One-Sided Fluxes – A Magnetic Curiosity?

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Abstract—It is shown that a previously unknown class of magnetization patterns exists in planar structures which have the unique property that all the flux escapes from one surface with none leaving the other side. A simple case is a constant amplitude rotating vector magnetization where the sense of rotation dictates which surface has no flux. More complicated magnetization patterns are elucidated. The likelihood that the one-sided flux phenomenon occurs partially in the normal write, the contact-printing and the print-through processes of tape recording is discussed. It is concluded that significant improvements in tape recording performance would ensue if means could be found to enhance the effect.

INTRODUCTION

This paper is concerned with the symmetry of the magnetic flux emanating from tapes, disks, and other planar magnetic structures. It is shown theoretically that a previously unknown class of magnetization patterns exists which has the remarkable property that all the flux escapes from one surface with none leaving the other side. Such one-sided fluxes are often quite surprising and inconsistent with "intuitive" concepts.

In the first part of the paper the magnetic theory is developed. Two proofs are given that a one-sided flux occurs when the magnetization is a constant amplitude rotating vector, the sense of rotation dictating which surface has no flux. The theory is then expanded to include general two-dimensional magnetization patterns; it transpires that in principle an infinite set of one-sided flux magnetization patterns exists. An example of a three-dimensional magnetization pattern is introduced. Finally, the physical realizability of such one-sided fluxes is discussed.

The second part of the paper deals with practical situations in which the one-sided flux phenomena may occur. In recording tapes a partial effect may occur in the writing process, and as Daniel[1] recently pointed out, the contact printing and print-through processes. Little direct evidence for one-sided fluxes has been reported. Until such time as deliberate efforts are made to exploit the effect, it remains a magnetic curiosity.

I. THEORETICAL CONSIDERATIONS

Rotating Magnetization Vector Case

We consider a planar structure, of thickness d, lying in the x, z plane. The upper and lower surfaces are at $y = 0$ and $-d$, respectively. Discussion is restricted to two-dimensional magnetization patterns in which the z component is either constant or zero. Suppose the magnetization is the superposition of two sinusoids in quadrature

$$m_x = m_0 \sin kx \qquad m_y = m_0 \cos kx \qquad m_z = 0 \tag{1}$$

where k is the wavenumber, $2\pi/\lambda$. We wish to solve the boundary value problem for the scalar potentials and fields above and below the sheet.

The potentials within the sheet obey Poisson's equation

$$\nabla^2 \phi_{\text{inside}} = m_0\, k \cos kx \tag{2a}$$

and Laplace's equation above and below,

$$\nabla^2 \phi_{\text{above}} = 0 \tag{2b}$$

$$\nabla^2 \phi_{\text{below}} = 0\,. \tag{2c}$$

Since the particular solution of (2a) is $-(m_0/k)\cos kx$, the general solutions are of the form

$$\phi_{\text{above}} = \left\{A\, e^{-ky} + B\, e^{+ky}\right\} \cos kx \tag{3a}$$

$$\phi_{\text{inside}} = \left\{C\, e^{-ky} + D\, e^{+ky} - \frac{m_0}{k}\right\} \cos kx \tag{3b}$$

$$\phi_{\text{below}} = \left\{E\, e^{-ky} + F\, e^{+ky}\right\} \cos kx\,. \tag{3c}$$

They are subject to six boundary conditions. The fields and potentials must go to zero as y becomes infinite; that is,

$$\phi_{\text{above}} = \phi_{\text{below}} = 0, \qquad \text{when } y = \pm\infty, \text{ respectively.} \tag{4a}$$

The tangential fields must match on the sheet upper and lower surfaces, thus

$$\phi_{\text{above}} = \phi_{\text{inside}}, \qquad \text{when } y = 0 \tag{4b}$$

$$\phi_{\text{below}} = \phi_{\text{inside}}, \qquad \text{when } y = -d\,. \tag{4c}$$

The normal flux density must be continuous on the upper and lower surfaces, thus

$$-\frac{\partial \phi_{\text{above}}}{\partial y} = -\frac{\partial \phi_{\text{inside}}}{\partial y} + m_0 \cos kx, \qquad \text{when } y = 0 \tag{4d}$$

$$-\frac{\partial \phi_{\text{below}}}{\partial y} = -\frac{\partial \phi_{\text{inside}}}{\partial y} + m_0 \cos kx, \qquad \text{when } y = -d. \tag{4e}$$

The reader may verify that the solutions consonant with these restrictions are

$$\phi_{\text{above}} = 0 \tag{5a}$$

$$\phi_{\text{inside}} = \frac{m_0}{k}\left(e^{ky} - 1\right) \cos kx \tag{5b}$$

$$\phi_{\text{below}} = \frac{m_0}{k}\left(1 - e^{kd}\right) e^{ky} \cos kx\,. \tag{5c}$$

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The remarkable fact thus emerges that the scalar potential above the sheet is identically zero everywhere. Thus no flux emerges from the sheet's upper surface. All the flux emerges from the lower surface.

It may be shown that if the sense of magnetization rotation is reversed, so that, for example,

$$m_x = m_0 \sin kx \tag{6a}$$

$$m_y = -m_0 \cos kx \tag{6b}$$

$$m_z = 0 \tag{6c}$$

the potentials then become

$$\phi'_{\text{above}} = \frac{-m_0}{k}(1 - e^{-kd})\, e^{-ky} \cos kx \tag{7a}$$

$$\phi'_{\text{inside}} = \frac{m_0}{k}(e^{-k(d+y)} - 1) \cos kx \tag{7b}$$

$$\phi'_{\text{below}} = 0 \tag{7c}$$

and no flux escapes from the lower surface.

When the magnetization, viewed in the negative z-direction, rotates clockwise the flux is all below the sheet (Eq. 5); when anticlockwise, the flux is all above the sheet (Eq. 7).

These findings may well rest uncomfortably with the reader. Considerable insight and perhaps a more physical understanding of the situation may be provided by the following second proof. Consider the sheet to be longitudinally magnetized according to $m_x = m_0 \sin kx$ only. Figure 1a shows schematically this magnetization, the $-\nabla \cdot M$ volume poles induced and the symmetry of the fluxes leaving the upper and lower surfaces. Now suppose the sheet is magnetized perpendicularly according to $m_y = m_0 \cos kx$ only. Figure 1b shows this magnetization, the $M \cdot n$ surface poles induced and the symmetry of the outside fluxes. In Fig. 1c we show the result of the superposition of the two magnetizations; again it becomes clear that a clockwise rotation of the magnetization vector produces a one-sided flux with no flux crossing the upper surface. Notice that, while the fluxes from the x and y components add in phase below the tape, they are π out of phase and cancel completely above the tape. The point of view may be taken that, with regard to the upper region, the surface poles and the volume poles cancel each other exactly.

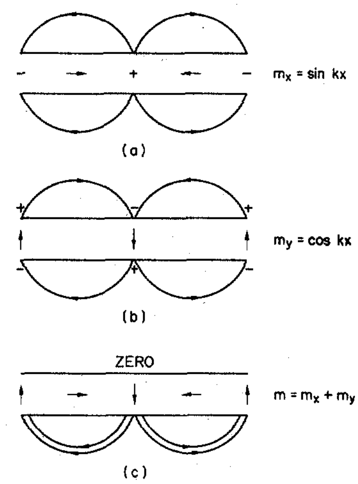

Fig. 1. (a) Flux from longitudinally magnetized tape. (b) Flux from perpendicularly magnetized tape. (c) Flux from rotating vector magnetized tape.

Extension to General Magnetization Patterns

Suppose now that one component of the magnetization is specified and we pose the question, "is it possible to find the other magnetization component so that a one-sided flux ensues?" In principle, the answer is always affirmative.

Suppose that the x component is an arbitrary, real function of x, $m_x = f(x)$, and we seek the matching y component which yields the one-sided flux. The function $f(x)$ contains certain sine and cosine harmonics. Clearly the y component sought must have just the same amplitude harmonics shifted in phase by $\pi/2$, so that the sines become cosines, etc. Upon adding this y component, the sheet then contains a number of superimposed clockwise rotating magnetization vectors, all of which yield one-sided fluxes separately. Obviously their sum is also a one-sided flux.

The mathematical transformation which changes the phase of all harmonics by $\pi/2$ is called the Hilbert transform and is defined by [2]

$$\hat{f}(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x')\,(x - x')^{-1}\, dx' \tag{9}$$

where it is understood that at $x' = x$ the Cauchy principal value is to be taken. Extensive tables of Hilbert transform pairs, $f(x)$ and $\hat{f}(x)$, exist. [3]

We see, therefore, that for every conceivable x component there exists a particular y component which produces a one-sided flux. The converse is also true, so that when either

$$m_x = f(x) \quad \text{or} \quad m_x = \hat{f}(x) \tag{10a}$$

$$m_y = \hat{f}(x) \quad \text{or} \quad m_y = -f(x) \tag{10b}$$

a one-sided flux ensues.

The reader with interests in communication theory will note the similarity between the general one-sided flux criterion given above and certain other known "one-sided" results. Writing the Fourier transform pairs as $f(x) \leftrightarrow F(k)$ we may cite the causality criterion, [2]

$$F(k) - i\,\hat{F}(k) \leftrightarrow \begin{cases} f(x), & \text{when } x \geqslant 0 \\ 0, & \text{when } x < 0 \end{cases} \tag{11}$$

and its dual, the single sideband criterion,

$$f(x) - i\,\hat{f}(x) \leftrightarrow \begin{cases} F(k), & \text{when } k \geqslant 0 \\ 0, & \text{when } k < 0 \end{cases} \tag{12}$$

The similarity may be formalized by writing the results expressed in (10) as

$$f(x) - i\,\hat{f}(x) \leftrightarrows \begin{cases} \phi(y), & \text{when } y \geqslant 0 \\ 0, & \text{when } y < -d \end{cases} \tag{13}$$

where the operator i denotes $\pi/2$ spatial rotation of the magnetization vector and the symbol $\leftrightarrows$ represents the linear transformation connecting the magnetization and the flux outside the sheet.

The linearity of Maxwell's equations permits an extension of the above two-dimensional (z invariant) results to three dimensions. Suppose we have, separately, two sheets magnetized according to

$$m_x = f_1(x) \quad \text{and} \quad m_x = 0 \tag{14a}$$

$$m_y = \hat{f}_1(x) \quad \text{and} \quad m_y = \hat{f}_2(z) \tag{14b}$$

$$m_z = 0 \quad \text{and} \quad m_z = f_2(z). \tag{14c}$$

Clearly both yield one-sided fluxes. Furthermore, they may be added together so that we may conclude that the three-dimensional patterns

$$m_x = \alpha f_1(x) \tag{15a}$$

$$m_y = \alpha \hat{f}_1(x) + \beta f_2(z) \tag{15b}$$

$$m_z = \beta f_2(z) \tag{15c}$$

also gives a one-sided flux for arbitrary α and β. It may be shown that more general three-dimensional magnetization patterns giving one-sided fluxes exist.

Reverting to the two-dimensional cases we conclude with a discussion of physical realizability. We have proved that, for an arbitrary function $f(x)$, a one-sided flux follows if the in- and out-of-plane components of magnetization are $f(x)$ and $\hat{f}(x)$. In order to be physically realizable the magnitude of the magnetization vector cannot exceed the saturation value m_s anywhere, thus

$$[f(x)]^2 + [\hat{f}(x)]^2 \leqslant m_s^2. \tag{16}$$

The condition places constraints upon the allowable functions; it is necessary that both $f(x)$ and $\hat{f}(x)$ remain finite everywhere. Whereas in principle any bounded $f(x)$ may be matched with its Hilbert transform, physical realizability limits $f(x)$ to functions with bounded Hilbert transforms. For continuous functions, the Hilbert transforms are bounded providing

$$|f(x)| < \infty, \quad \text{for all } x \tag{17}$$

$$f(x) \to 0, \quad \text{as } x \to \pm\infty. \tag{18}$$

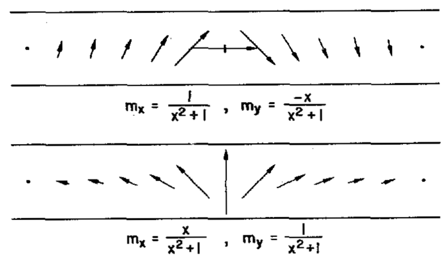

Fig. 2. Two magnetization patterns yielding one-sided fluxes. No flux exists above the planes.

As an example of a realizable Hilbert transform pair consider

$$f(x) = \frac{1}{1+x^2} \tag{19a}$$

$$\hat{f}(x) = -\frac{x}{1+x^2}. \tag{19b}$$

A diagram of the corresponding pair of vector magnetization patterns is shown in Fig. 2. The reader may test his "intuition" and speculate whether the fluxes emerge from the upper or lower surfaces.

As an example of impermissible functions, let us consider the integrals of the pair just given

$$g(x) = \int_0^x f(x)\,dx = \tan^{-1} x \tag{20a}$$

$$\hat{g}(x) = \int_0^x \hat{f}(x)\,dx = -(1/2)\log_e(1+x^2). \tag{20b}$$

Here $g(x)$ fails condition (18) and $\hat{g}(x)$, being unbounded, is not physically realizable.

II. PRACTICAL MANIFESTATIONS

The Normal Writing Process

In magnetic recording the medium (tape or disk) is magnetized by subjecting it to the fringing field of a gapped write head. It is usually assumed that this produces a predominantly longitudinal (in plane) magnetization pattern. The fluxes recoverable in the read head are, at the limits of long and short wavelengths, given by [4]

$$\phi \to \frac{2\mu}{\mu+1}\, 2\pi M_R\, d \qquad \text{(long wavelengths)} \tag{21a}$$

$$\phi \to \frac{2\mu}{\mu+1}\, \frac{2\pi M_R}{k}\, e^{-ka} \qquad \text{(short wavelengths)} \tag{21b}$$

where μ is the read head permeability and a is the read head-to-tape spacing. If the means could be found to add to this longitudinal magnetization an equal, and properly phased, perpendicular (out-of-plane) component, the above fluxes would

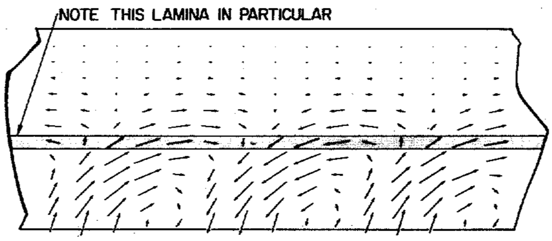

Fig. 3. Actual magnetization pattern recorded on tape (after Tjaden and Leyten[5]).

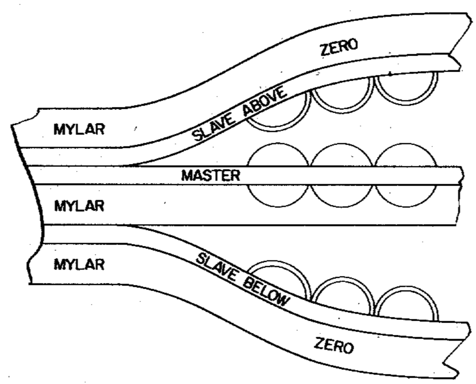

Fig. 4. Contact printing (and, with upper tape inverted, print-through) configuration.

be doubled. For high permeability heads the limits then become

$$\phi \to 8\pi M_R \, d \qquad \text{(long wavelengths)} \qquad (22a)$$

$$\phi \to \frac{8\pi M_R}{k} e^{-ka} \qquad \text{(short wavelengths).} \qquad (22b)$$

How can this be achieved? Presently, no well-defined techniques suggest themselves. As has been emphasized by repeated investigations, it is very difficult to conceive of practical write heads whose fringing field is substantially different from the norm. Similarly, the scope for making tapes with radically different properties seems to be rather limited.

A rational starting point for inquiry appears in Tjaden and Leyten's work.[5] In Fig. 3, copied from Tjaden and Leyten, we show a vector magnetization pattern measured in the Philips 5000:1 scale-up recorder. The reader will note that not only is there evidence of a rotating magnetization vector, but also that it is rotating in the correct sense to increase the read head flux! With our current deplorable lack of understanding of the full vector writing process, not even an intuitive explanation of this finding can be given. Nevertheless, experiments might well be undertaken to determine which factors promote the magnetization rotation.

The Contact Printing Process

In contact printing the slave tape is magnetized by the fringing field of a prerecorded master tape. The process may be enhanced by either magnetic of thermal excitations; comprehensive theoretical analyses are available for both methods.[6]

Consider the master tape fringing field. It may be shown from (5) and (7) and is clear from Fig. 1 that the fringing field appears to rotate when observed in a plane normal to the tape plane. The sense of rotation (as defined in Part I) is clockwise above and anticlockwise below the master tape. This remains true without regard to the vector direction, magnitude or phase of the master tape magnetization.

After contact printing has occurred, we might naively expect, therefore, the slave tape magnetization to rotate in the same direction as the master tape field. Thus a slave tape placed below the master tape would have a magnetization vector rotating anticlockwise and vice versa for the slave tape above. It is intriguing to note that, in accordance with the one-sided flux rules deduced in Part I, these senses of rotation of the slave tape magnetization are such that all their fluxes appear on the master tape side only as shown in Fig. 4.

In reality, however, two factors spoil the simple scheme just postulated. First, the slave tape magnetization actually acquired is proportional to the difference between the master tape field and the slave tape's own demagnetizing field. Second, most tapes are magnetically anisotropic so that the anhysteretic or thermoremanant susceptibilities (χ) are not equal in the x and y directions. For both reasons, the x and y magnetization components are not usually equal. The complete two-component, cross-coupled problem has been solved.[7] The results show that the fluxes on the master tape side of the slave tape are proportional to the following factors:

$$\left\{ \chi_x + \frac{\chi_y}{1 + 4\pi\chi_y} \right\} \qquad \text{(long wavelengths)} \qquad (23a)$$

$$\frac{\sqrt{(\chi_x + 1)(\chi_y + 1)} - 1}{\sqrt{(\chi_x + 1)(\chi_y + 1)} + 1} \qquad \text{(short wavelengths).} \qquad (23b)$$

Upon comparing these fluxes for two cases, $\chi_y = 0$ and $\chi_y \neq 0$, it is seen that the addition of y components of magnetization always increases the slave tape flux. Thus, although generally the x and y components of the slave tape magnetization are not equal, they are always in phase quadrature. A fraction of the slave tape magnetization always rotates. Rotation of the slave total magnetization vector can only occur in the limit of short wavelengths for isotropic ($\chi_x = \chi_y$) tape. Although one-sided fluxes will no doubt exist in this case it should be borne in mind that, due to spacing loss effects at short wavelengths, in no case would any flux emerge from the reverse (away from master tape) side of the slave tape.

The Print-Through Process

Print-through is the unintentional contact printing which occurs when reels of prerecorded tape are stored. The physical process is analogous to contact printing and consequently all the theoretical apparatus of contact printing is directly applicable to print-through. A major difference is that the adjacent layers of tape are separated by the relatively thick tape base film so that print-through at short wavelengths is precluded. Furthermore, the values of susceptibilities pertaining are so low that, at long wavelengths, very little "shearing" of the y component occurs and, therefore, the limiting expression (Eq. 23a) given for contact printing at long wavelengths becomes

$$\chi_x \pm \chi_y . \tag{25}$$

Here the + sign refers to the master tape side of the slave and vice versa. As discussed above, it is clear that, while the x and y components are not usually equal in magnitude, they are in phase quadrature. Only a fraction of the magnetization rotates and only a fraction of the flux is one-sided.

Experimentally, in accord with (25), it is found that a larger print-through signal is always measured on the tape layers which faced the master. For tape reels wound oxide side in, the "pre-prints" always exceed the "post-prints." In standard γFe_2O_3 tapes, a 3-4 dB difference is measured at wavelengths of 5-10 mils, which is more than can be accounted for by spacing loss effects: clearly a fraction of the flux is one-sided. In cobalt doped γFe_2O_3 "high energy" tapes, the predominance of a random cubic magnetocrystalline energy renders the tapes almost magnetically isotropic and, satisfyingly enough, much larger differences of 8-9 dB have been observed; in this case about one half of the print-through flux must be one-sided.

Next, we consider the print-through expected for a perfectly isotropic ($\chi_x = \chi_y$) tape. In this case the print-through magnetizations are full vector rotating and the fluxes one-sided so that no print-through signal is expected for the tape layers facing away from the master tape. Dependent upon the winding method (oxide in or out), isotropic tapes would have the special property of having either no "pre-prints" or not "post-prints."

Finally, we remark upon the intriguing possibility that tapes completely without print-through are conceivable. Suppose that the tape is isotropic and has been prerecorded with a rotating vector magnetization so that its flux is one-sided. No matter which way this tape is wound, no print-through can occur. If the "pre-print" is zero due to isotropy of the tape, the "post-print" is zero due to the one-sided master tape flux and vice versa.

CONCLUSIONS

It has been demonstrated theoretically that there exists a class of two-dimensional magnetization patterns which yield one-sided fluxes. It is hoped that the analogy between this effect and certain well-known results in communications theory may stimulate further theoretical work.

Some evidence exists that one-sided fluxes exist in both the normal write and the print-through processes of tape recording. If the effects could be enhanced, significant improvements in tape recorder performance would ensure. Not only might the signal-to-noise ratio be increased but also, perhaps more importantly, print-through might be eliminated.

Finally, we may point out that it is possible that one-sided fluxes occur, and may be detectable, in a wide variety of other magnetic structures. Thus there may be magnetization patterns which produce no Bitter pattern at all or perhaps there exist "bubble" structures with anamolously small (or large) detectable fluxes.

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