Física 5^{to}
Solución 1^{er} Parcial

Parte A:

1. (a)

$$
\begin{aligned}
F_e &= k_e \frac{e^2}{r^2} = 8.99 \times 10^9 \frac{Nm^2}{C^2} (\frac{1.6 \times 10^{-19}\, C}{5 \times 10^{-11}\, m})^2 = 9 \times 10^{-8}\, N. \\
F_g &= G \frac{m_1 m_2}{r^2} = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2} \frac{(9.11 \times 10^{-31}\, kg)\,(1.67 \times 10^{-27}\, kg)}{(5 \times 10^{-11}\, m)^2} = \\
&= 4 \times 10^{-47}\, N. \\
\frac{F_e}{F_g} &= \frac{9.2 \times 10^{-8}}{4.1 \times 10^{-47}} = 2 \times 10^{39}
\end{aligned}
$$

(b)

$$
\begin{aligned}
a_e &= \frac{F_e}{m_e} = \frac{8.2 \times 10^{-8}\, N}{9.11 \times 10^{-31}\, kg} = 1 \times 10^{23} \frac{m}{s^2} \\
a_g &= \frac{F_e}{m_e} = \frac{3.6 \times 10^{-47} N}{9.11 \times 10^{-31}\, kg} = 4 \times 10^{-17} \frac{m}{s^2}
\end{aligned}
$$

2.

$$
\begin{aligned}
F_{13} &= k_e \frac{(15 \times 10^{-6})\, q_3}{(2-x)^2} \\
F_{23} &= k_e \frac{(6.0 \times 10^{-6})\, q_3}{x^2}
\end{aligned}
$$

Igualando ambas fuerzas y simplificando, obtenemos

$$
\begin{aligned}
& 3x^2 + 8x - 8 = 0 \\
\Rightarrow \quad & x = -\frac{4}{3} \pm \frac{2\sqrt{10}}{3}
\end{aligned}
$$

La única solución física es la positiva: $x = 0.77$ m.

Comprobación:

$$
\begin{aligned}
\frac{15}{(2-x)^2} &= 9.9934 \\
\frac{6}{x^2} &= 9.9934
\end{aligned}
$$

3. (a)

$$\begin{aligned}
\vec{F}_{23} &= k_e \frac{q_2\, q_3}{r_{23}^2} = 8.99 \times 10^9 \, \frac{Nm^2}{C^2} \, \frac{(-2 \times 10^{-9}\, C)(5.99 \times 10^{-9}\, C)}{(4\, m)^2} = \\
&= 6.73 \times 10^{-9}\, N. \\
F_{23_x} &= -6.73 \times 10^{-9}\, N. \\
F_{23_y} &= 0\, N.
\end{aligned}$$

(b)

$$\begin{aligned}
\vec{F}_{13} &= k_e \frac{q_1\, q_3}{r^2} = 8.99 \times 10^9 \, \frac{Nm^2}{C^2} \, \frac{(6 \times 10^{-9}\, C)(5.99 \times 10^{-9}\, C)}{(5\, m)^2} = \\
&= 1.29 \times 10^{-8}\, N. \\
F_{13_x} &= F_{13} \cos\theta = 1.29 \times 10^{-8}\ 0.8\, N = 1.03 \times 10^{-8}\, N. \\
F_{13_y} &= F_{13} \sin\theta = 1.08 \times 10^{-8}\ 0.6\, N = 7.76 \times 10^{-9}\, N.
\end{aligned}$$

(c)

$$\begin{aligned}
F_x &= F_{23_x} + F_{13_x} = -6.73 \times 10^{-9} + 1.03 \times 10^{-8} = 3.60 \times 10^{-9}\, N. \\
F_y &= F_{23_y} + F_{13_y} = 0 + 7.76 \times 10^{-9} = 7.76 \times 10^{-9}\, N. \\
F &= \sqrt{F_x^2 + F_y^2} = 8.55 \times 10^{-9}\, N. \\
\theta &= \arctan \frac{F_y}{F_x} = \tan^{-1} \frac{7.76}{3.60} = 65.1^\circ
\end{aligned}$$

4.

$$\begin{aligned}
\vec{F}_{12} &= k_e \frac{q_1\, q_2}{r_{12}^2} = 8.99 \times 10^9 \, \frac{Nm^2}{C^2} \, \frac{(5 \times 10^{-9}\, C)(6 \times 10^{-9}\, C)}{(3 \times 10^{-1}\, m)^2} = 3.00 \times 10^{-6}\, N. \\
\vec{F}_{13} &= k_e \frac{q_1\, q_3}{r_{13}^2} = 8.99 \times 10^9 \, \frac{Nm^2}{C^2} \, \frac{(5 \times 10^{-9}\, C)(3 \times 10^{-9}\, C)}{(1 \times 10^{-1}\, m)^2} = 1.35 \times 10^{-5}\, N. \\
F &= \sqrt{F_{12}^2 + F_{13}^2} = 1.38 \times 10^{-5}\, N. \\
\theta &= \arctan(\frac{F_{13}}{F_{12}}) + 180^\circ = 78^\circ + 180^\circ = 258^\circ
\end{aligned}$$

Parte B:

5. (a)

$$\begin{aligned}
\sum F_y &= T - mg - F_e = 0 \\
&\Rightarrow T = mg + F_e = mg + k_e \frac{q_1 q_2}{d^2} \\
&= (7.5 \times 10^{-3}\, kg)(9.8\, \frac{m}{s^2}) + 8.99 \times 10^9\, \frac{Nm^2}{C^2}\, \frac{(32 \times 10^{-9}\, C)(58 \times 10^{-9}\, C)}{(2 \times 10^{-2} m)^2} \\
&= 0.115\, N.
\end{aligned}$$

(b)

$$\begin{aligned}
T_{max} &= mg + F_e \\
&\Rightarrow F_e = T_{max} - mg
\end{aligned}$$

$$\begin{aligned}
k_e \frac{q_1 q_2}{{d_{min}}^2} &= T_{max} - mg \\
d_{min} &= \sqrt{\frac{k_e\, q_1\, q_2}{T_{max} - mg}} \\
&= \sqrt{\frac{8.99 \times 10^9\, \frac{Nm^2}{C^2}\, (32 \times 10^{-9}\, C)(58 \times 10^{-9}\, C)}{0.180\, N - (7.5 \times 10^{-3}\, kg)(9.8\, \frac{m}{s^2})}} = \\
&= 1.25 \times 10^{-2}\, m = 1.25\, cm
\end{aligned}$$

6.

$$\begin{aligned}
F_1 &= 8.99 \times 10^9\, \frac{Nm^2}{C^2}\, \frac{(2 \times 10^{-6}\, C)(4 \times 10^{-6}\, C)}{(0.5\, m)^2} = 0.288\, N \\
F_2 &= 8.99 \times 10^9\, \frac{Nm^2}{C^2}\, \frac{(2 \times 10^{-6}\, C)(7 \times 10^{-6}\, C)}{(0.5\, m)^2} = 0.503\, N \\
F_x &= F_1 - F_2 \cos 60^\circ = 0.288 - \frac{1}{2}\, 0.503 = 3.65 \times 10^{-2}\, N = 0.0360\, N \\
F_y &= -F_2 \sin 60^\circ = -\frac{\sqrt{3}}{2}\, 0.503 = -0.436\, N \\
F_{tot} &= \sqrt{F_x^2 + F_y^2} = 0.437\, N \qquad (1) \\
\Theta &= \tan^{-1}\left(\frac{F_y}{F_x}\right) + \pi = 1.49 + \pi = 85^\circ + 180^\circ = 265^\circ
\end{aligned}$$

7.

$$\begin{aligned}
\sum F_x &= T\sin 5^\circ - F_e \Rightarrow F_e = T\sin 5^\circ \\
\sum F_y &= T\cos 5^\circ - mg \Rightarrow T = \frac{mg}{\cos 5^\circ} \\
k_e \frac{q_1\, q_2}{d^2} = F_e &= \frac{mg}{\cos 5^\circ}\sin 5^\circ = mg\tan 5^\circ
\end{aligned}$$

$$d = 2\,L\,\sin 5^\circ$$

Reemplazamos todo, y consideramos a $q_1 = q_2$:

$$q = 2\,L\,\sin 5^\circ \sqrt{\frac{mg\tan 5^\circ}{k_e}} = 7.2\,nC.$$

8.

$$\begin{aligned}
T &= F_e = k_e\,\frac{q_1\,q_2}{r^2} = 8.99\times 10^9\,\frac{Nm^2}{C^2}\,\frac{(8\times 10^{-7}\,C)(6\times 10^{-7}\,C)}{(5\times 10^{-2}\,m)^2} = 1.73\,N \\
T &= kx \Rightarrow k = \frac{T}{x} = \frac{1.73}{0.035} = 49.3\,\frac{N}{m}
\end{aligned}$$

9.

$$\begin{aligned}
\sum F_x &= F_e - F_s = 0 \Rightarrow F_s = F_e \\
kx &= k_e\,\frac{q_1\,q_2}{d^2} \Rightarrow k = \frac{k_e\,q_1\,q_2}{x\,d^2} = \\
&= \frac{(8.99\times 10^9\,\frac{Nm^2}{C^2})\,(2.70\times 10^{-6}\,C)(8.60\times 10^{-6}\,C)}{(5.00\times 10^{-3}\,m)(9.00\times 10^{-2}\,m)^2} = \\
&= 5.15\times 10^3\,\frac{N}{m}
\end{aligned}$$