

Student understanding of symmetry and Gauss's law of electricity

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We investigate the difficulties that students in calculus-based introductory physics courses have with the concepts of symmetry, electric field, and electric flux which are important for applying Gauss's law. The determination of the electric field using Gauss's law requires determining the symmetry of a particular charge distribution and predicting the direction of the electric field everywhere if a high symmetry exists. Effective application of Gauss's law implicitly requires understanding the principle of superposition for electric fields. Helping students learn when Gauss's law can be readily applied to determine the strength of the electric field, and then helping them learn to determine the appropriate shape of Gaussian surfaces if sufficient symmetry exists, can help develop their reasoning and problem-solving skills. We administered free-response and multiple-choice questions and conducted interviews with individual students using a think-aloud protocol to elucidate the difficulties that students have with the concepts of symmetry, electric field, and electric flux. We also developed a multiple-choice test that targets these conceptual issues to obtain quantitative information about their difficulties and administered it to 541 students in the introductory calculus-based physics courses and to upper-level undergraduates in an electricity and magnetism course and to graduate students enrolled in a teaching assistant seminar course. We find that undergraduate students have many common difficulties with these concepts. © 2006 American Association of Physics Teachers.
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I. INTRODUCTION

A major goal of most calculus-based introductory physics courses is to help students develop problem solving and reasoning skills.¹⁻⁴ Gauss's law of electricity is an important topic in the second semester of most calculus-based introductory physics courses. Learning to reason whether Gauss's law can be exploited in a particular situation to determine the electric field, without having to evaluate complicated integrals, can provide an excellent context for helping students develop a good grasp of symmetry considerations. Unfortunately, students often memorize a collection of formulas for the magnitude of the electric field for various geometries, without paying attention to symmetry considerations. Students apply these formulas without being able to differentiate between electric field and electric flux. They have difficulty identifying situations where Gauss's law is useful and over-generalize results obtained for a highly symmetric charge distribution to situations where they are not applicable. Most textbooks do not sufficiently emphasize symmetry considerations or the chain of reasoning required to determine if Gauss's law is useful for calculating the electric field.

To investigate student understanding of the concepts of symmetry, electric field, and electric flux, we administered free-response and multiple-choice questions and conducted interviews with 15 individual students using a think-aloud protocol.⁵ We then developed a multiple-choice test with 25 conceptual questions that addresses these issues and administered it to 541 students in the introductory calculus-based physics courses in eight different classes to obtain a quantitative understanding of the nature of the difficulties. The test was also administered as a pre- and post-test to undergraduates enrolled in an upper-level electricity and magnetism (E&M) course and to graduate students enrolled in a teaching assistant (TA) seminar course. The tests and interviews explored the extent to which students have become proficient in exploiting symmetry and in making conceptual predictions

about the magnitude and direction of the electric field for a given charge distribution using Coulomb's or Gauss's laws. The test also explores whether students can distinguish between electric field and electric flux, identify situations in which Gauss's law can readily be used to calculate the electric field strength from the information about the electric flux, and the shapes of the Gaussian surfaces that would be appropriate in those cases.

II. PREVIOUS INVESTIGATIONS RELATED TO ELECTRICITY AND MAGNETISM

Investigation of student difficulties related to a particular physics concept is important for designing instructional strategies to reduce them.⁶⁻¹⁰ Prior investigations related to electricity and magnetism have included difficulties with general introductory concepts, electrical circuits, and superposition of the electric field.⁶⁻¹⁰ Maloney *et al.*⁶ developed and administered a 32 question multiple-choice test (the Conceptual Survey of Electricity and Magnetism) that surveys many important concepts covered in the introductory physics courses and is suitable for both calculus- and algebra-based courses. They found that students have common difficulties with fundamental concepts related to electricity and magnetism. McDermott *et al.*⁷ performed an in-depth investigation of the difficulties students have with electrical circuits and developed exemplary tutorials and inquiry-based curriculum to significantly reduce these difficulties among introductory physics students and pre- and in-service teachers.¹² Engelhard and Beichner⁸ have developed conceptual assessments related to electrical circuits. Rainson *et al.*⁹ investigated difficulties with the superposition of electric fields by administering written questions. Belcher *et al.*¹⁰ and Christian and Belloni¹¹ have developed visualization tools to improve student understanding of physics concepts including those related to electricity and magnetism.

III. METHOD

Our investigation of student difficulties in discerning symmetry and applying Gauss's law was performed using two methods: the design and administration of free-response and multiple-choice questions to elicit difficulties in a particular context and in-depth audio-taped interviews with individual students using a think-aloud protocol⁵ while they solved those problems. A principal advantage of written tests is that they can be administered to large student populations. Multiple-choice tests are easy and economical to administer and to grade, have objective scoring, and are amenable to statistical analysis that can be used to compare student populations or instructional methods. The main drawback is that thought processes are not revealed completely by the answers alone. However, when combined with interviews with a subset of individual students, well-designed tests can serve as excellent educational tools. Conceptual multiple-choice tests have already been designed to assess student understanding of force, energy and momentum, and electricity and magnetism.^{6,8,13–15} They show that students' knowledge of physics is often fragmented and context dependent and that students share common difficulties.

IV. BACKGROUND

Gauss's law allows us to relate the net electric flux through a closed surface to the net charge enclosed by the surface.

$$\Phi_E = Q_{\text{enclosed}}/\epsilon_0. \quad (1)$$

Equation (1) implies that if we know the net electric flux through a closed surface, we can readily find the net charge inside it. And if we know the net charge inside a closed surface, we can readily find the net electric flux through it.

In general, Eq. (1) does not mean that we can use Gauss's law to readily find the magnitude of the electric field $|\mathbf{E}|$ at a point. Only in situations where the charge distribution has very high symmetry can we find $|\mathbf{E}|$ from the net electric flux Φ_E . Although there are only three types of symmetry (spherical, cylindrical, and planar) for which Gauss's law can readily be exploited to determine the electric field at various points from the information about the electric flux, students need help in identifying when these symmetries are present.

The net electric flux through a closed surface is given by

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint |\mathbf{E}| \cos \theta |d\mathbf{A}|. \quad (2)$$

The net electric flux Φ_E over a closed (Gaussian) surface can be exploited to determine the electric field magnitude $|\mathbf{E}|$ at an arbitrary point P on the surface only if the following conditions are satisfied:

1. We can determine the exact direction of \mathbf{E} relative to the area vector at every point on the closed surface by symmetry (only $\theta=0$, 180° or $\pm 90^\circ$ are associated with sufficiently high symmetry).
2. In some cases, we can divide the closed surface into subsections (for each subsection the electric flux can be readily calculated for example, the side and two caps of a cylinder) such that one of the following is true:
 - (a) $|\mathbf{E}|$ is the same everywhere on the subsection due to the symmetry of the charge distribution.

- (b) \mathbf{E} and the area vector (outward normal to the surface) are perpendicular ($\theta=90^\circ$) so that there is no electric flux through that subsection.

Thus, to determine if the information about the net electric flux through a closed surface can be exploited to determine $|\mathbf{E}|$ at a point P , we may choose a Gaussian (closed imaginary) surface such that it contains the point P where we want to determine $|\mathbf{E}|$ and $|\mathbf{E}| \cos \theta$ is known (by symmetry) to have a constant value on each subsection of the surface so that it can be pulled out of the flux integral in Eq. (2). Then $\int |d\mathbf{A}|$ equals the total area of the subsection of the surface.

Although students can calculate the electric field without regard to symmetry using Coulomb's law and the principle of superposition, determining the electric field using Gauss's law requires an explicit focus on the symmetry of the charge distribution. The principle of superposition is also a prerequisite for employing Gauss's law successfully to determine the electric field (albeit implicit in the actual application of Gauss's law) and helps determine if sufficient charge symmetry exists in a particular situation. In addition to considerations of symmetry, the area vector and the electric flux are new concepts that are introduced in the context of Gauss's law. Students must be able to distinguish the electric flux from the electric field, a task that is very difficult. Students need to learn that the electric flux is a scalar which is positive and negative depending on the relative directions of the electric field and area vector, while the electric field at a point is a vector. They need to learn that the electric flux and the electric field have different dimensions, that Gauss's law applies only to closed surfaces, and that for any closed surface information about the net enclosed charge is sufficient to determine the net electric flux through it. On the other hand, the determination of the electric field at various points due to a charge distribution depends on the way in which the charges are distributed and can depend on both the charges inside and outside the closed surface. For example, the electric field at various points on a Gaussian surface may vary from point to point, even though the net electric flux is zero (and hence the net enclosed charge is zero).

V. DISCUSSION OF STUDENT DIFFICULTIES

We first administered free-response and multiple-choice questions and interviewed individual students in several calculus-based introductory physics courses about concepts related to symmetry and Gauss's law. These investigations provided useful insight into common difficulties students have with these concepts and also provided guidelines for the development of a 25 question multiple-choice test that was then used as a tool to obtain quantitative information about the extent to which students have common difficulties with these concepts. Appendix A provides a summary of the development of the multiple-choice test. The final version of the test is in Appendix B. This 50 min test was administered to 541 students in the calculus-based introductory physics courses in eight different classes at the University of Pittsburgh. Five additional student volunteers were interviewed (in addition to those interviewed earlier for a total of 15 individual interviews) to obtain a greater understanding of their difficulties. Two of these classes were honors courses. The average score on the test was 49%. Table I shows the percent of students who selected the choices (a)–(e) on Problems 1–25. The correct responses are italicized. Although

Table I. Percentage of introductory calculus-based physics students (total number of students 541) who selected choices (a)–(e) on Problems (1)–(25) on the test. The correct response for each question has been italicized. The average score was 49%.

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
(a)	55	3	20	0	14	3	5	4	7	20	4	57	55	12	40	4	0	47	2	22	28	37	35	10	32
(b)	35	13	57	30	0	11	6	50	66	0	5	6	10	24	8	10	33	7	3	14	5	15	2	15	33
(c)	5	69	15	5	0	8	81	5	12	19	57	30	8	2	43	18	25	27	34	6	17	13	53	27	22
(d)	1	7	7	15	0	58	6	35	5	50	6	7	6	60	4	42	17	13	56	53	32	17	3	19	12
(e)	4	8	1	50	86	20	2	6	10	11	28	0	21	2	5	26	25	6	5	5	18	18	7	29	1

some questions have a strong single distractor, others have several distractor choices that are equally popular. In eight of the test questions, students had to determine the correctness of three statements. Students identified the correctness of some of the statements but not others as shown in Table I (we will discuss these issues in detail later in this section). The reliability coefficient¹⁶ α , which is a measure of the internal consistency of the test, is 0.8, which is considered good by the standards of test design. The point biserial discrimination¹⁶ (PBD) quantifies the ability of a question to discriminate between students who did well overall and those who did not. This discrimination index for 16 questions was more than 0.4. Only four questions had a PBD value less than 0.3 (only one question with less than 0.2), which is also good by the standards of test design.¹⁶ The reliability coefficient and discrimination indices show that the conceptual difficulties with symmetry and Gauss's law found using the test are meaningful.

Table II shows the concepts that were covered and the questions in the multiple-choice test that addressed them. The table provides only one of the several ways questions can be classified. Some of the categories in Table II are subsets of other categories. We have found these subcategories convenient for classifying student difficulties. The categorization of concepts in Table II is based on student difficulties and does not necessarily reflect the way experts would categorize those problems. For example, one of the concept categories in which we placed Problems 13 and 15 is “difference between the electric field and electric flux.” Although the word “flux” is never explicitly mentioned in these ques-

tions, our investigation shows that students often believed that because there was no charge enclosed inside the Gaussian surfaces that are exclusively contained in the hollow region, the electric field must be zero everywhere inside the hollow region. They were often interpreting zero net electric flux through a closed surface to imply zero electric field at all points of the surface. We classified Problems 19, 21, and 23 in the same concept category “difference between the electric field and electric flux.” As shown in Table I, the most common misconception in Problem 19 was due to confusion between the electric field and the electric flux. In Problem 21 the most common incorrect response was choice (a) because students believed that in order to determine the net electric flux through a closed surface, knowledge of the charge enclosed was not sufficient and charges had to be symmetrically distributed (a requirement for determining the electric field). The most common incorrect response for Problem 23 was choice (a), which also arises due to the difficulty in distinguishing between the electric field and flux. Similarly, we placed Problem 14 in the category “recognizing symmetry to determine if it is easy to exploit Gauss's law...,” because many students incorrectly believed that even in experiment 1, Gauss's law can be used to infer that the magnitude of the electric field is the same at points A, B, and C.

A. The electric charge and electric flux are scalars

The most common difficulty with Problem 1 was mistakenly thinking that the electric flux and/or electric charge are vectors. In interviews, students justified their response about

Table II. Concepts covered and the questions that addressed them in the multiple-choice test.

Concepts	Problem number
Electric flux	1, 7, 8, 9, 12, 18, 20, 21, 23
Recognizing the symmetry of the charge distribution	2, 3, 4, 6, 10, 11, 12, 13, 14, 15, 16, 17, 22, 24, 25
Symmetry of the object versus symmetry of the charge distribution	6, 10, 14, 15, 16
Coulomb's law, superposition, and symmetry considerations sufficient	2, 3, 4, 14, 16
Difference between the electric field and the electric flux	1, 7, 8, 12, 13, 15, 18, 19, (21, 22), 23
Relevance of a closed surface in Gauss's law	5, 21, 22
Recognizing the symmetry to determine if it is easy to exploit Gauss's law or exploiting Gauss's law to determine the electric field	6, 10, 11, 12, 13, 14, 15, 16, 17, 22, 24, 25
Appropriate Gaussian surface for determining the electric field for a given charge distribution	6, 11, 22
Electric field inside hollow nonconducting objects with different charge distribution	13, 15, 25

why the electric flux is a vector by using the following facts: The expression for flux involves a scalar product of two vectors. Instead of identifying $\cos \theta$ as the angle between the electric field and the area vector, many students concluded that the flux is a vector because it involves a $\cos \theta$. Students pointed to the fact that the electric flux can have both positive and negative signs. When asked if it would make sense to say that the electric flux points at 30° south of west, students often avoided a direct response. Their response implied that for a physical quantity to be a vector, it was not necessary to be able to specify the exact direction. Rather, because the electric field lines “going out” of a closed surface contribute positively and those “going in” contribute negatively to the total electric flux through a closed surface, flux must be a vector. To justify why the electric charge should be a vector, students often made similar claims that positive charges point outward and negative charges point inward. It was clear from the responses that students were often referring to the electric field but calling it “charge.”

B. The principle of superposition

The performance of many students was closely tied with their understanding of the principle of superposition. For example, Problems 2–4 are related to symmetry and require the use of the superposition principle to compare the electric field at various points for a given charge distribution. Many students had difficulty with the principle of superposition and could not differentiate between the electric field due to individual charges at a point and the net electric field. Interviews suggest that some believed that only the nearest charge will contribute to the electric field at a point. Others believed that the magnitude of the electric field at the desired points in Problem 2 (and 3) should be the same because they were the same perpendicular distance from the straight line joining the three charges. Some students provided more detailed reasoning. Instead of viewing it as a problem involving the addition of three electric field vectors, these students often made guesses by looking at the distances of points A, B, and C from the three charges and hoping that the electric field will somehow work out to be the same at the three points. They claimed that in Problem 2, point A is closer to one charge and farther away from the other two charges than point B which is equidistant from the two charges and not as far away from the third charge as point A. Therefore, the electric field at points A and B will be the same if we take into account all three charges. Because charge is uniformly distributed on the finite sheet in Problem 3, this type of confusion was even more prevalent. The most common distractor in Problem 4 implies a similar difficulty. Many students believed that if charges are uniformly distributed on an insulating equilateral triangle, the magnitude of the electric field will be the same everywhere on a concentric imaginary triangle.

C. The electric field inside a hollow nonconducting object

Problems 13, 15, and 25 assess student understanding of the electric field inside hollow nonconducting objects of different shapes due to charges on their surface or charges outside. Problem 13 was the most difficult question on the test and only 21% of the introductory students responded correctly. 55% of the students believed that the electric field

inside a nonconducting hollow cube with charge uniformly distributed on its surface will be zero everywhere. Interviews suggest that some students believed that the hollow region inside is always shielded from the charges on the surface or charges outside. This notion of shielding was retained by the students despite being reminded by the interviewer that the object on which charges are distributed is not conducting. Some students explicitly said that the net effect of all the charges outside must work out to be zero everywhere inside the hollow region. One student went on to claim that he has always been amazed at how Gauss’s law can be used to prove that the electric field in the hollow region inside a closed object is always zero everywhere, a result that appears to be counterintuitive to him. Some students even drew spherical or cubic Gaussian surfaces inside a hollow cube to argue that because there is no charge enclosed, the electric field will be zero everywhere according to Gauss’s law. Similar to Problem 13, the most common difficulty with Problem 15 was assuming that the electric field inside the sphere in experiment 1 is also zero everywhere. In interviews and free-response questions, students used reasoning similar to Problem 13.

For Problem 25, choices (b)–(d) were popular due to the difficulties with the principle of superposition and the electric field inside a hollow nonconducting sphere. Interviews suggest that students who believed that the electric field at point A is not zero often thought that the sphere with the uniform surface charge $+Q$ will produce a larger electric field at that point because one of its sides is only a distance L away compared to the point charge $+Q$ which is a distance $2L$ away. At the end of the interview, in response to the query by some students, the interviewer discussed with them that for point A, the charge on the sphere can be thought of as a point charge at the center of the sphere. Students often noted that this fact is very nonintuitive because of the proximity of one end of the sphere to point A. Students who claimed that the net electric field at point B is zero often referred to the shielding of the inside of the sphere from the charges on the sphere and charges outside of the sphere (similar to Problems 13 and 15) when they were explicitly asked by the interviewer why the point charge near the sphere does not produce an electric field at point B. Even when the interviewer reminded students that the sphere was nonconducting, they often maintained that the point charge cannot have any influence inside the sphere. Some students said that they could not explain exactly why the nonconducting sphere will produce shielding, but that they remember that the electric field must somehow cancel in the hollow region for all shapes and charge distributions. Further prodding showed that due to a lack of thorough understanding, these students were often overgeneralizing or confusing two different facts: the symmetry argument that shows (using Gauss’s law) that the electric field for a sphere with a uniform surface charge is zero everywhere inside regardless of whether the sphere is conducting or insulating, and/or the fact that the electric field inside a conductor is zero in equilibrium regardless of the shape of the conductor.

D. The underlying symmetry of a charge distribution

Many students have difficulty realizing that it is the symmetry of the charge distribution (and not the symmetry of the object on which the charges are embedded) that is important in determining whether Gauss’s law can be applied to calcu-

late the electric field at a point. For example, students had to determine when a Gaussian surface would be convenient for determining the electric field at a point on its surface in Problems 6, 11, and 22. In Problems 11 and 22 they had to identify the shape of appropriate Gaussian surfaces that would make it easy to use Gauss's law to calculate the electric field due to an infinite uniform sheet of charge and line of charge, respectively. In Problem 11 the most common distractor was (e), and many students believed that all surfaces will work because they are all symmetric. However, the calculation for the sphere is not easy because the area vector and the electric field make different angles for different infinitesimal areas on the sphere. In Problem 6 students were asked to evaluate the validity of three general statements without being given a specific charge distribution that produced the electric field. Many students chose (e) and believed that the Gaussian surface must be chosen to take advantage of the symmetry of the object enclosed, regardless of how the charges are distributed on that object.

The idea of whether the symmetry of the charge distribution or the symmetry of the object on which charges are distributed is important for being able to determine the electric field using Gauss's law is also explored in Problems 10 and 14–16. In Problem 10 Gauss's law can readily be used to determine the electric field in cases (i) and (iii) because of the spherical symmetry of the charge distribution on nonpolarizable objects but not in case (ii). In Problems 14–16 students were presented with an insulating sphere on which there are six point charges distributed in a way that the adjacent charges are equidistant. Although the charges are on a spherical object, the charge distribution does not have spherical symmetry. Interviews and written responses suggest that many students incorrectly believed that a spherical symmetry exists in this case for exploiting Gauss's law to calculate the electric field readily. The most common difficulty with Problem 14 was assuming that the magnitude of the electric field in experiment 1 would be the same at the three points shown. In interviews and free-response questions, some students explained this response by claiming that because the six point charges and the three points are symmetrically situated, the field magnitude must be the same at the three points; others explained their response by claiming that for a point outside, the six point charges on the sphere can be thought to be point charges at the center of the sphere. Problem 16 is one of the most difficult questions on the test. It was easy for most students to rule out (i), but it was difficult for them to evaluate the validity of (ii) and (iii) because of the difficulty of recognizing the underlying symmetry of the charge distribution.

Problems 17 and 22 also probe the extent to which students can discern the underlying symmetry of the charge distribution. In Problem 17 the last three distractors were popular and students often believed that we can use Gauss's law to find the electric field at a point outside due to a cube or finite cylinder with uniform surface charge. In the interviews students sometime recalled using Gauss's law for these surfaces. More prodding showed that they were either confusing the fact that those surfaces can be used as Gaussian surfaces for appropriate charge distributions or the fact that for an infinite cylinder (but not a finite cylinder) it is possible to exploit Gauss's law to find the electric field. It appears that many students have not thought carefully about the principle of superposition and its implication for the electric field due to a charge distribution and were applying

memorized knowledge whose applicability was forgotten. In Problem 22 students had to choose the Gaussian surfaces that would help them determine the electric field at point P readily due to the infinite line of charge. All of the alternative choices were selected with an almost uniform frequency. Students were often unsure about the symmetry concepts relevant for making appropriate decisions and those who chose option (c) were often quite confident that the magnitude of the electric field due to the infinite line must be the same at every point on the cube as well.

E. Electric field and electric flux

Problems 7, 8, and 9 are related to electric flux and the distinction between the electric field and flux. The most common distractor in Problem 8 was (d). Some students were quite assertive in their interview and incorrectly claimed that if the magnitude of the electric flux through one closed surface is smaller than another, the magnitude of the electric field at points on the first surface must be smaller too. All the distractors were popular for Problem 9. Interviews suggest that students who chose (c) often believed that although the net charge enclosed is the same for surfaces A and B, the electric flux through surface A must be larger because it is closer to the positive charge at the center. Interviews also suggest that students who selected option (d) often believed that although the net charge enclosed is the same for surfaces A and B, the electric flux through surface B must be larger because it has a larger area.

Similar difficulty in differentiating and relating the electric field and electric flux was manifested in response to Problems 12, 18, 21, 22, and 23. The most common difficulty with Problem 12 was assuming that both electric flux and electric field can be determined using Gauss's law; 30% of the students chose option (c). Interviews suggest that students were often confident about their choice of (c) because the surface area of the Gaussian sphere was given and they believed that $\Phi = |\mathbf{E}|A$ is always true (instead of $\Phi = \oint \mathbf{E} \cdot d\mathbf{A}$). Problem 18 is more abstract than most other questions in which a specific physical situation is explicitly given. In this question students had to evaluate the validity of three statements and the main difficulty was not being clear about the relation and distinction between the electric field and electric flux. The most common difficulty with Problem 19 was assuming that the electric field is zero at point B on the side surface of the cube, although the problem statement explicitly mentions that the cube is in a uniform electric field of 20 N/C. In interviews and free-response questions, some students explicitly claimed that the area vector of the side surface is perpendicular to the direction of the electric field lines. Therefore, the electric field must be zero at point B. This kind of confusion between the electric field at a point and the contribution to the electric flux from a certain area was found in other questions as well.

Response to Problems 21 and 22 suggest that many students are not comfortable with the statement of Gauss's law that relates the net flux through a closed surface to the net charge enclosed. They have difficulty differentiating between the electric flux through a closed surface and the electric field at a point on the surface. For example, in Problem 21 many students chose (i) [or (i) and (iii)] and claimed in the interviews and free-response questions that only those surfaces can be used to determine the net electric flux through them because the other surfaces did not have the correct symmetry.

Table III. Percentage of students in the upper-level undergraduate E&M course (total number of students 33) who selected choices (a)–(e) on the pretest (before instruction in the upper-level course). The correct response for each question is italicized. The average score was 44%.

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
(a)	42	9	15	3	30	0	9	0	9	24	3	43	49	9	49	9	0	58	0	15	15	27	42	12	27
(b)	49	15	<i>61</i>	36	0	3	18	52	<i>67</i>	0	15	18	15	24	3	21	27	3	6	18	13	15	3	27	30
(c)	6	52	9	9	0	3	<i>64</i>	12	15	21	24	33	15	9	36	27	30	9	24	9	15	12	42	30	21
(d)	3	21	15	18	0	82	0	24	6	<i>45</i>	18	3	3	55	3	33	6	18	<i>64</i>	55	<i>36</i>	30	3	12	21
(e)	0	3	0	<i>33</i>	<i>70</i>	12	9	12	3	9	39	3	<i>18</i>	3	9	9	36	12	6	3	21	15	9	<i>18</i>	0

The most common distractor in Problem 23 was (a), which was chosen by 35% of the students. These students believed that the net charge enclosed in a region is largest if the number of field lines penetrating the region is greatest. They did not pay attention to the direction of the electric field lines which is crucial for determining the net flux through a closed surface and hence the net charge enclosed using Gauss's law.

F. Other difficulties

Students were sometimes unsure about the distinction between open and closed surfaces and that Gauss's law is only applicable to closed surfaces. Problems 5, 21, and 22 at least partly assess whether students understand this distinction. Some students incorrectly believed that Gauss's law applies to any symmetrical surface even if it is not closed. For example, in response to Problem 21, these students claimed that the electric flux Φ due to an infinitely long line of charge (with uniform linear charge density λ) is $\lambda L/\epsilon_0$ even for the two-dimensional square sheet.

In response to Problem 24 many students selected some of the statements as correct but only 29% identified that all three statements are correct. Many had difficulty determining the validity of statement (ii) which can be checked by drawing a Gaussian surface in region A that includes the center and using Gauss's law. In the interview a student insisted that there can be a point charge in region A even if the electric field is zero. When asked to explain, the student drew a positive point charge with the electric field lines radially outward and said: "Well...the field due to a point charge will cancel out because it points in all directions." It was clear from the explanation that the student was confused about the electric field line representation.

G. Performance of upper-level undergraduates

We also administered the 25-item multiple-choice test as a pre- and post-test to students enrolled in a sophomore-junior level E&M course in which vector calculus was used extensively; 33 students took the pretest and 28 students took the post-test. Tables III and IV show the percentage of students

who made the various choices on various questions on the pretest and post-test, respectively. The average pre- and post-test scores are 44 and 49%, respectively; this difference is not statistically significant. The fact that the upper-level undergraduates did not perform better than the introductory students (compare Tables I and IV) suggests that the higher mathematical sophistication of the course with vector calculus did not help students acquire a better conceptual understanding of the superposition principle, symmetry, and Gauss's law.

H. Performance of graduate students

To calibrate the test we also administered it (over two consecutive years) to a total of 33 physics graduate students who were enrolled in a seminar course for teaching assistants (TAs). Most of them were first year graduate students who were simultaneously enrolled in the first semester of the graduate E&M course. Students were told ahead of time that they would be taking a test related to electrostatics concepts. They were asked to take the test seriously, but it did not count for their course grade. The average test score for the graduate students was approximately 75% with the reliability coefficient $\alpha \approx 0.8$.¹⁶ The better performance of graduate students compared to the undergraduates is statistically significant. The minimum score obtained by a graduate student (an American student) was 28% and the maximum score obtained by two Chinese graduate students was 100%.

Table V shows the percentage of graduate students who selected the various choices on the test. Many of the conceptual difficulties that the graduate students displayed are similar to those of the introductory students. Problems 13 and 17 were the only ones on which graduate students performed less than 50% with misconceptions similar to those of the undergraduates. The PBD for Problems 13 and 17 were 0.5 and 0.6, respectively, which shows that the graduate students who performed well overall on the test did well on those questions. Comparison of Tables I and V shows that on an average, the graduate students outperformed the introductory students on all other questions except Problem 2, but this

Table IV. Percentage of students in the upper-level undergraduate E&M course (total number of students 28) who selected the choices (a)–(e) on the post-test (after instruction in the upper-level course). The correct response for each question is italicized. The average score was 49%.

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
(a)	53	11	11	4	32	0	11	11	4	21	0	53	75	0	64	4	0	39	0	7	7	50	36	4	28
(b)	39	29	<i>61</i>	46	0	11	11	32	<i>68</i>	4	4	11	0	32	4	7	7	7	4	21	4	4	4	7	32
(c)	0	57	11	7	0	4	68	7	18	11	68	25	4	4	25	7	43	32	36	14	32	18	53	18	0
(d)	4	0	14	4	0	82	11	36	7	<i>50</i>	0	7	7	<i>64</i>	7	46	11	11	53	53	53	14	0	11	36
(e)	4	4	4	39	<i>68</i>	4	0	14	4	14	28	4	<i>14</i>	0	0	36	39	11	7	4	4	14	7	<i>61</i>	4

Table V. Percentage of physics graduate students enrolled in a course for teaching assistants (total number of students 33) who selected the choices (a)–(e) on Problems (1)–(25) on the test. The correct response for each question has been italicized. The average score was 75%.

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
(a)	73	3	12	0	12	3	3	6	0	12	0	79	55	0	36	6	0	73	3	3	6	73	18	0	55
(b)	27	9	79	18	0	3	3	88	91	0	3	0	0	12	3	6	45	0	0	3	3	3	0	6	9
(c)	0	64	9	0	0	6	91	0	0	6	94	12	9	3	61	9	40	24	0	0	0	12	82	9	12
(d)	0	3	0	9	0	76	0	6	6	70	0	6	0	85	0	27	0	0	94	94	88	12	0	30	24
(e)	0	21	0	73	88	12	3	0	3	12	3	3	36	0	0	52	15	3	3	0	3	0	0	55	0

difference is not statistically significant. The PBD for this question is 0.5, so the graduate students who performed well overall on the test did well on this question.

VI. SUMMARY

We find that the undergraduate students, including those in the upper-level course with higher mathematical sophistication, have common difficulties related to the superposition principle, symmetry concepts, and Gauss's law. The knowledge deficiencies can be broadly divided into three levels with increasing difficulty: lack of knowledge related to a particular concept, knowledge that is retrieved from memory but cannot be interpreted correctly, and knowledge that is retrieved and interpreted at the basic level but cannot be used to draw inferences in specific situations. Our investigation shows evidence that students' difficulties were due to knowledge deficiencies across all three levels. Because many questions required students to predict the outcomes for specific setups, they necessitated a transition from a mathematical representation to a concrete case. Therefore, deficiencies at the latter two levels were frequently observed. Instructional strategies that focus on improving student understanding of these concepts should take into account these difficulties. The multiple-choice test that we developed can help assess the effectiveness of strategies to improve student understanding of these concepts. We are developing and evaluating tutorials to help students with these concepts.

ACKNOWLEDGMENTS

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APPENDIX A: SUMMARY OF THE MULTIPLE-CHOICE TEST DESIGN

During the design of the multiple-choice test, we paid particular attention to the issues of reliability and validity.¹⁶ Reliability refers to the relative degree of consistency between testing if the test procedures are repeated for an individual or group. Validity refers to the appropriateness of the test score interpretation. The test design began with the development of a test blueprint that provided a comprehensive framework for planning decisions about the desired test attributes. The degree of specificity in the test plan was useful for creating questions. We tabulated the scope and extent of the content

covered and the level of cognitive complexity desired. We used previous free-response and multiple-choice questions administered to students as a guide and identified the desired performance and a description of conditions/contexts under which the performance was expected to occur.

We classified the cognitive complexity using a simplified version of Bloom's taxonomy:¹⁷ specification of knowledge, interpretation of knowledge, and application of knowledge in different situations. The performance targets and table of content and cognitive complexity were shown to five physics faculty members at the University of Pittsburgh for review. Modifications were made to the weights assigned to various concepts and to the performance targets based on faculty feedback. The performance targets were converted to approximately 30 free-response questions. These questions required students to provide reasoning for their responses. The free-response questions were administered (in groups of 10–20) to students in the calculus-based courses. Often, some students in a class were given one set of questions and others were given another set in order to sample student responses on most of the questions. We also tape-recorded interviews with five introductory student volunteers using a think-aloud protocol.⁵ Thirty multiple-choice questions were then designed using the most frequent incorrect student responses for the free-response questions and interviews as a guide for making the distractor choices. Choosing the four distractors to conform to the common difficulties is essential for increasing the discriminating properties of the questions. Five physics faculty members were asked to review the multiple-choice questions and comment on their appropriateness and relevance for calculus-based introductory physics and to detect ambiguity in question wording. A review form was developed to aid the faculty in reviewing the questions. The faculty also classified each question on a scale from very appropriate to least appropriate. Further modifications were made based on their recommendations. Then, a multiple-choice test was assembled using 25 questions that closely matched the initial table delineating the scope of the content and cognitive complexity. The same faculty members who earlier reviewed the questions were shown the test several times and modifications were made based on the feedback during the iterations.

The 50 min multiple-choice test was administered after instruction in Gauss's law to students in the calculus-based courses at Pittsburgh. Five student volunteers who had not taken the test earlier were interviewed individually and asked to respond to the test questions using the think-aloud protocol. These interviews provided us with an opportunity to clarify issues in depth. The reliability index α for the test was approximately 0.8 which is good by the standards of test-design.¹⁶ Item analysis of student responses was performed to judge whether each question functioned as

expected.¹⁶ In addition to the calculation of difficulty and discrimination of questions,¹⁶ item analysis included creating a table to count the number of students selecting each distractor in the upper and lower quartiles. Item analysis was very useful to determine whether individual questions and distractors functioned as expected. Based upon the item analysis and interviews, the test questions were modified further before being administered to the students.

APPENDIX B: THE MULTIPLE-CHOICE TEST

Instructions: Select *one* of the five choices (a)–(e) for each of the 25 problems. In all questions all physical objects are insulating (nonconducting); all insulators are nonpolarizable; ϵ_0 is the permittivity of free space; Gauss's law: $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}}/\epsilon_0$; and the sign convention is that for all closed surfaces, consider outward flux as positive.

Problem 1. Choose *all* of the following physical variables that are vectors:

- (i) Electric field
- (ii) Electric flux
- (iii) Electric charge

- (a) (i) only.
- (b) (i) and (ii) only.
- (c) (i) and (iii) only.
- (d) (ii) and (iii) only.
- (e) (i), (ii), and (iii).

Problem 2. Three identical point charges $+Q$ are arranged in a line as shown in Fig. 1. Points A, B, and C are along a parallel line. You do *not* know the lengths L and d . The three charges produce an electric field. Without knowledge of L and d , what can you infer about the electric field at points A, B, and C?

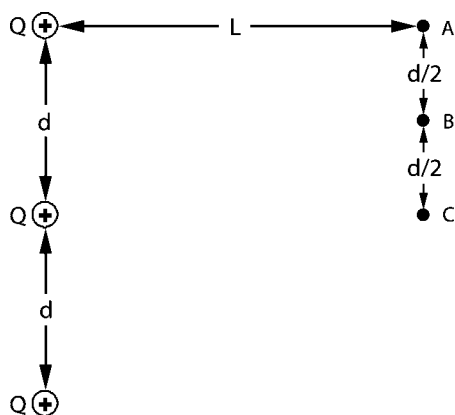


Fig. 1. Charge distribution for Problem 2.

- (a) Both the magnitude and direction of the field are the same at points A, B, and C.
- (b) The magnitude of the field is the same at points A, B, and C but the directions are different.

- (c) The exact direction of the field can be predicted only at point C.
- (d) The exact direction of the field can be predicted only at points A and C.
- (e) The exact direction of the field cannot be predicted at any of the three points.

Problem 3. Consider a horizontal square sheet, length L on each side, on which positive charge is uniformly distributed with charge per unit area (surface charge density σ C/m²) (see Fig. 2). You measure the electric field at two points, each at a height $h=L/2$ above the sheet: point C is directly above the center of the sheet and point B is off center.

Which one of the following statements is true about the field due to the finite sheet of charge, observed at points B and C?

- (a) The fields at points B and C have the same magnitude and same direction.
- (b) The fields at points B and C have different magnitudes and different directions.
- (c) The fields at points B and C have the same magnitude but different directions.
- (d) The fields at points B and C have different magnitudes but the same direction.
- (e) We cannot compare the fields at points B and C without knowing the numerical value of σ .

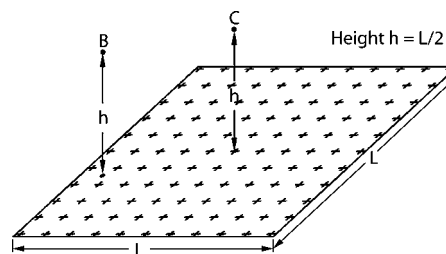


Fig. 2. A horizontal square sheet of charge for Problem 3.

Problem 4. You perform two experiments (E1 and E2) in which you distribute charge $+3Q$ differently on an equilateral triangle made with thin insulating rods. E1: You put identical charges, $+Q$ each, in three localized blobs on the vertices of the triangle. E2: You distribute charge $+3Q$ uniformly on the triangle. The dashed triangle in Fig. 3 shows an imaginary equilateral triangle concentric with the insulating triangle. Which one of the following statements is true about the electric field magnitude at points on the dashed imaginary triangle due to the $+3Q$ charge?

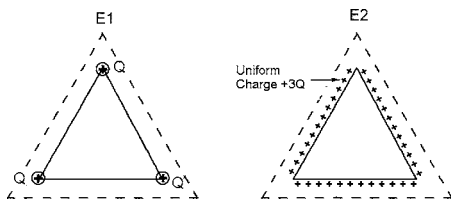


Fig. 3. Charge distribution for Problem 4.

- It is the same everywhere on the dashed triangle only in experiment E1.
- It is the same everywhere on the dashed triangle only in experiment E2.
- In each experiment, it is the same everywhere on the dashed triangle, but the magnitudes differ in the two experiments.
- In each experiment, it is the same everywhere on the dashed triangle, and the magnitudes are equal in the two experiments.
- Both in experiment E1 and in experiment E2 it varies from point to point on the dashed triangle.

Problem 5. For Gauss's law to be valid, the Gaussian surface used *must* be a

- Highly symmetrical surface.
- Spherical surface.
- Cylindrical surface.
- Open surface.
- Closed surface.

Problem 6. Choose all of the following statements that must be true about a Gaussian surface in order for Gauss's law to be convenient for calculating the electric field at a point on the surface:

- The electric field direction should be easy to predict at every point on the surface.
 - The Gaussian surface must be chosen to take advantage of the symmetry of the charge distribution.
 - The Gaussian surface must be chosen to take advantage of the symmetry of the object enclosed inside it regardless of how the charges are distributed on that object.
- (i) only.
 - (ii) only.
 - (iii) only.
 - (i) and (ii) only.
 - (i) and (iii) only.

Problem 7. In Fig. 4, a point charge $+Q_1$ is at the center of an imaginary spherical surface and another point charge $+Q_2$ is outside it. Point P is on the surface of the sphere. Let Φ_S be the net electric flux through the sphere and \vec{E}_P be the electric field at point P on the sphere. Which one of the following statements is true?

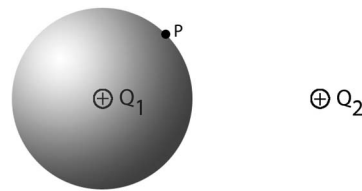


Fig. 4. Charge distribution for Problem 7.

- Both charges $+Q_1$ and $+Q_2$ make nonzero contributions to Φ_S but only the charge $+Q_1$ makes a nonzero contribution to \vec{E}_P .
- Both charges $+Q_1$ and $+Q_2$ make nonzero contributions to Φ_S but only the charge $+Q_2$ makes a nonzero contribution to \vec{E}_P .
- Only the charge $+Q_1$ makes a nonzero contribution to Φ_S but both charges $+Q_1$ and $+Q_2$ make nonzero contributions to \vec{E}_P .
- Charge $+Q_1$ makes no contribution to Φ_S or \vec{E}_P .
- Charge $+Q_2$ makes no contribution to Φ_S or \vec{E}_P .

Problem 8. Your friend measures the electric flux through three closed surfaces (1), (2), and (3) to be 1, 2, and $-3 \text{ Nm}^2/\text{C}$, respectively. Choose all of the following statements that can be inferred from these measurements:

- The area of surface (3) is largest.
- The magnitude of the net charge enclosed inside surface (3) is largest.
- The electric field everywhere on surface (1) is weaker than on surface (2).

- (i) only.
- (ii) only.
- (i) and (ii) only.
- (ii) and (iii) only.
- (i), (ii), and (iii).

Problem 9. Shown in Fig. 5 are three concentric spherical Gaussian surfaces A, B, and C with a positive point charge $+Q$ at their center. A second, but negative point charge $-Q$ is enclosed only by surface C.

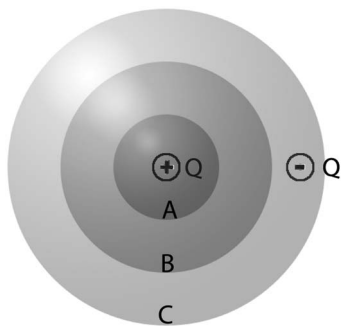


Fig. 5. Gaussian surfaces for Problem 9.

Which is a correct statement about the magnitudes of the electric flux Φ_S through the three surfaces?

- (a) $\Phi_A = \Phi_B = \Phi_C$.
- (b) $\Phi_A = \Phi_B > \Phi_C$.
- (c) $\Phi_A > \Phi_B > \Phi_C$.
- (d) $\Phi_B > \Phi_A > \Phi_C$.
- (e) None of the above.

- Problem 10. Choose all of the following cases for which the electric field at any point outside the object can be calculated easily from Gauss's law. (In each case, assume that the insulators are nonpolarizable and no other charges are present anywhere.) See Fig. 6.
- (i) Insulating sphere with a uniform charge throughout its volume.
 - (ii) Insulating dumbbell with a uniform charge throughout its volume.
 - (iii) Insulating dumbbell with only one of the two spherical balls at the end uniformly charged throughout its volume.

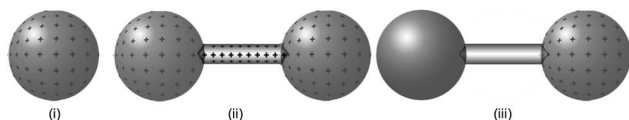


Fig. 6. Diagram for Problem 10.

- (a) (i) only.
- (b) (ii) only.
- (c) (i) and (ii) only.
- (d) (i) and (iii) only.
- (e) (i), (ii), and (iii).

- Problem 11. Consider three possible Gaussian surfaces (a sphere, a cube, and a cylinder) which extend half above and half below an infinite horizontal sheet of uniform charge density as shown in Fig. 7.

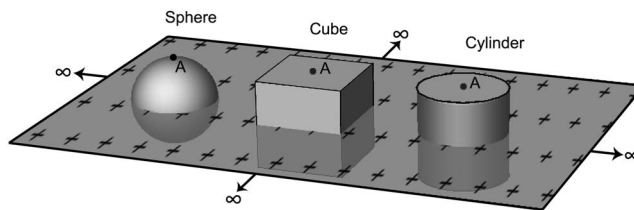


Fig. 7. Diagram for Problem 11.

Point A is located at the top center of each Gaussian surface. For which of the Gaussian surfaces will Gauss's law help us to easily calculate the electric field at point A due to the sheet of charge?

- (a) Only the sphere is symmetric enough.
- (b) Only the cylinder, because the side walls have zero flux and it has circular symmetry.
- (c) Only the cylinder and the cube, because any shape with the side walls perpendicular to the sheet and end caps parallel to the sheet will work.
- (d) Only the sphere and the cylinder, because they have circular cross sections.
- (e) All surfaces will work since they are symmetric.

- Problem 12. A thin insulating rod of length 1 m, with charge $Q = +100$ nC (nanocoulombs) uniformly distributed over it, is symmetrically situated inside a spherical Gaussian surface with a total surface area of $A = 15$ m² (see Fig. 8). No other charges are present anywhere.

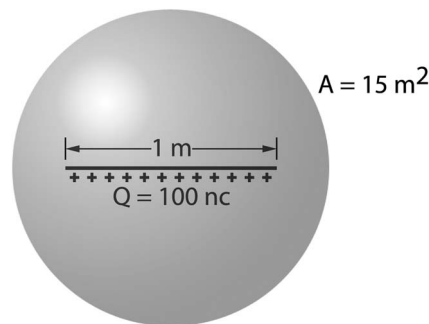


Fig. 8. Diagram for Problem 12.

We can use Gauss's law to conclude that:

- (i) The magnitude of the net electric flux through the Gaussian surface is $\Phi_S = Q/\epsilon_0$.
- (ii) The electric field magnitude at any point on the surface is

$$|\vec{E}| = \Phi_S/A = Q/(\epsilon_0 A).$$

Which of the above statements are true?

- (a) (i) only.
- (b) (ii) only.
- (c) Both (i) and (ii).
- (d) Neither (i) nor (ii).
- (e) Not enough information.

Problem 13. The surface of a thin-walled cubic insulating (nonconducting) box is given a uniformly distributed positive surface charge. Which one of the following can be inferred about the electric field everywhere inside the insulating box due to this surface charge using Gauss's law?

- (a) Its magnitude everywhere inside must be zero.
- (b) Its magnitude everywhere inside must be nonzero but uniform (the same).
- (c) Its direction everywhere inside must be radially outward from the center of the box.
- (d) Its direction everywhere inside must be perpendicular to one of the sides.
- (e) None of the above.

Setup for Problems 14 and 15

You perform two experiments (see Fig. 9) in which you distribute charge $+6Q$ differently on the surface of an isolated hollow insulating (nonconducting) sphere:

Experiment 1: You put identical charges, $+Q$ each, on the spherical surface in six localized blobs (you can consider them point charges) such that the adjacent blobs are equidistant from each other. Experiment 2: You distribute charge $+6Q$ uniformly on the surface of the sphere.

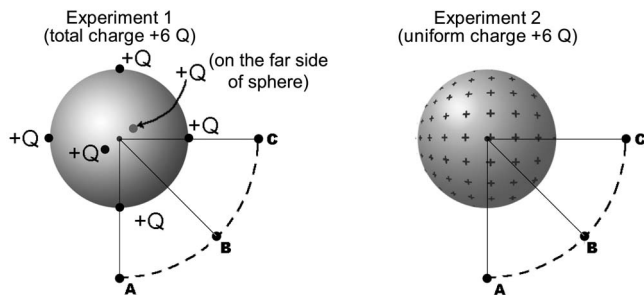


Fig. 9. Diagram for Problems 14 and 15.

Problem 14. In experiments 1 and 2, points A, B, and C are equidistant from the center and lie in the same equatorial plane of the sphere. In experiment 1, points A and C are straight out from two of the charges and point B is in between points A and C as shown. Which one of the following statements is true about the electric field magnitudes at points A, B, and C due to the $+6Q$ surface charge?

- (a) In each experiment, the field magnitude is the same at points A, B, and C, but the magnitudes differ in the two experiments.

- (b) In each experiment, the field magnitude is the same at points A, B, and C, and the magnitudes are equal in the two experiments.
- (c) In experiment 1, the field magnitude is the same at points A, B, and C, but not in experiment 2.
- (d) In experiment 2, the field magnitude is the same at points A, B, and C, but not in experiment 1.
- (e) None of the above.

Problem 15. Which one of the following is a true statement about the electric field magnitude inside the hollow insulating sphere due to the $+6Q$ surface charge (see Fig. 9)?

- (a) It is zero everywhere inside the sphere in both experiments.
- (b) It is nonzero everywhere inside the sphere in both experiments.
- (c) In experiment 1, it has a magnitude that varies from point to point inside the sphere, but it is zero everywhere inside the sphere in experiment 2.
- (d) In experiment 1, it has the same nonzero magnitude everywhere inside the sphere, but it is zero everywhere inside the sphere in experiment 2.
- (e) None of the above.

Problem 16. Six positive point charges, $+Q$ each, are placed on an isolated hollow insulating sphere such that the adjacent point charges are equidistant (same arrangement as in experiment 1 in Problem 15). A spherical and a cubic Gaussian surface concentric with the insulating sphere are shown in Fig. 10. Choose *all* of the following statements that are true about the electric field due to this charge distribution:

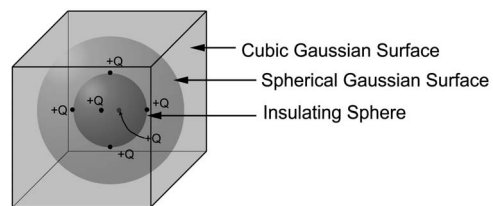


Fig. 10. Diagram for Problem 16.

- (i) The electric field magnitude is the same everywhere on the cubic Gaussian surface because the cube has the same symmetry as that of the charge distribution.

- (ii) The electric field magnitude is the same everywhere on the spherical Gaussian surface because the sphere has the same symmetry as the insulating sphere.
 (iii) The electric field is radially outward (straight out from the center) everywhere on the spherical Gaussian surface.

- (a) (i) only.
 (b) (ii) only.
 (c) (iii) only.
 (d) (ii) and (iii) only.
 (e) None of the above.

Problem 17. Shown in Fig. 11 are three thin-walled insulating objects with a net charge $+Q$ uniformly distributed on their surfaces: a cube, a sphere, and an open ended cylinder of length L (no caps) and diameter L . The objects are distant from each other so that each may be considered electrically isolated.

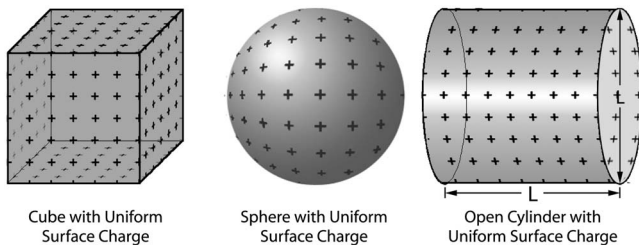


Fig. 11. Diagram for Problem 17.

We can easily use Gauss's law to find the electric field due to the uniform surface charge at a point outside due to:

- (a) The cube only.
 (b) The sphere only.
 (c) The sphere and cylinder only.
 (d) The sphere and cube only.
 (e) All three objects.

Problem 18. Choose *all* of the following statements that are true (Note: This question does not refer to a particular charge distribution so a statement is true only if there are no exceptions):

- (i) If the electric field at every point on a Gaussian surface is zero, the net electric flux through the surface *must* be zero.
 (ii) If there is no charge enclosed inside a Gaussian surface, the electric field everywhere on the surface *must* be zero.
 (iii) If the net electric flux through a Gaussian surface is zero, the electric field everywhere on the surface *must* be zero.

- (a) (i) only.
 (b) (ii) only.
 (c) (i) and (ii) only.
 (d) (i) and (iii) only.

- (e) (ii) and (iii) only.

Setup for Problems 19 and 20

A cubic Gaussian surface with 1 m on a side is oriented with two horizontal and four vertical faces, as shown in Fig. 12. It is in a uniform electric field of 20 N/C which is di-

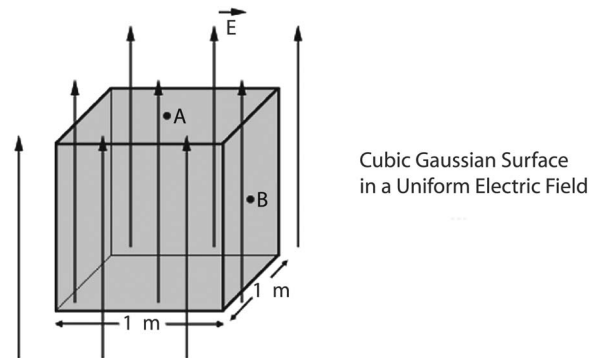


Fig. 12. Diagram for Problems 19 and 20.

rected vertically upward. Point A is on the top surface and point B on a side surface of the cubic Gaussian surface.

Problem 19. Which one of the following statements is true about the electric field at points A and B?

- (a) The field is zero at both points A and B.
 (b) The field is zero at point A but not at point B.
 (c) The field is zero at point B but not at point A.
 (d) The field is nonzero at both points A and B and its direction is the same at the two points.
 (e) The field is nonzero at both points A and B but its direction is different at the two points.

Problem 20. Choose all of the following statements that are true about the electric flux.

- (i) The net flux through the whole cubic surface is zero.
 (ii) The magnitude of the flux through the top face of the cubic surface is $20 \text{ N m}^2/\text{C}$.
 (iii) The magnitude of the net flux through the whole cubic surface is $20 \text{ N m}^2/\text{C}$.

- (a) (i) only.
 (b) (ii) only.
 (c) (iii) only.
 (d) (i) and (ii) only.
 (e) (ii) and (iii) only.

Setup for Problems 21 and 22

Shown in Fig. 13 are four imaginary surfaces coaxial with an isolated infinitely long line of charge (with uniform linear charge density $\lambda \text{ C/m}$):

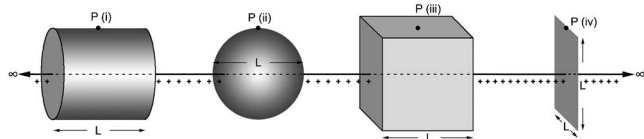


Fig. 13. Diagram for Problems 21 and 22.

- (i) A closed cylinder of length L .
- (ii) A sphere of diameter L .
- (iii) A closed cubic box with side L .
- (iv) A two-dimensional square sheet with side L . The plane of the sheet is perpendicular to the line of charge.

Problem 21. Choose all of the above surfaces through which the net electric flux is $\Phi_S = \lambda L / \epsilon_0$:

- (a) (i) only.
- (b) (i) and (ii) only.
- (c) (i) and (iii) only.
- (d) (i), (ii), and (iii) only.
- (e) (i), (ii), (iii), and (iv).

Problem 22. Choose all of the above surfaces which can be used as Gaussian surfaces to easily find the electric field magnitude (due to the infinite line of charge) at a point P shown on the surface using Gauss's law:

- (a) (i) only.
- (b) (i) and (ii) only.
- (c) (i) and (iii) only.
- (d) (i), (ii), and (iii) only.
- (e) (i), (ii), (iii), and (iv).

Problem 23. The diagram in Fig. 14 shows the electric field lines in a region. Sadly, you do not know the field inside the three regions (i), (ii), and (iii). This cross-sectional drawing is qualitatively correct. Which region (or regions) carries a net charge of the greatest magnitude?

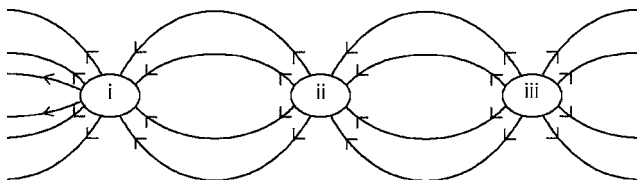


Fig. 14. Diagram for Problem 23.

- (a) (i) only.
- (b) (ii) only.
- (c) (iii) only.
- (d) (ii) and (iii) which have equal net charge.
- (e) (i), (ii), and (iii) which have equal net charge.

Problem 24.

Shown in Fig. 15 are four regions A, B, C, and D (separated by spherical surfaces). The electric field is zero in regions A (innermost) and D (outermost). The electric field in regions B and C is radially outward and inward, respectively.

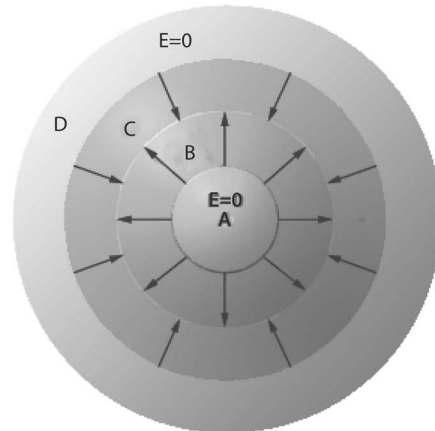


Fig. 15. Diagram for Problem 24.

Choose all of the following statements that must be true:

- (i) The combined net charge enclosed in all the regions shown must be zero.
- (ii) There *cannot* be a point charge at the center of region A.
- (iii) There must be a negative surface charge between regions B and C.

- (a) (i) only.
- (b) (i) and (ii) only.
- (c) (i) and (iii) only.
- (d) (ii) and (iii) only.
- (e) (i), (ii), and (iii).

Problem 25.

In Fig. 16 a point charge $+Q$ is near a thin hollow insulating (nonconducting) sphere of radius L that has an equal amount of charge $+Q$ uniformly distributed on its surface. No other charges are around.

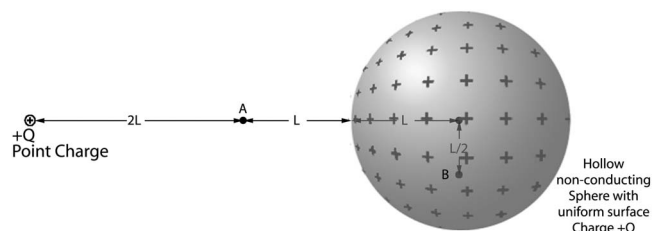


Fig. 16. Diagram for Problem 25.

Which one of the following is a true statement about the net electric field (due to the point charge and the surface charge on the hollow insulating sphere) at points A (outside the sphere at a distance $2L$ from the center) and B (inside the sphere at a distance $L/2$ from the center)?

- (a) The electric field is zero at point A but is nonzero at point B.
- (b) The electric field is nonzero at point A but is zero at point B.
- (c) The electric field is nonzero at both points A and B.
- (d) The electric field is zero at both points A and B.
- (e) It is impossible to answer this question without knowing the numerical value of Q .

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