$$\left(\frac{dQ}{dt}\right)_{con} = -\frac{k_T A}{d}(T - T_0) \qquad (11)$$

where k_T is the coefficient of thermal conductivity of the material. Using Eq. (5) to relate again dQ and dT, we can cast Eq. (11) into the dimensionless form

$$\frac{d\mathcal{T}}{d\tau} = 1 - \mathcal{T} \qquad (12)$$

where now (and for the rest of this article) we define the reduced time $\tau \equiv \frac{t}{t_c}$ in terms of the characteristic "conduction time"

$$t_c = \frac{mcd}{k_T A} \qquad (13)$$

The form of Eq. (12) is precisely that of Newton's law, and the solution is

$$\mathcal{T} = 1 + (\mathcal{T}_m - 1)e^{-\tau} \qquad (14)$$

or equivalently (in terms of original variables) Eq. (1a) with rate constant k identified with t_c^{-1}.

To test relations (12) through (14) on a system for which conduction ought ideally to be the only significant cooling mechanism, we cut a small rectangular block of white Styrofoam and covered it with a thin wrap of aluminum foil for which the emissivity is very low ($\epsilon \sim 0.02$). Highly reflective surfaces by definition do not absorb radiation, and poor absorbers make poor emitters, a fact that often seems paradoxical to those encountering it for the first time. We inserted the digital thermometer probe down the long axis of the block and set the block (fastened vertically to a chemical stand) into a pot of water. When the water was boiling vigorously and the display of the thermometer was registering 100 °C, we removed the block from the water, set the stand on the kitchen counter (in the absence of wind!), and recorded the temperature with resolution of 0.1 °C in intervals of one minute as before. The experimental points are plotted with small circles (upper data set) in Fig. 3. The solid line through the circles is the exponential curve calculated from Eq. (14) and leads to a conduction time $t_c = 5.3$ minutes.

That the value obtained for t_c is reasonable may be seen by substituting into Eq. (13) the appropriate parameters for our Styrofoam block: $m = 0.02$ kg, $c = 1226$ J/kg·K, $d = 0.006$ m, $A = 0.0132$ m^2, $k_T = 0.029$ W/m·K. The theoretical result is 6.4 minutes.

In the second part of the experiment, designed to enhance the effects of radiation without changing any other property of the system, we simply painted the foil surface black, using flat lampblack paint, which to a large extent is an oil emulsion of soot. That the blackening of the surface markedly affected the cooling rate is shown by the locus of diamond plotting symbols (lower data set) in Fig. 3.

It is important to note (although the graph does not show) that an exponential fit to the "black" data is as poor as before. The dashed line in Fig. 3 is an exponential curve parametrically adjusted (not fit) to match visibly well the overall pattern of data points. That even this attempt is poor can be seen in the logarithmic plots of Fig. 4. Newton's law does not work particularly well here. How, then, can we account for these results? If not Newton's nor Stefan's, then what or whose law applies?

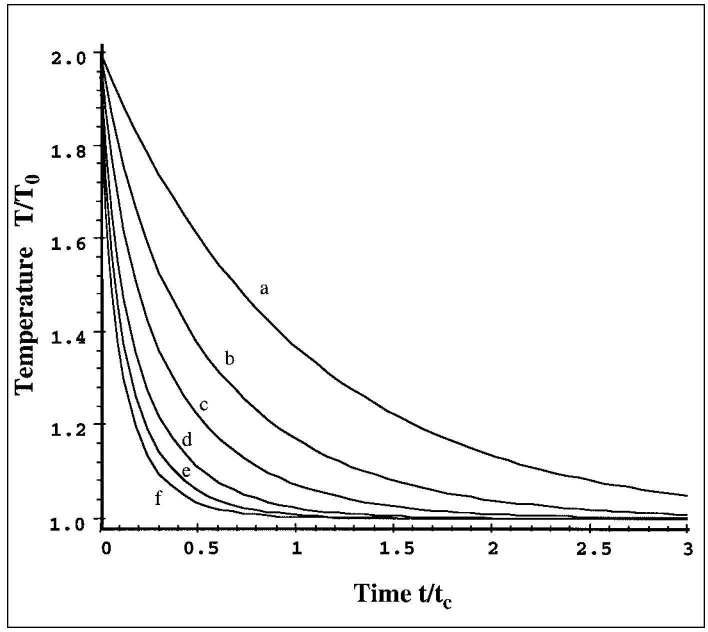

Fig. 5. Numerically calculated cooling curves for a system with both conductive and radiative energy loss. The one adjustable parameter is γ = (a) 0 [no radiation], (b) 0.1, (c) 0.25, (d) 0.5, (e) 0.75, (f) 1.0.

Combining the radiation law (4) and the conduction law (11) together with the temperature heat relation (5) we obtain the following dimensionless cooling law

$$\frac{d\mathscr{T}}{d\tau} = (1+\gamma) - \mathscr{T} - \gamma\mathscr{T}^4 \qquad (15)$$

in which the parameter γ is the ratio of the conduction and radiation times

$$\gamma = \frac{t_c}{t_r} = \frac{\epsilon\sigma d T_0^{\,3}}{k_T} \qquad (16)$$

Although Eq. (15) may look more-or-less manageable, it cannot be integrated analytically to yield an exact closed-form expression. Nevertheless, it can be integrated numerically. Figure 5 shows a sequence of cooling curves, solutions of Eq. (15) obtained with the symbolic computation software *Maple*, showing the transition from pure conduction with $\gamma = 0$ (i.e., Newton's law) to strong radiation $\gamma = 1$. A discussion of the numerical procedure, which is one of the Runge-Kutta methods, would take us too far afield but can be sought in appropriate reference books.[15]

It is also possible to derive an approximate solution to Eq. (15) that works quite well in the low-temperature regime, i.e., when $\mathscr{T}_m$ is not much in excess of one. In that case we treat the radiative part of (15) (i.e., the terms containing γ) by the approximation in Eq. (10) to obtain an exponential solution of the form of (la), but now with rate constant $k = 1+4\gamma$. This approximation is then substituted back into (15) to obtain, after a bit of patient effort, the rather interesting expression

$$\mathscr{T} = 1+(\mathscr{T}_m - 1)e^{-\tau}\, e^{-\frac{a}{k}(1-e^{-k\tau})} \qquad (17)$$

with $a = 4\gamma\,[1 + \frac{3}{2}(\mathscr{T}_m - 1)]$. Note that Eq. (17) involves the exponential of an exponential, a law ostensibly quite different from Newton's (14) or Stefan's (8).

The solid line through the "black" data in Figs. 3 and 4 is the theoretical curve calculated from Eq. (17) using the same value of t_c as obtained for the "white" data, since the rate of heat conduction is determined by the conductivity of the Styrofoam and should not be significantly affected by a thin layer of surface paint. Theory and experiment are in excellent accord when γ, the only adjustable parameter, takes the value 0.21. Then, from Eq. (16), with $T_0 = 301.1$ K, we find the emissivity of the block to be about $\epsilon = 0.7$.

Remarks

The studies summarized in this paper were undertaken as part of a high-school physics program emphasizing the inclusion of meaningful research opportunities (in lieu of "cookbook" laboratory exercises) as described by the senior author elsewhere.[16] By collaborating as partners in an endeavor of mutual interest, both student and teacher acquired some useful lessons in the workings of science and intricacies of history.

Puzzled by the frequent reference in math and physics textbooks to a law of Newton's of whose origin we knew nothing, and by the apparent unquestioning credence with which the law was reported to hold widely, we tracked down Newton's paper. To our surprise we found that, far from demonstrating a physical law, the investigation of cooling, whose corroborative details Newton did not even bother to report, was to Newton solely an auxiliary procedure in the more important task of creating a practical temperature scale—a procedure, moreover, that, for all we can tell from the written account, was based on a mathematical relation that Newton merely assumed to be true.

Our investigation impressed upon us another lesson as well. If there is anything in history that bears comparison with the universality and immutability of a physical law, it is this: Whatever the subject of inquiry, someone, somewhere, has studied it previously! Following the editorial review of our original manuscript, we were apprised of two articles on Newton's thermometry by A. P. French.[17] These articles discuss Newton's experimental procedure, compare many of the melting points measured by Newton with their current values on the centigrade scale, and give an estimate of the ratio of convective to radiative effects. They do not, however, solve the radiative heat-loss equations explicitly and test them experimentally, which were our primary objectives. French's conclusion concerning the conceptual basis of Newton's thermometric procedure coincided with our own: "It seems that he [Newton] *basically assumed* both that the expansion of the linseed oil was proportional to temperature change and that his theoretical law of cooling was correct." [Italics added.]

As to the validity of Newton's law, however, our own kitchen experiments indicate that, where energy loss by radiation contributes significantly—even when the temperatures involved are relatively low—an exponential variation does not make a particularly good model. Exceptions to Newton's law are not hard to find. The cooling of a hot burner on an electric range is very well accounted for by Stefan's law. The cooling of a piece of black Styrofoam—an object with high emissivity and low thermal conductivity—is accounted for by "the Silvermen's law" [if we may so call Eq. (17)].

And therein lies perhaps the most important lesson of all: Abide Rumford's advice quoted earlier, and you cannot go too far astray for too long.

References

1. R. E. Larson, R. P. Hostetler, and B. H. Edwards, *Calculus of a Single Variable* (Houghton Mifflin, Boston, 1998) p. 310.
2. Paul G Hewitt, *Conceptual Physics,* 8th ed., (Addison-Wesley, Reading, MA, 1998) pp. 279-280.
3. See, for example, M. W. Zemansky, *Heat and Thermodynamics,* 5th ed. (McGraw-Hill, New York, 1968) p. 134. Newton's calculation of the speed of sound in air lacked the factor γ, the ratio of specific heat capacities at constant pressure and constant volume, which reflects the fact that heat cannot be exchanged between the sound wave and ambient medium within the span of one period (reciprocal of the sound frequency). This error was subsequently corrected by Laplace.
4. W. F. Magie, *A Sourcebook in Physics* (McGraw-Hill, 1935), pp. 125-128.
5. R. S. Westfall, *Never At Rest* (Cambridge, London, 1980), p. 527. The paper cited is "Scala graduum caloris," *Philosophical Transactions*, **22**, 824-829 (1700-1701).
6. For additional discussion of Newton's experimental genius, see M. P. Silverman, *Waves and Grains: Reflections on Light and Learning* (Princeton University Press, 1998), Chap. 5: "Newton's Two-Knife Experiment."
7. If 12 degrees Newton (°N) corresponds to body temperature (37 °C), then the hottest sustainable bath of 17 °N corresponds to 52 °C. Newton's value is actually quite good. The two authors, while living in Japan, experienced total immersion in the skin-searing temperatures of Japanese baths fed by hot springs. The hottest of such baths in Japan is said to be in the town of Kusatsu and is recorded at 57.8 °C.
8. In his paper Newton explains a geometrical progression as one in which the second element is twice as great as the first, the third twice as great as the second, etc. Equation (1c) implies an analogous scaling, except that the scale factor is not 2, but $e^{k\Delta t}$.
9. This paper ("Convection of Heat"), among other Rumford writings, is also to be found in Magie's *Sourcebook* (pp. 146-161).
10. Extech Model 421305 digital thermocouple thermometer; ambient operating range 0-50 °C; measurement range -50-1300 °C; resolutions of 0.1 °C and 1 °C depending on range.
11. Given the common origin (0 °C for the freezing point of water) and linearity of the Newton scale, it follows that $\frac{37\ ^\circ\text{C}}{12\ ^\circ\text{N}} = \frac{456\ ^\circ\text{C}}{x\ ^\circ\text{N}}$ or $x = 148$ on the Newton scale.
12. See, for example, R. A. Serway and J. S. Faughn, *College Physics*, 5th ed. (Saunders, New York, 1999), p. 356.
13. This follows readily from the more familiar expression $\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$.
14. E. Hecht, *Physics: Algebra/Trig*, 2nd ed. (Brooks/Cole, New York, 1997), p. 1020.
15. See, for example, A Heck, *Introduction to Maple*, 2nd ed. (Springer, 1997), p. 537.
16. M. P. Silverman, "Self-directed learning: A heretical experiment in teaching physics," *Am. J. Phys.* **63**, 495-508 (1995). See also Ref. 6, Chapter 15.
17. A. P. French, "Isaac Newton's thermometry," *Phys. Teach.* **31**, 208 (1993); "Newton's thermometry: The role of radiation," ibid., p. 310.