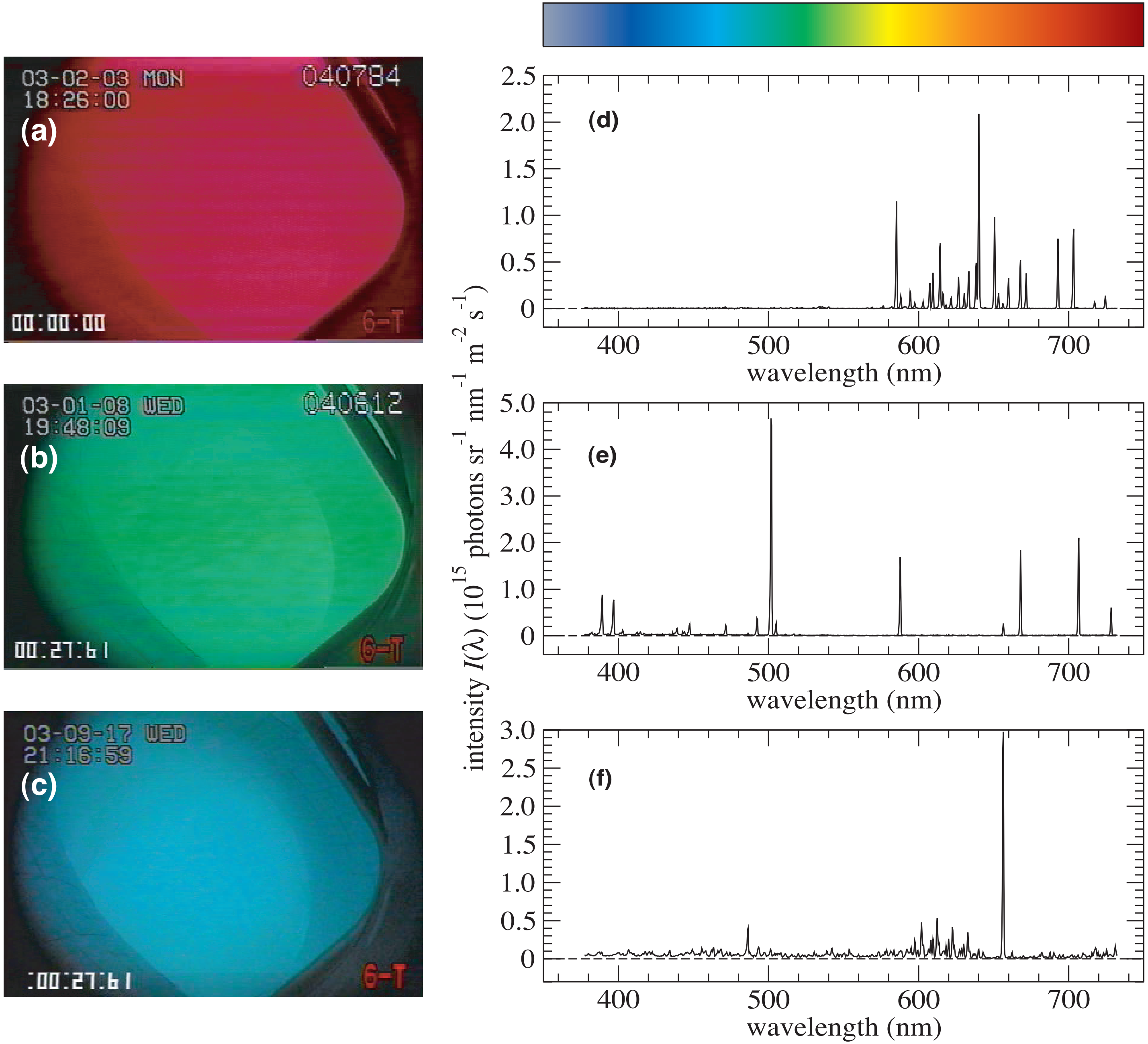
Introduction to the plasma spectroscopy

Motoshi Goto
National Institute for Fusion Science

- Einstein coefficients
- energy state designation under L-S coupling scheme
- population mechanism for excited states — collisional-radiative model
- effect of external field — perturbation theory

introduction

glow discharge in LHD

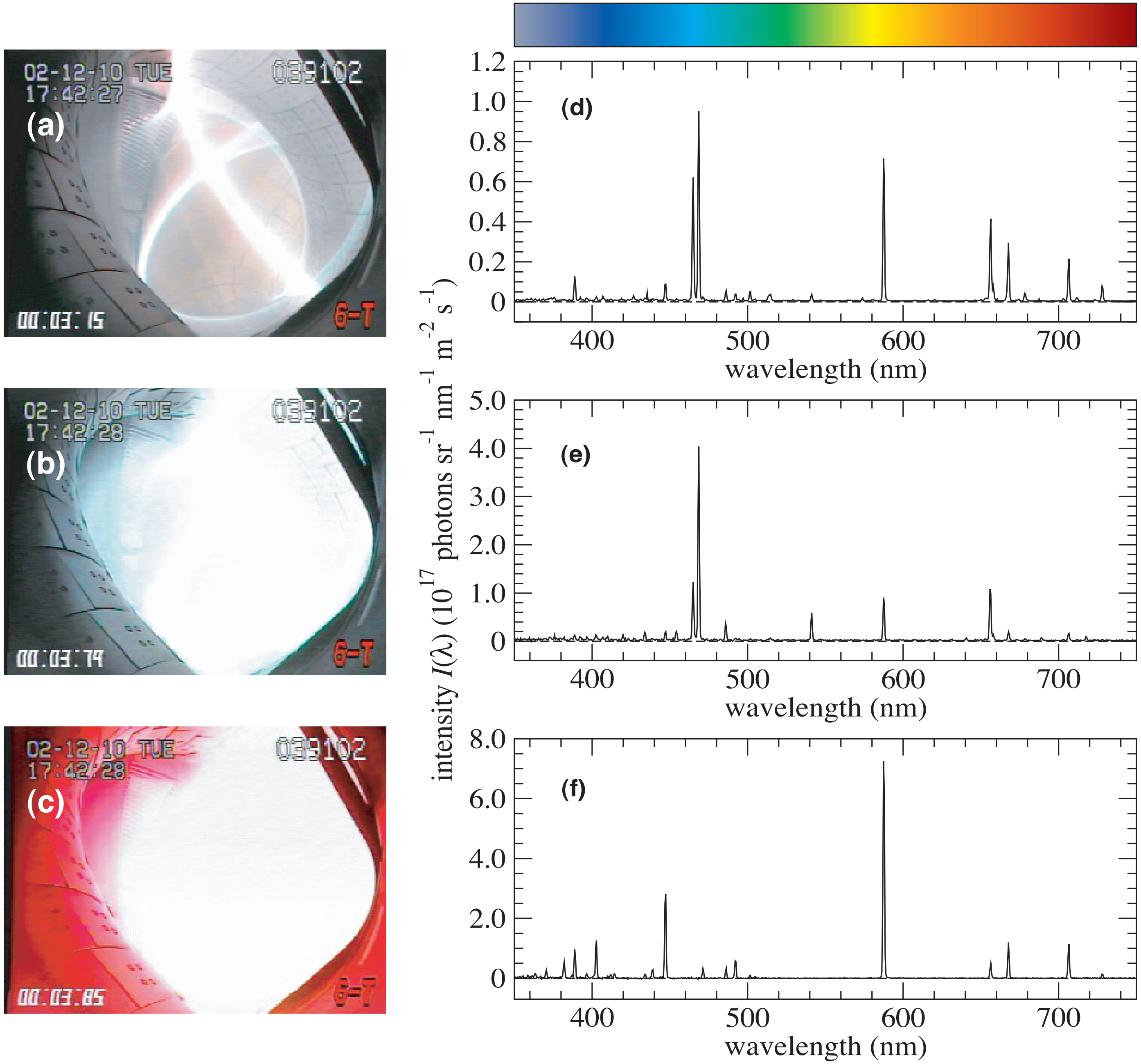


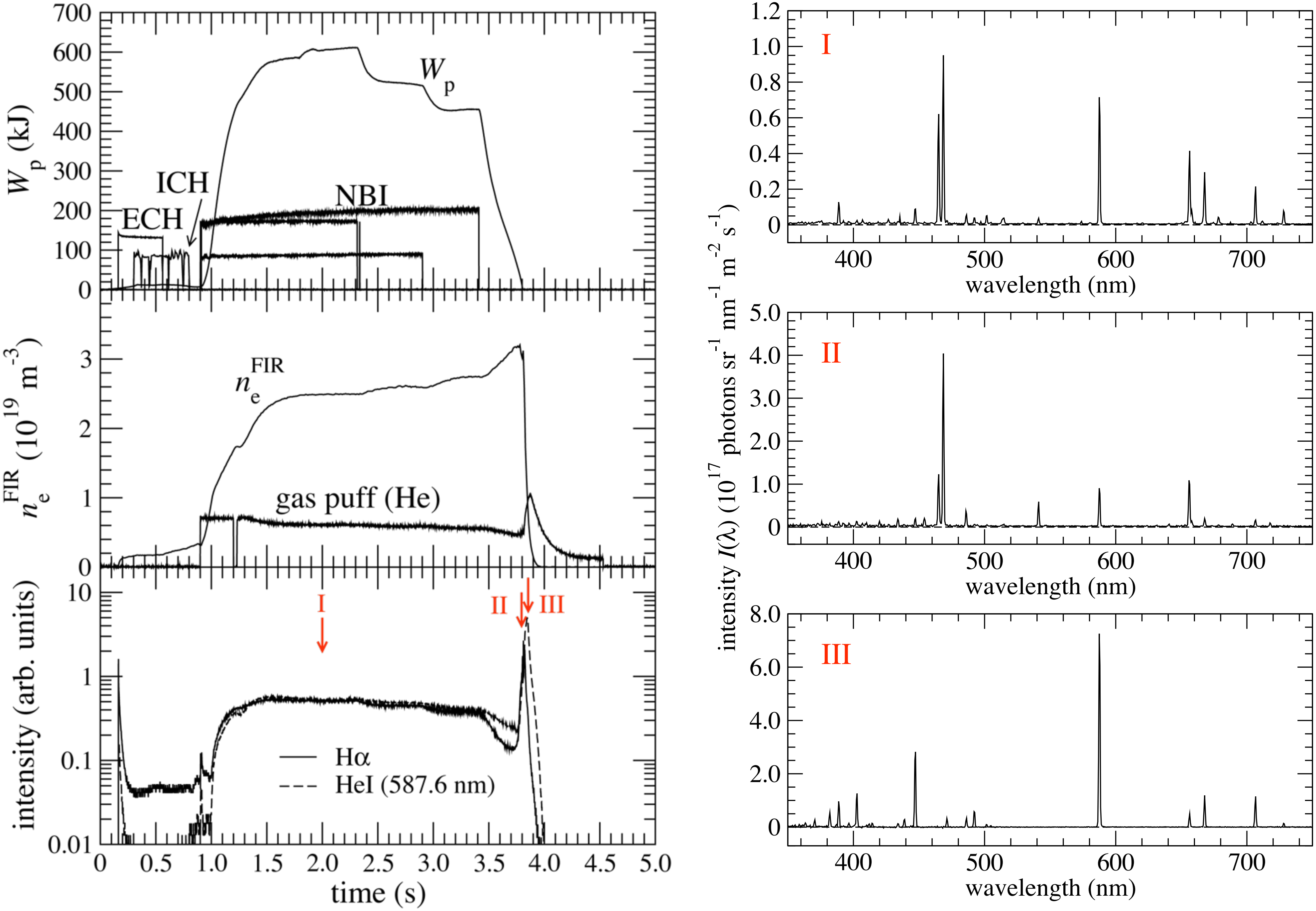
Ne

He

H_2

main discharge with helium gas

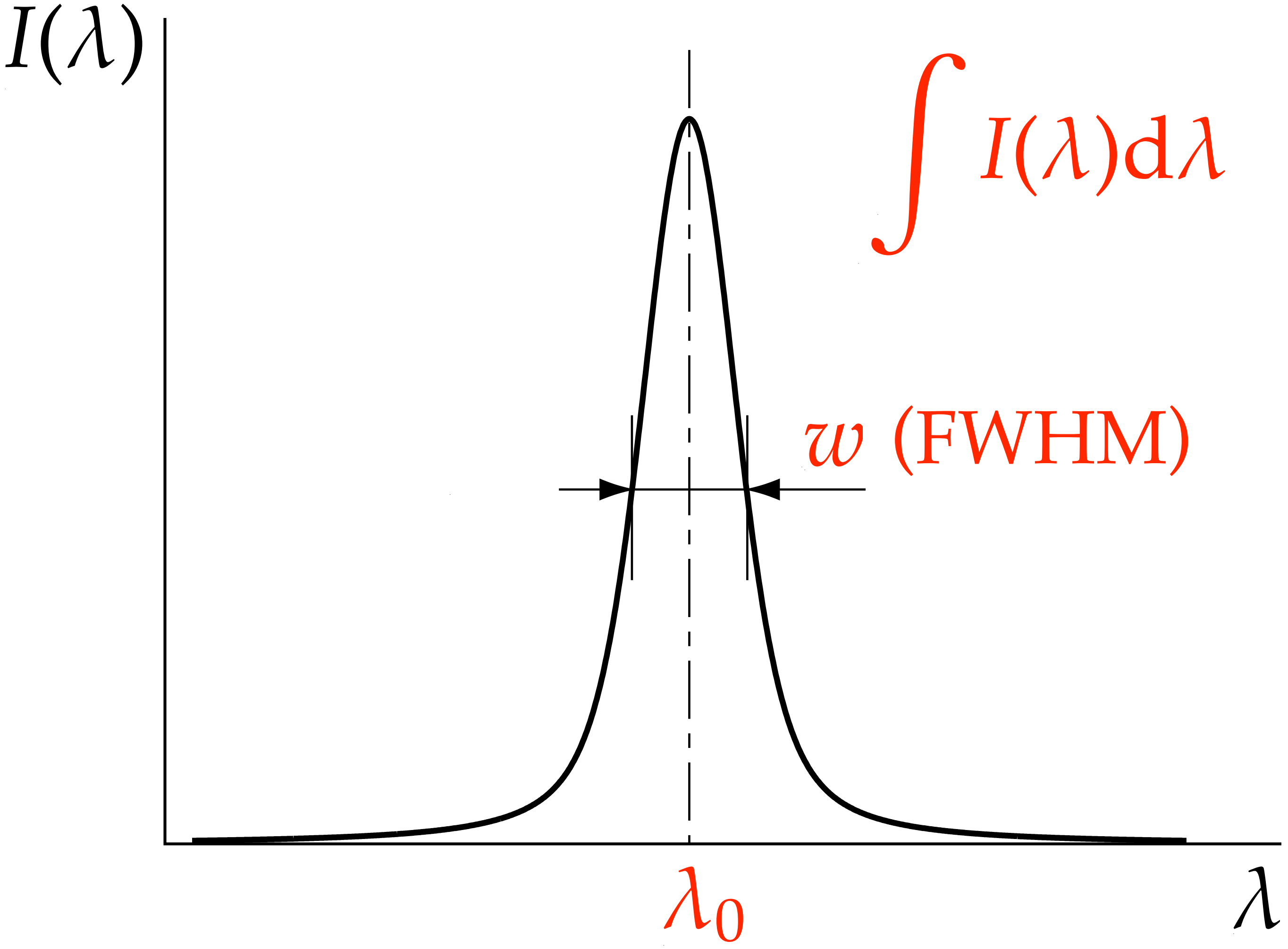



W_p (kJ)
700
600
500
400
300
200
100
0
W_p
ECH
ICH
NBI
n_e^{FIR} (10^{19} m^{-3})
3
2
1
0
n_e^{FIR}
gas puff (He)
intensity (arb. units)
10
1
0.1
0.01
I
II
III
Hα
HeI (587.6 nm)
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
time (s)
intensity $I(\lambda)$ (10^{17} photons sr^{-1} nm^{-1} m^{-2} s^{-1})
I
1.2
1.0
0.8
0.6
0.4
0.2
0
400
500
600
700
wavelength (nm)
II
5.0
4.0
3.0
2.0
1.0
0
400
500
600
700
wavelength (nm)
III
8.0
6.0
4.0
2.0
0
400
500
600
700
wavelength (nm)

- plasma shows various colors with different working gas
- quantitative treatment of color is, however, difficult
- spectrum is used for quantitative analysis instead

emission lines

properties of emission line



what can be known?

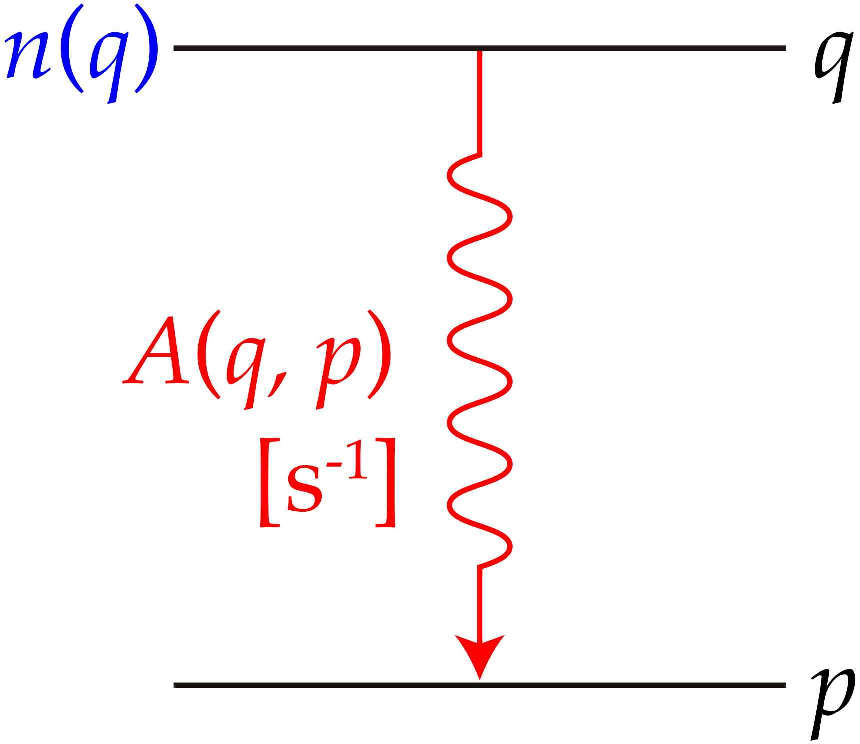
observable		obtainable
shift		ion velocity
broadening	Doppler	T_i
	Stark	n_e
splitting	Zeeman	magnetic field
	Stark	electric field
intensity ratios intensity distribution		T_e, n_e ionizing or recombining
intensity		n_i

high resolution measurement

low resolution measurement

line intensity

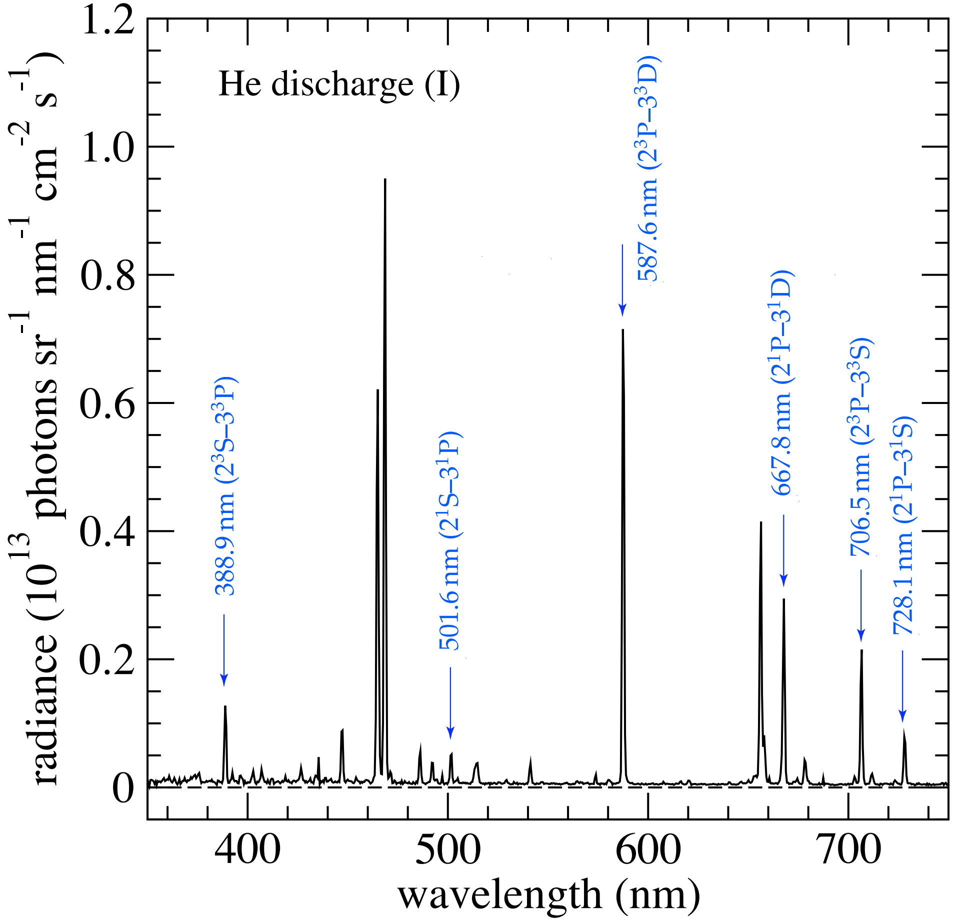
- intensity is the product of population density and spontaneous transition probability or Einstein A coefficient

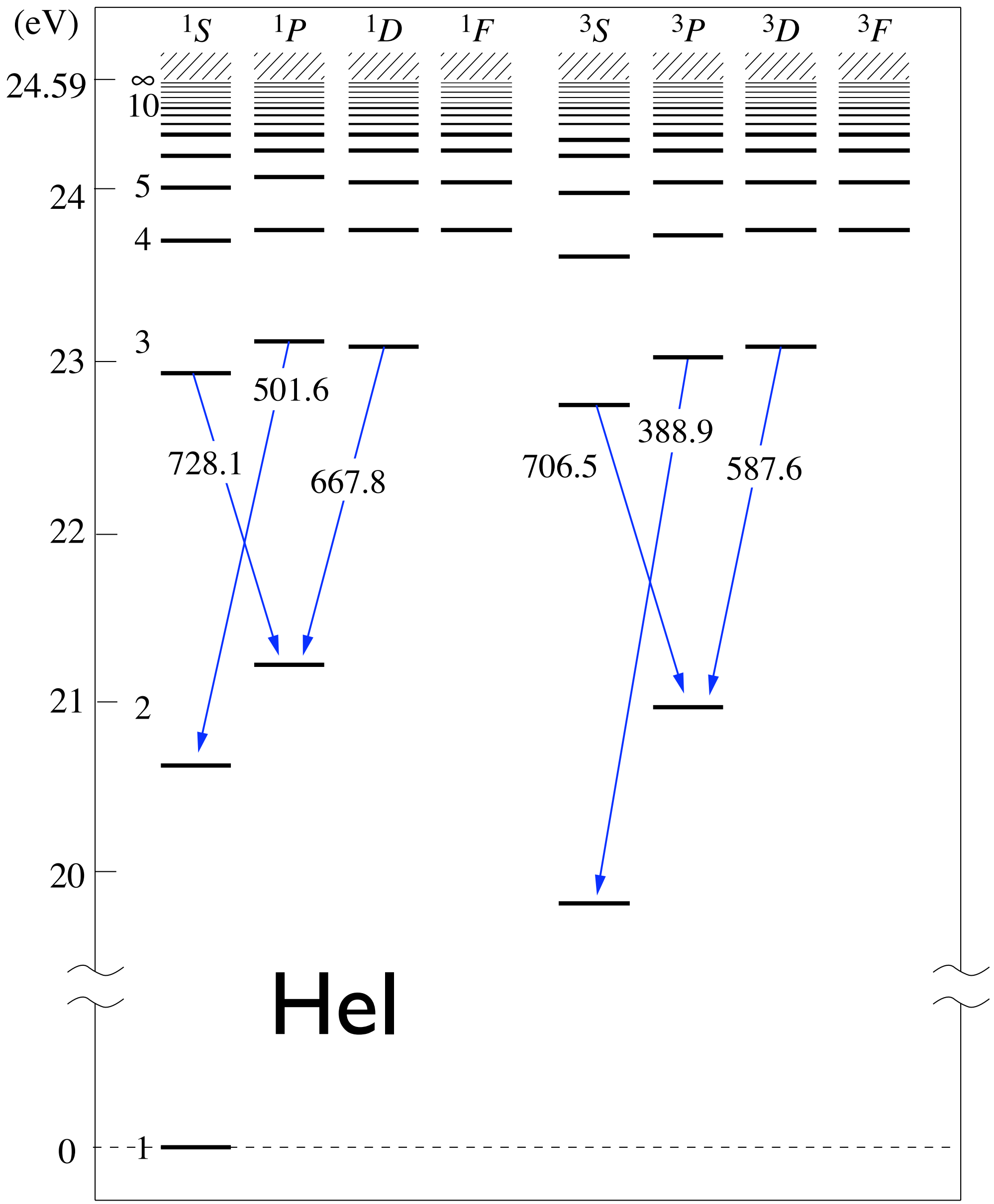


$$I_{qp} = n(q)A(q,p)$$

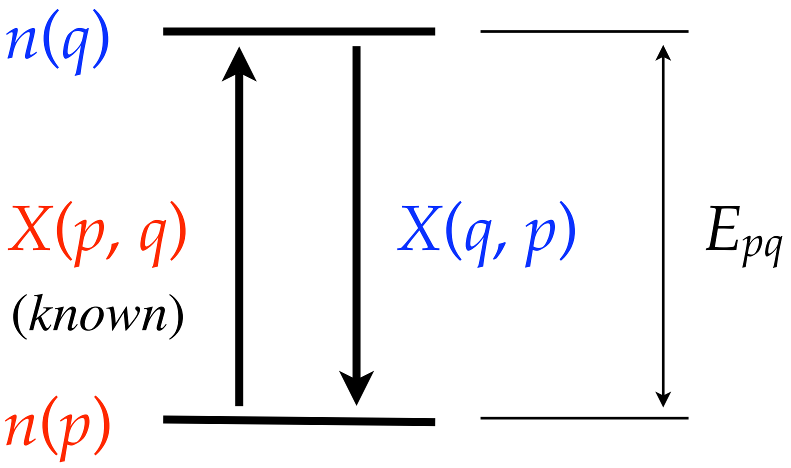
$$\left[\mathrm{m}^{-3}\mathrm{s}^{-1}\right]$$

A coefficient is constant and proper to individual transitions





detailed balance



under thermodynamic equilibrium

$$n(p)X(p, q) = n(q)X(q, p)$$

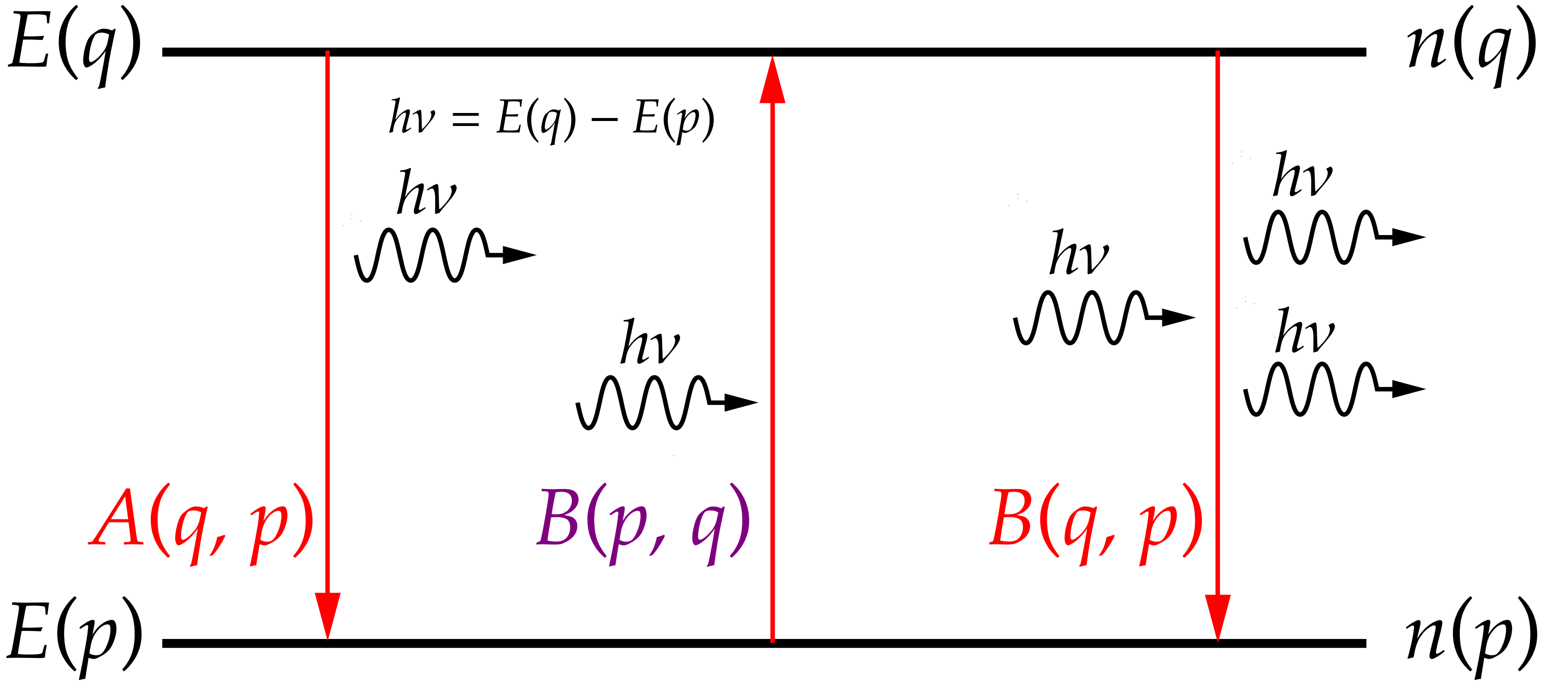
$n(q)$ and $n(p)$ should obey Boltzmann distribution

$$\frac{n(p)}{n(q)} = \frac{g(p)}{g(q)} \exp\left[\frac{E_{pq}}{kT}\right]$$

$X(q, p)$ must be

$$X(q,p) = X(p,q)\frac{g(p)}{g(q)} \exp\left[\frac{E_{qp}}{kT}\right]$$

Einstein coefficients



$B(p, q)$ can be calculated, then how $A(q, p)$ and $B(q, p)$ are derived?

$$A(q,p)n(q) + B(q,p)n(q)I_\nu = B(p,q)n(p)I_\nu$$

under thermodynamic equilibrium

population ratio is subject to Boltzmann distribution

$$\frac{n(q)}{n(p)} = \frac{g(q)}{g(p)} \exp\left[-\frac{h\nu}{kT}\right]$$

the balance equation is rewritten as

$$I_\nu = \frac{\dfrac{A(q,p)}{B(q,p)}}{\dfrac{B(p,q)}{B(q,p)} \dfrac{g(p)}{g(q)} \exp\left(\dfrac{h\nu}{kT}\right) - 1}$$

this should be equivalent to Planck's black-body equation

$$I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(h\nu/kT\right) - 1}$$

finally, interrelationships among Einstein coefficients are derived as

$$B(q,p) = \frac{g(p)}{g(q)} B(p,q)$$

$$A(q,p) = \frac{2h\nu^3}{c^2} B(q,p) = \frac{2h\nu^3}{c^2} \frac{g(p)}{g(q)} B(p,q)$$

level notations

(angular momentum theory)

electron states for hydrogen

orbital angular momentum quantum number

principal quantum number

l n	0 s (2)	1 p (6)	2 d (10)	3 f (14)	4 g (18)	5 h (22)
1 (2)	1s					
2 (8)	2s	2p				
3 (18)	3s	3p	3d			
4 (32)	4s	4p	4d	4f		
5 (50)	5s	5p	5d	5f	5g	
6 (72)	6s	6p	6d	6f	6g	6h

$m_l = -l, \ldots, l$

$m_s = \pm 1/2$

spin quantum number $s = 1/2$

( ) indicates statistical weight

PERIODIC TABLE

Atomic Properties of the Elements

National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce

Physics Laboratory
physics.nist.gov

Standard Reference Data Group
www.nist.gov/srd

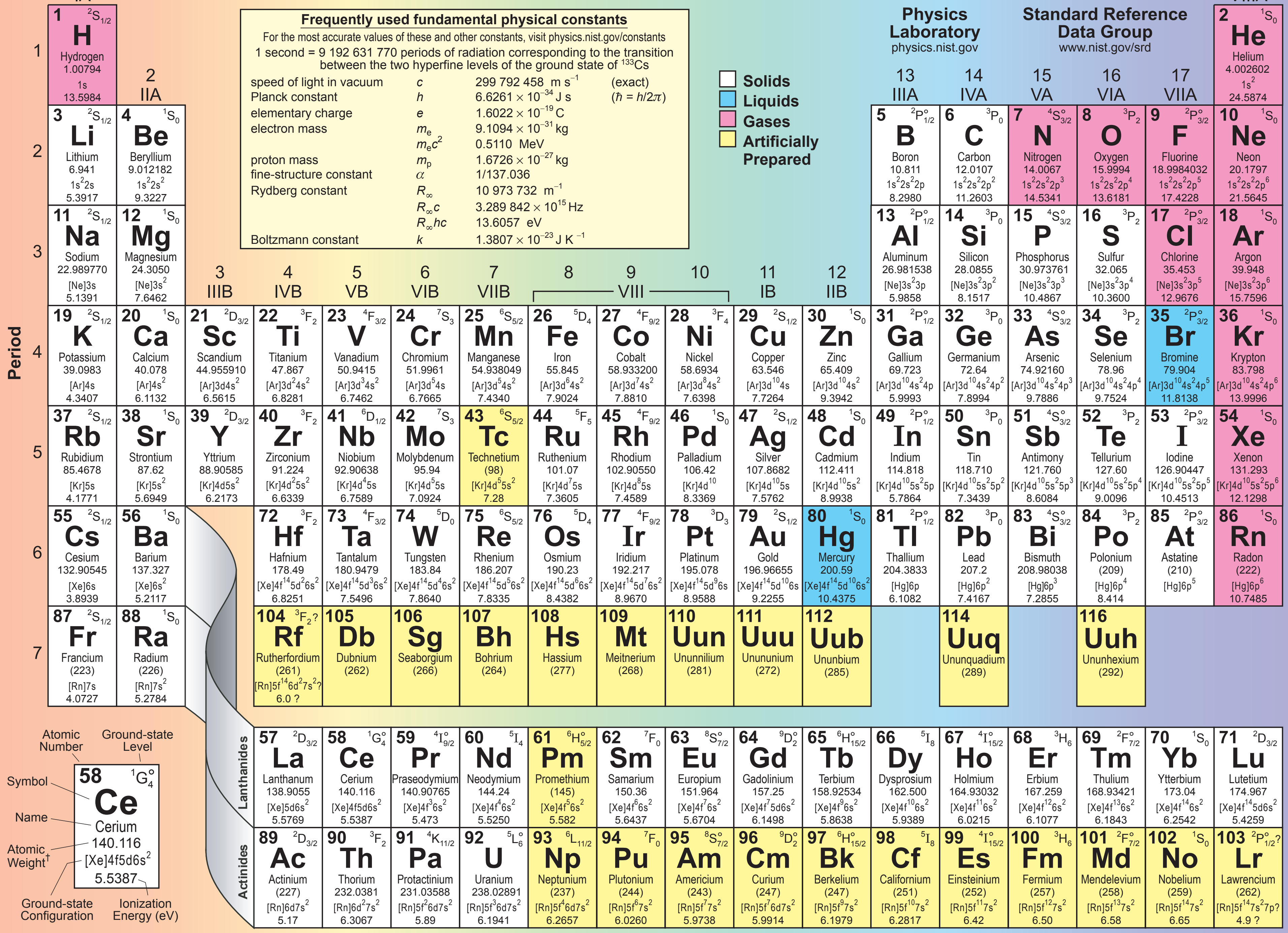
Frequently used fundamental physical constants

For the most accurate values of these and other constants, visit physics.nist.gov/constants

1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum	c	299 792 458 m s^{-1}	(exact)
Planck constant	h	6.6261×10^{-34} J s	($\hbar = h/2\pi$)
elementary charge	e	1.6022×10^{-19} C	
electron mass	m_e	9.1094×10^{-31} kg	
	$m_e c^2$	0.5110 MeV	
proton mass	m_p	1.6726×10^{-27} kg	
fine-structure constant	α	1/137.036	
Rydberg constant	R_∞	10 973 732 m^{-1}	
	$R_\infty c$	$3.289\ 842 \times 10^{15}$ Hz	
	$R_\infty hc$	13.6057 eV	
Boltzmann constant	k	1.3807×10^{-23} J K^{-1}	

☐ Solids
🟦 Liquids
🟥 Gases
🟨 Artificially Prepared



Period	Group 1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8	9 VIII	10	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
1	1 $^2S_{1/2}$ **H** Hydrogen 1.00794 $1s$ 13.5984																	2 $^1S_0$ **He** Helium 4.002602 $1s^2$ 24.5874
2	3 $^2S_{1/2}$ **Li** Lithium 6.941 $1s^22s$ 5.3917	4 $^1S_0$ **Be** Beryllium 9.012182 $1s^22s^2$ 9.3227											5 $^2P^\circ_{1/2}$ **B** Boron 10.811 $1s^22s^22p$ 8.2980	6 $^3P_0$ **C** Carbon 12.0107 $1s^22s^22p^2$ 11.2603	7 $^4S^\circ_{3/2}$ **N** Nitrogen 14.0067 $1s^22s^22p^3$ 14.5341	8 $^3P_2$ **O** Oxygen 15.9994 $1s^22s^22p^4$ 13.6181	9 $^2P^\circ_{3/2}$ **F** Fluorine 18.9984032 $1s^22s^22p^5$ 17.4228	10 $^1S_0$ **Ne** Neon 20.1797 $1s^22s^22p^6$ 21.5645
3	11 $^2S_{1/2}$ **Na** Sodium 22.989770 [Ne]$3s$ 5.1391	12 $^1S_0$ **Mg** Magnesium 24.3050 [Ne]$3s^2$ 7.6462											13 $^2P^\circ_{1/2}$ **Al** Aluminum 26.981538 [Ne]$3s^23p$ 5.9858	14 $^3P_0$ **Si** Silicon 28.0855 [Ne]$3s^23p^2$ 8.1517	15 $^4S^\circ_{3/2}$ **P** Phosphorus 30.973761 [Ne]$3s^23p^3$ 10.4867	16 $^3P_2$ **S** Sulfur 32.065 [Ne]$3s^23p^4$ 10.3600	17 $^2P^\circ_{3/2}$ **Cl** Chlorine 35.453 [Ne]$3s^23p^5$ 12.9676	18 $^1S_0$ **Ar** Argon 39.948 [Ne]$3s^23p^6$ 15.7596
4	19 $^2S_{1/2}$ **K** Potassium 39.0983 [Ar]$4s$ 4.3407	20 $^1S_0$ **Ca** Calcium 40.078 [Ar]$4s^2$ 6.1132	21 $^2D_{3/2}$ **Sc** Scandium 44.955910 [Ar]$3d4s^2$ 6.5615	22 $^3F_2$ **Ti** Titanium 47.867 [Ar]$3d^24s^2$ 6.8281	23 $^4F_{3/2}$ **V** Vanadium 50.9415 [Ar]$3d^34s^2$ 6.7462	24 $^7S_3$ **Cr** Chromium 51.9961 [Ar]$3d^54s$ 6.7665	25 $^6S_{5/2}$ **Mn** Manganese 54.938049 [Ar]$3d^54s^2$ 7.4340	26 $^5D_4$ **Fe** Iron 55.845 [Ar]$3d^64s^2$ 7.9024	27 $^4F_{9/2}$ **Co** Cobalt 58.933200 [Ar]$3d^74s^2$ 7.8810	28 $^3F_4$ **Ni** Nickel 58.6934 [Ar]$3d^84s^2$ 7.6398	29 $^2S_{1/2}$ **Cu** Copper 63.546 [Ar]$3d^{10}4s$ 7.7264	30 $^1S_0$ **Zn** Zinc 65.409 [Ar]$3d^{10}4s^2$ 9.3942	31 $^2P^\circ_{1/2}$ **Ga** Gallium 69.723 [Ar]$3d^{10}4s^24p$ 5.9993	32 $^3P_0$ **Ge** Germanium 72.64 [Ar]$3d^{10}4s^24p^2$ 7.8994	33 $^4S^\circ_{3/2}$ **As** Arsenic 74.92160 [Ar]$3d^{10}4s^24p^3$ 9.7886	34 $^3P_2$ **Se** Selenium 78.96 [Ar]$3d^{10}4s^24p^4$ 9.7524	35 $^2P^\circ_{3/2}$ **Br** Bromine 79.904 [Ar]$3d^{10}4s^24p^5$ 11.8138	36 $^1S_0$ **Kr** Krypton 83.798 [Ar]$3d^{10}4s^24p^6$ 13.9996
5	37 $^2S_{1/2}$ **Rb** Rubidium 85.4678 [Kr]$5s$ 4.1771	38 $^1S_0$ **Sr** Strontium 87.62 [Kr]$5s^2$ 5.6949	39 $^2D_{3/2}$ **Y** Yttrium 88.90585 [Kr]$4d5s^2$ 6.2173	40 $^3F_2$ **Zr** Zirconium 91.224 [Kr]$4d^25s^2$ 6.6339	41 $^6D_{1/2}$ **Nb** Niobium 92.90638 [Kr]$4d^45s$ 6.7589	42 $^7S_3$ **Mo** Molybdenum 95.94 [Kr]$4d^55s$ 7.0924	43 $^6S_{5/2}$ **Tc** Technetium (98) [Kr]$4d^55s^2$ 7.28	44 $^5F_5$ **Ru** Ruthenium 101.07 [Kr]$4d^75s$ 7.3605	45 $^4F_{9/2}$ **Rh** Rhodium 102.90550 [Kr]$4d^85s$ 7.4589	46 $^1S_0$ **Pd** Palladium 106.42 [Kr]$4d^{10}$ 8.3369	47 $^2S_{1/2}$ **Ag** Silver 107.8682 [Kr]$4d^{10}5s$ 7.5762	48 $^1S_0$ **Cd** Cadmium 112.411 [Kr]$4d^{10}5s^2$ 8.9938	49 $^2P^\circ_{1/2}$ **In** Indium 114.818 [Kr]$4d^{10}5s^25p$ 5.7864	50 $^3P_0$ **Sn** Tin 118.710 [Kr]$4d^{10}5s^25p^2$ 7.3439	51 $^4S^\circ_{3/2}$ **Sb** Antimony 121.760 [Kr]$4d^{10}5s^25p^3$ 8.6084	52 $^3P_2$ **Te** Tellurium 127.60 [Kr]$4d^{10}5s^25p^4$ 9.0096	53 $^2P^\circ_{3/2}$ **I** Iodine 126.90447 [Kr]$4d^{10}5s^25p^5$ 10.4513	54 $^1S_0$ **Xe** Xenon 131.293 [Kr]$4d^{10}5s^25p^6$ 12.1298
6	55 $^2S_{1/2}$ **Cs** Cesium 132.90545 [Xe]$6s$ 3.8939	56 $^1S_0$ **Ba** Barium 137.327 [Xe]$6s^2$ 5.2117		72 $^3F_2$ **Hf** Hafnium 178.49 [Xe]$4f^{14}5d^26s^2$ 6.8251	73 $^4F_{3/2}$ **Ta** Tantalum 180.9479 [Xe]$4f^{14}5d^36s^2$ 7.5496	74 $^5D_0$ **W** Tungsten 183.84 [Xe]$4f^{14}5d^46s^2$ 7.8640	75 $^6S_{5/2}$ **Re** Rhenium 186.207 [Xe]$4f^{14}5d^56s^2$ 7.8335	76 $^5D_4$ **Os** Osmium 190.23 [Xe]$4f^{14}5d^66s^2$ 8.4382	77 $^4F_{9/2}$ **Ir** Iridium 192.217 [Xe]$4f^{14}5d^76s^2$ 8.9670	78 $^3D_3$ **Pt** Platinum 195.078 [Xe]$4f^{14}5d^96s$ 8.9588	79 $^2S_{1/2}$ **Au** Gold 196.96655 [Xe]$4f^{14}5d^{10}6s$ 9.2255	80 $^1S_0$ **Hg** Mercury 200.59 [Xe]$4f^{14}5d^{10}6s^2$ 10.4375	81 $^2P^\circ_{1/2}$ **Tl** Thallium 204.3833 [Hg]$6p$ 6.1082	82 $^3P_0$ **Pb** Lead 207.2 [Hg]$6p^2$ 7.4167	83 $^4S^\circ_{3/2}$ **Bi** Bismuth 208.98038 [Hg]$6p^3$ 7.2855	84 $^3P_2$ **Po** Polonium (209) [Hg]$6p^4$ 8.414	85 $^2P^\circ_{3/2}$ **At** Astatine (210) [Hg]$6p^5$	86 $^1S_0$ **Rn** Radon (222) [Hg]$6p^6$ 10.7485
7	87 $^2S_{1/2}$ **Fr** Francium (223) [Rn]$7s$ 4.0727	88 $^1S_0$ **Ra** Radium (226) [Rn]$7s^2$ 5.2784		104 $^3F_2$? **Rf** Rutherfordium (261) [Rn]$5f^{14}6d^27s^2$? 6.0 ?	105 **Db** Dubnium (262)	106 **Sg** Seaborgium (266)	107 **Bh** Bohrium (264)	108 **Hs** Hassium (277)	109 **Mt** Meitnerium (268)	110 **Uun** Ununnilium (281)	111 **Uuu** Unununium (272)	112 **Uub** Ununbium (285)		114 **Uuq** Ununquadium (289)		116 **Uuh** Ununhexium (292)		

Lanthanides	57 $^2D_{3/2}$ **La** Lanthanum 138.9055 [Xe]$5d6s^2$ 5.5769	58 $^1G^\circ_4$ **Ce** Cerium 140.116 [Xe]$4f5d6s^2$ 5.5387	59 $^4I^\circ_{9/2}$ **Pr** Praseodymium 140.90765 [Xe]$4f^36s^2$ 5.473	60 $^5I_4$ **Nd** Neodymium 144.24 [Xe]$4f^46s^2$ 5.5250	61 $^6H^\circ_{5/2}$ **Pm** Promethium (145) [Xe]$4f^56s^2$ 5.582	62 $^7F_0$ **Sm** Samarium 150.36 [Xe]$4f^66s^2$ 5.6437	63 $^8S^\circ_{7/2}$ **Eu** Europium 151.964 [Xe]$4f^76s^2$ 5.6704	64 $^9D^\circ_2$ **Gd** Gadolinium 157.25 [Xe]$4f^75d6s^2$ 6.1498	65 $^6H^\circ_{15/2}$ **Tb** Terbium 158.92534 [Xe]$4f^96s^2$ 5.8638	66 $^5I_8$ **Dy** Dysprosium 162.500 [Xe]$4f^{10}6s^2$ 5.9389	67 $^4I^\circ_{15/2}$ **Ho** Holmium 164.93032 [Xe]$4f^{11}6s^2$ 6.0215	68 $^3H_6$ **Er** Erbium 167.259 [Xe]$4f^{12}6s^2$ 6.1077	69 $^2F^\circ_{7/2}$ **Tm** Thulium 168.93421 [Xe]$4f^{13}6s^2$ 6.1843	70 $^1S_0$ **Yb** Ytterbium 173.04 [Xe]$4f^{14}6s^2$ 6.2542	71 $^2D_{3/2}$ **Lu** Lutetium 174.967 [Xe]$4f^{14}5d6s^2$ 5.4259
Actinides	89 $^2D_{3/2}$ **Ac** Actinium (227) [Rn]$6d7s^2$ 5.17	90 $^3F_2$ **Th** Thorium 232.0381 [Rn]$6d^27s^2$ 6.3067	91 $^4K_{11/2}$ **Pa** Protactinium 231.03588 [Rn]$5f^26d7s^2$ 5.89	92 $^5L^\circ_6$ **U** Uranium 238.02891 [Rn]$5f^36d7s^2$ 6.1941	93 $^6L_{11/2}$ **Np** Neptunium (237) [Rn]$5f^46d7s^2$ 6.2657	94 $^7F_0$ **Pu** Plutonium (244) [Rn]$5f^67s^2$ 6.0260	95 $^8S^\circ_{7/2}$ **Am** Americium (243) [Rn]$5f^77s^2$ 5.9738	96 $^9D^\circ_2$ **Cm** Curium (247) [Rn]$5f^76d7s^2$ 5.9914	97 $^6H^\circ_{15/2}$ **Bk** Berkelium (247) [Rn]$5f^97s^2$ 6.1979	98 $^5I_8$ **Cf** Californium (251) [Rn]$5f^{10}7s^2$ 6.2817	99 $^4I^\circ_{15/2}$ **Es** Einsteinium (252) [Rn]$5f^{11}7s^2$ 6.42	100 $^3H_6$ **Fm** Fermium (257) [Rn]$5f^{12}7s^2$ 6.50	101 $^2F^\circ_{7/2}$ **Md** Mendelevium (258) [Rn]$5f^{13}7s^2$ 6.58	102 $^1S_0$ **No** Nobelium (259) [Rn]$5f^{14}7s^2$ 6.65	103 $^2P^\circ_{1/2}$? **Lr** Lawrencium (262) [Rn]$5f^{14}7s^27p$? 4.9 ?

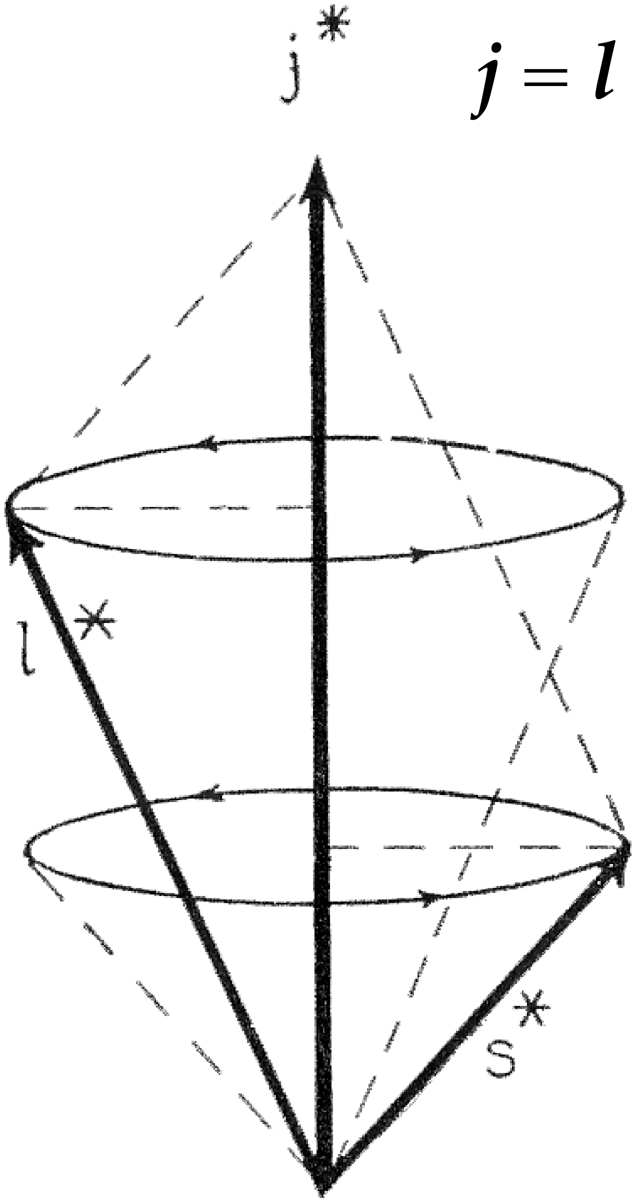
---

†Based upon ^{12}C. () indicates the mass number of the most stable isotope.

For a description of the data, visit physics.nist.gov/data

NIST SP 966 (September 2003)

coupling of angular momentum



$$\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s}\ (|\boldsymbol{s}|=1/2)$$

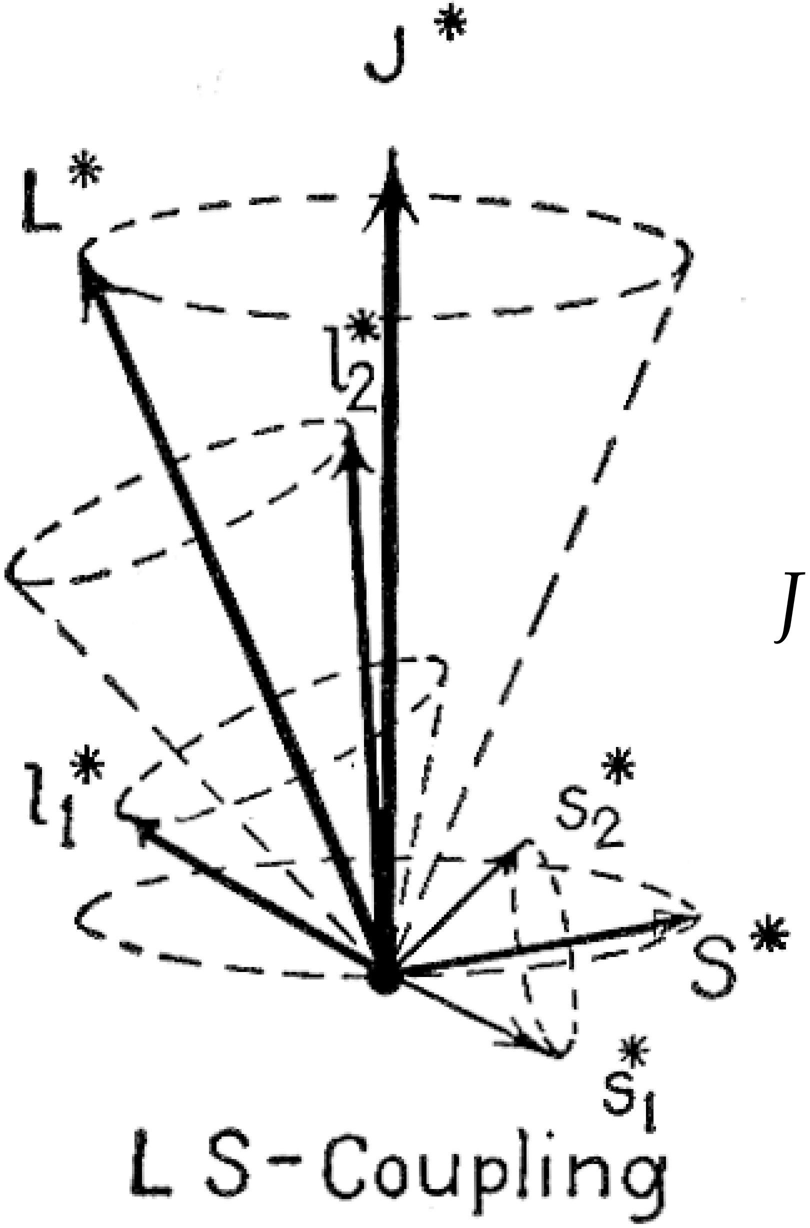
total angular momentum quantum number

$$j = \begin{cases} l \pm \dfrac{1}{2} & \text{for } l \geq 1 \\ \dfrac{1}{2} & \text{for } l = 0 \end{cases}$$

each j state has $2j+1$ magnetic substates m $(= -j, -j+1, \ldots, j)$

(H. E. White, *Introduction to Atomic Spectra*)

L-*S* coupling



(H. E. White, *Introduction to Atomic Spectra*)

$$\boldsymbol{L} = \boldsymbol{l}_1 + \boldsymbol{l}_2$$

$$\boldsymbol{S} = \boldsymbol{s}_1 + \boldsymbol{s}_2$$

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$$

$$J = \begin{cases} L - S, \ldots, L + S & \text{for } L \geq S \\ & (2S + 1 \text{ states}) \\ S - L, \ldots, S + L & \text{for } L < S \\ & (2L + 1 \text{ states}) \end{cases}$$

each J state has $2J+1$ magnetic substates M (= $-J$, $-J+1$, ..., J)

quantum numbers

variables	meanings	values	
n	principal	0, 1, 2, ...	
s	spin	1/2	
S		0, 1/2, 1, 3/2, 2, ...	
l	orbital	0, 1, 2, ...	s, p, d, f, ...
L			S, P, D, F, ...
j	total	0, 1/2, 1, 3/2, 2, ...	
J			
m	magnetic	0, ±1/2, ±1, ±3/2, ±2, ...	
M			

term designation

$$nl^k\, n'l'^{k'} \ldots\ {}^{2S+1}L_J$$

configuration	possible terms	isoelectronic sequence
3d	${}^2D_{3/2,5/2}$	H-like
1s4f	${}^1F_3$ ${}^3F_{2,3,4}$	He-like
$1s^22p$	${}^2P_{1/2,3/2}$	Li-like
$1s^22s^2$	${}^1S_0$	Be-like
$1s^22s2p(^3P^o)3p$	${}^2S_{1/2}$ ${}^2P_{1/2,3/2}$ ${}^2D_{3/2,5/2}$ ${}^4S_{3/2}$ ${}^4P_{1/2,3/2,5/2}$ ${}^4D_{1/2,3/2,5/2,7/2}$	B-like

one electronic configuration could have several terms

selection rules for transition

$$^{2S+1}L_J \text{ - } ^{2S'+1}L'_{J'}$$

$$\Delta S = 0$$

$$\Delta L = 0, \pm 1 \ (0 \rightarrow 0 \text{ excluded})$$

$$\Delta J = 0, \pm 1 \ (0 \rightarrow 0 \text{ excluded})$$

besides the rules for total quantum numbers,
$\Delta l = \pm 1$ is always required

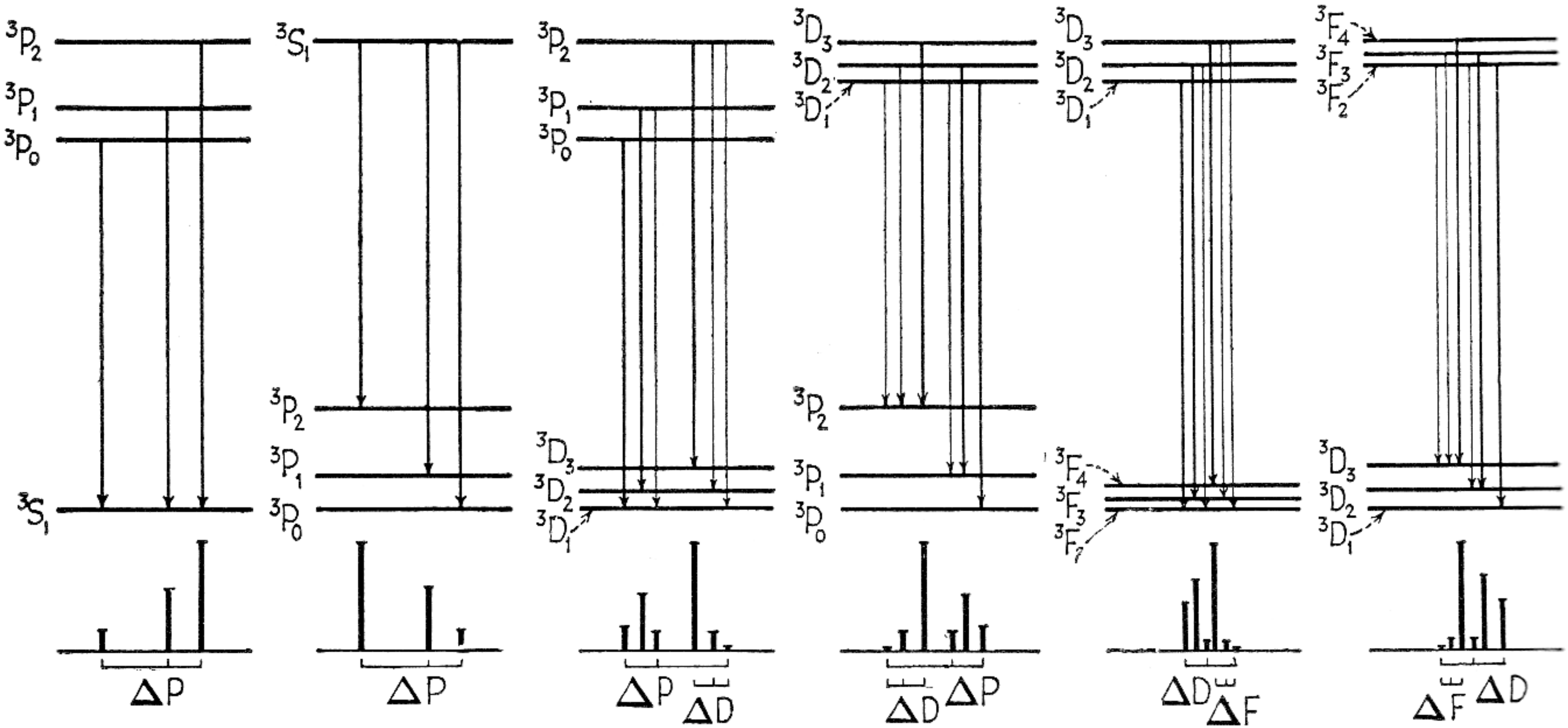
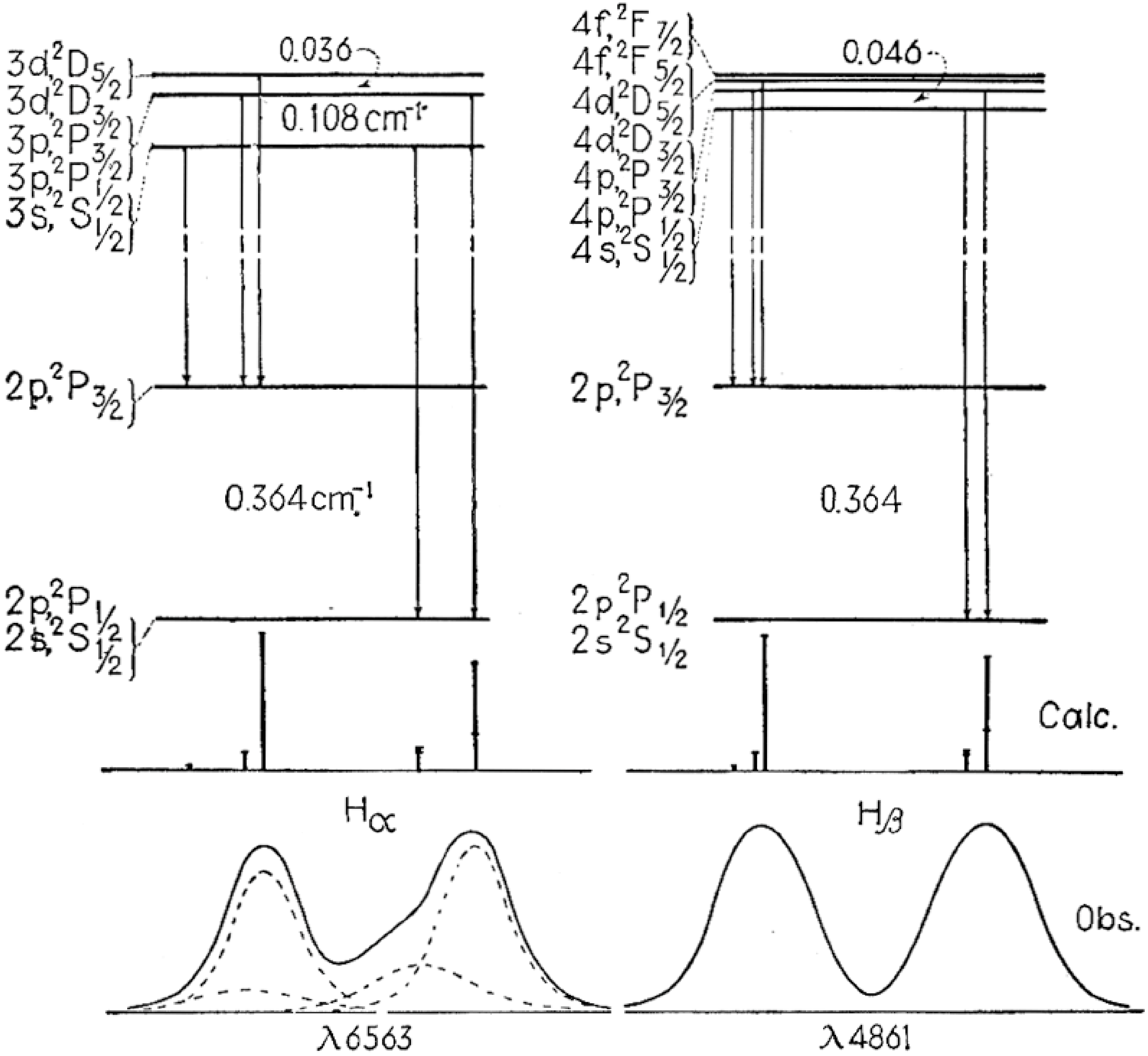


FIG. 11.4.—Triplet-triplet transitions showing selection rules and relative intensities.

(H. E. White, *Introduction to Atomic Spectra*)

$$I \propto \langle J_i \| D \| J_k \rangle = (-1)^{S+1+L_i+J_k} \sqrt{g_i g_k} \left\{ \begin{array}{ccc} L_i & J_i & S \\ J_k & L_k & 1 \end{array} \right\} \langle L_i \| D \| L_k \rangle$$

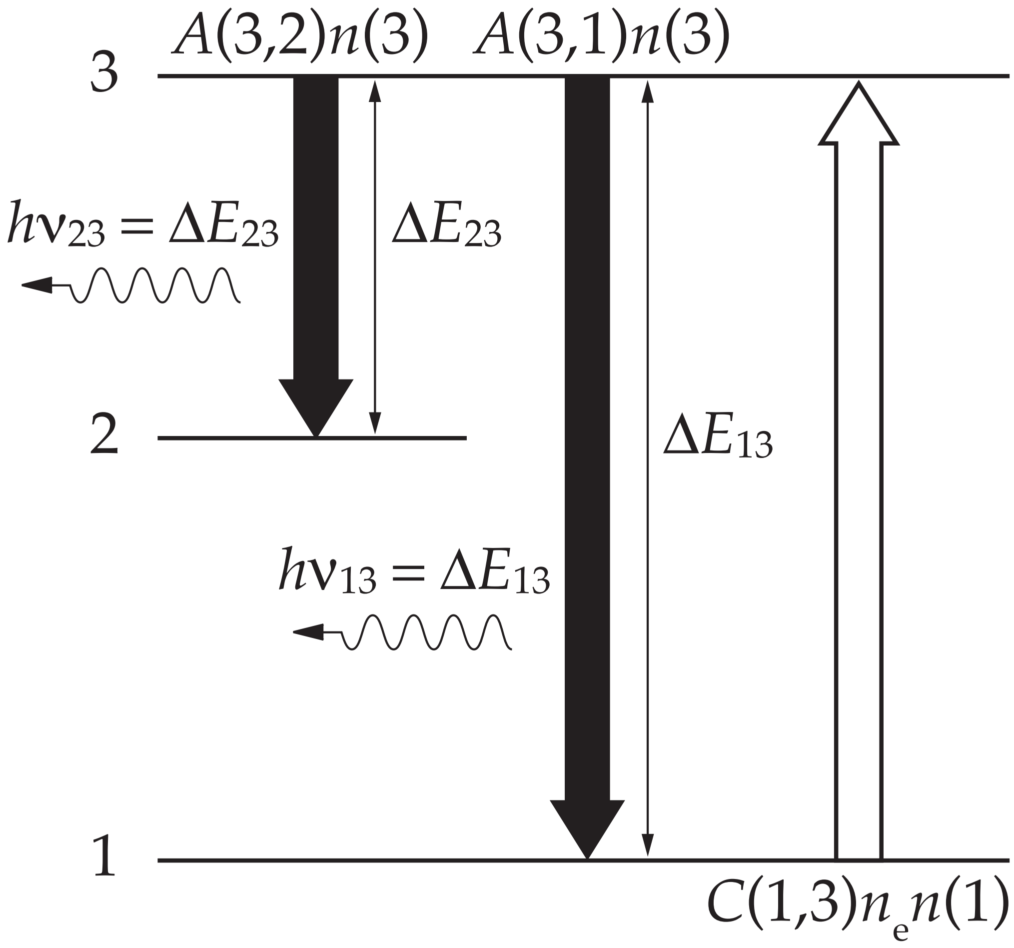


(H. E. White, *Introduction to Atomic Spectra*)

configuration			term(s)
$1s2s2p$	(Li-like)	⇒	$1s2s(^1S)2p\ ^2P_{1/2,3/2}$ $1s2s(^3S)2p\ ^2P_{1/2,3/2}$ $^4P_{1/2,3/2,5/2}$
$1s^22p^2$	(Be-like)	⇒	^1S, ^1D, ^3P

population mechanisms

corona equilibrium



$$C(1,3)n_e n(1) = [A(3,1) + A(3,2)]\, n(3)$$

more generally

$$C(1,p)n_e n(1) = \sum_{q<p} A(p,q) n(p)$$

$$n(p) = \frac{C(1,p)n_e}{\sum_{q<p} A(p,q)} n(1)$$

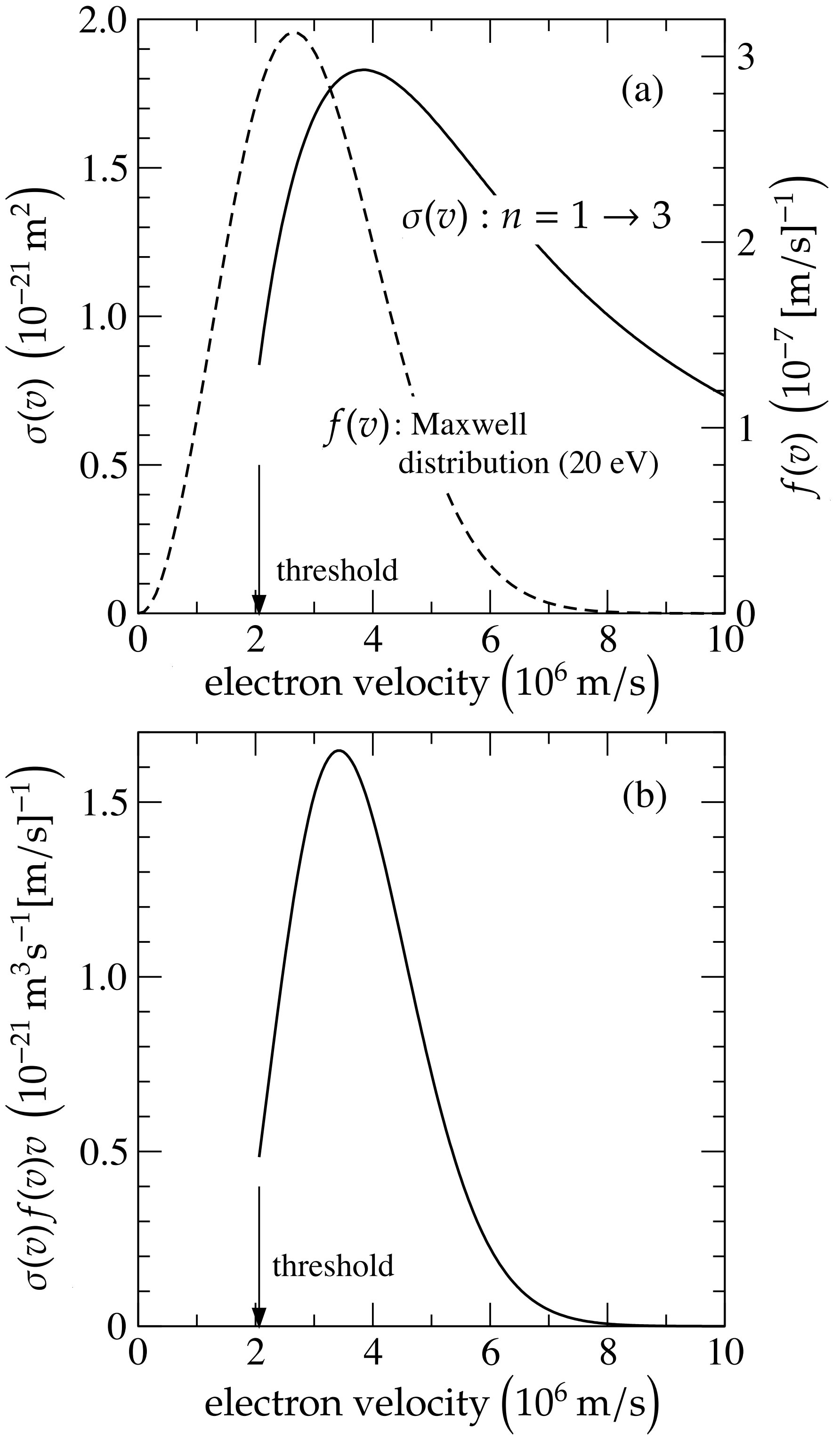
collisional excitation

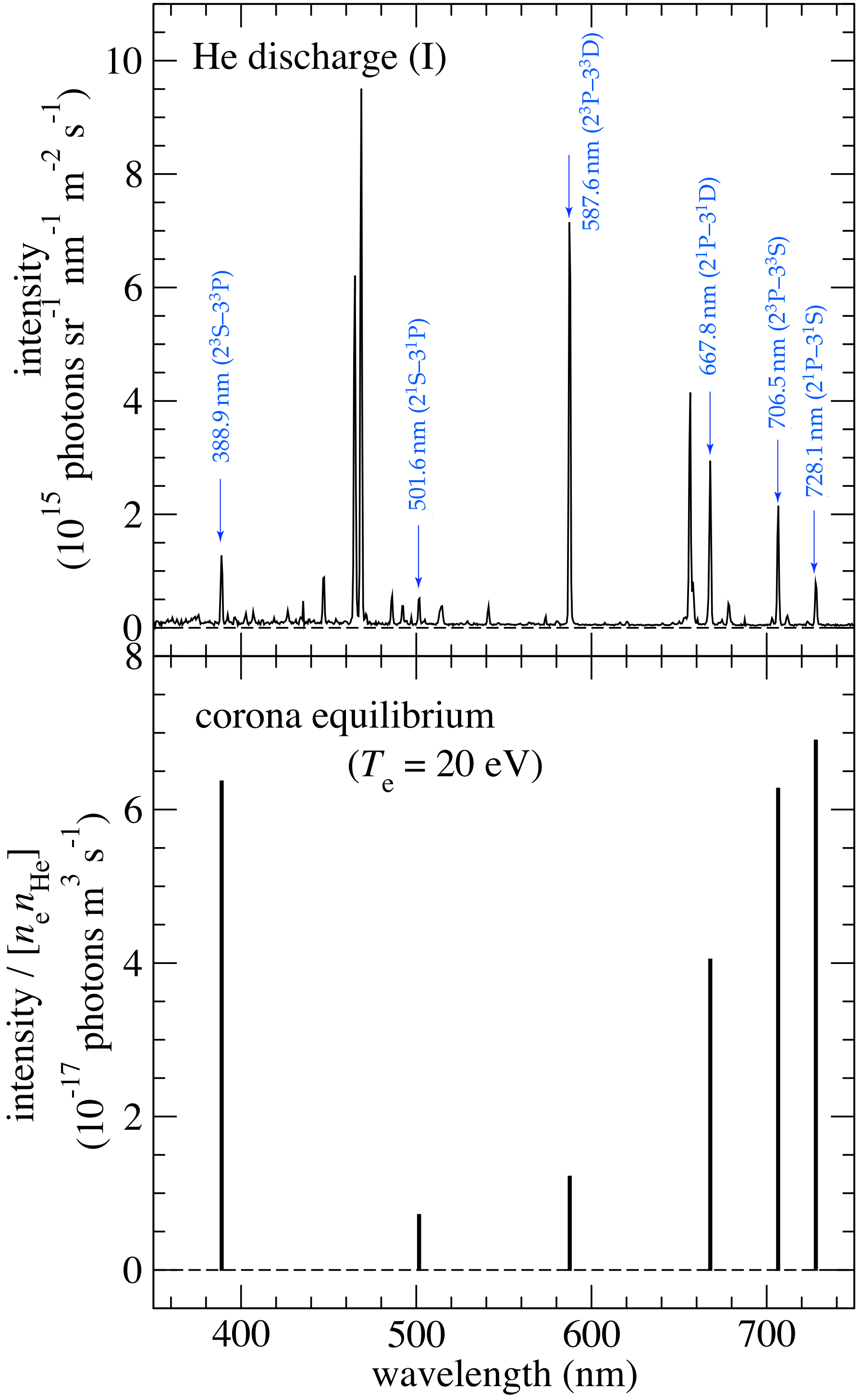
$$C(q,p)n_e n(1) \quad [\mathrm{m}^{-3}\mathrm{s}^{-1}]$$

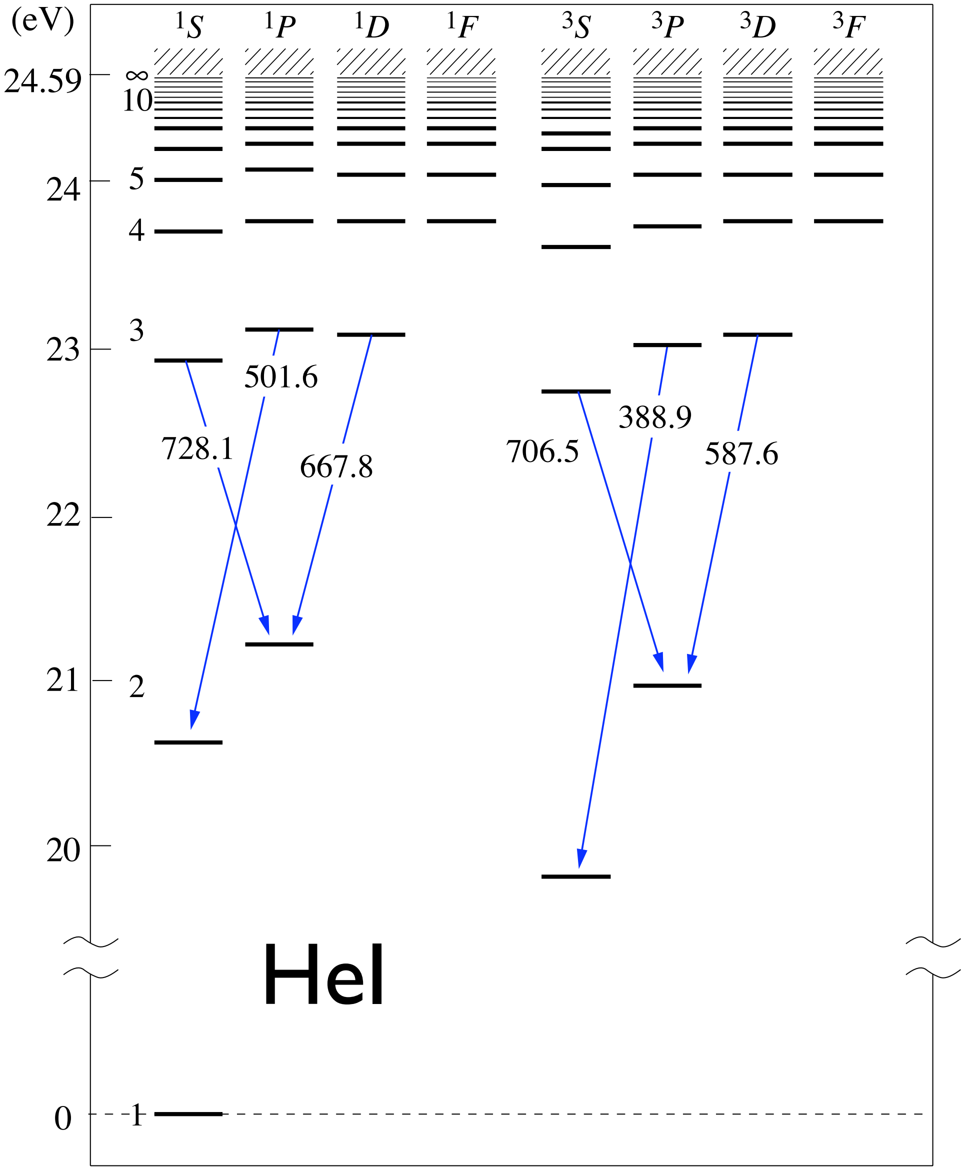
rate coefficients are obtained from cross section

$$C(q,p) = \int_0^\infty \sigma_{qp}(v) f(v) v \mathrm{d}v$$

$C(q,p)$ depends on T_e



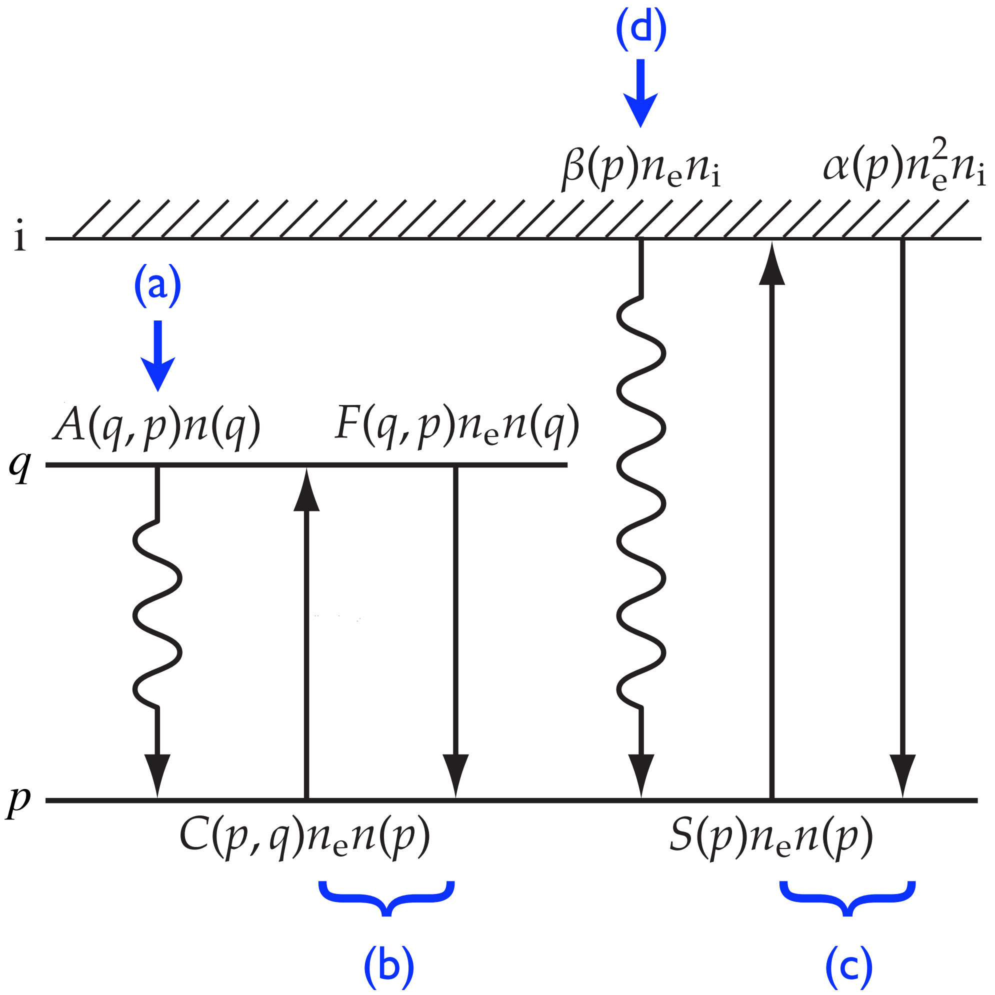


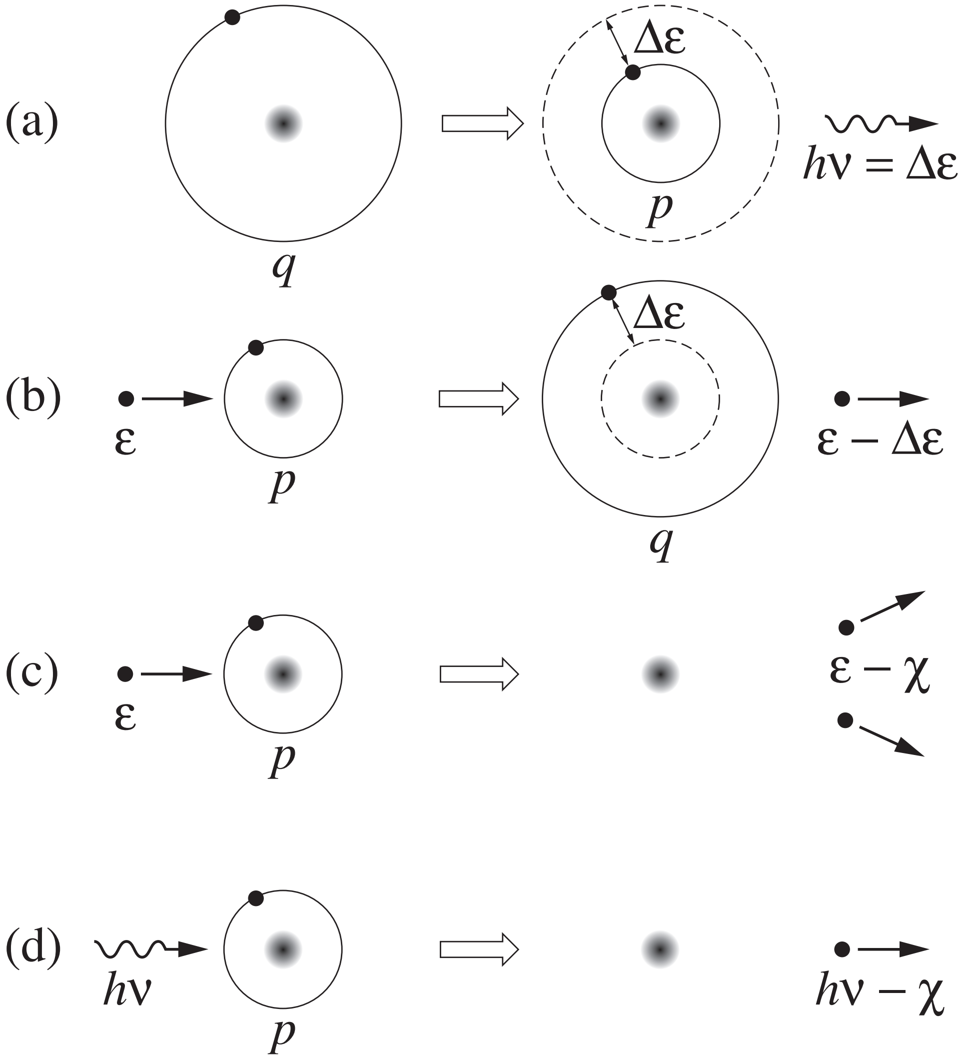


corona model does not explain observed spectrum

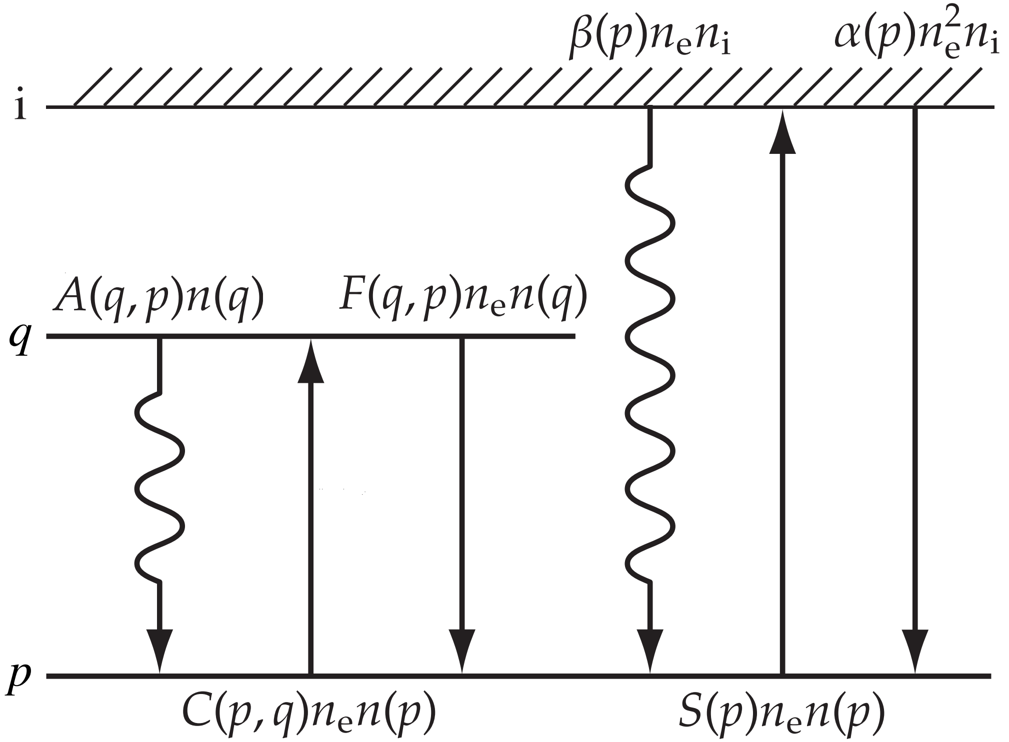
atomic processes

various processes should be taken into consideration





collisional-radiative model



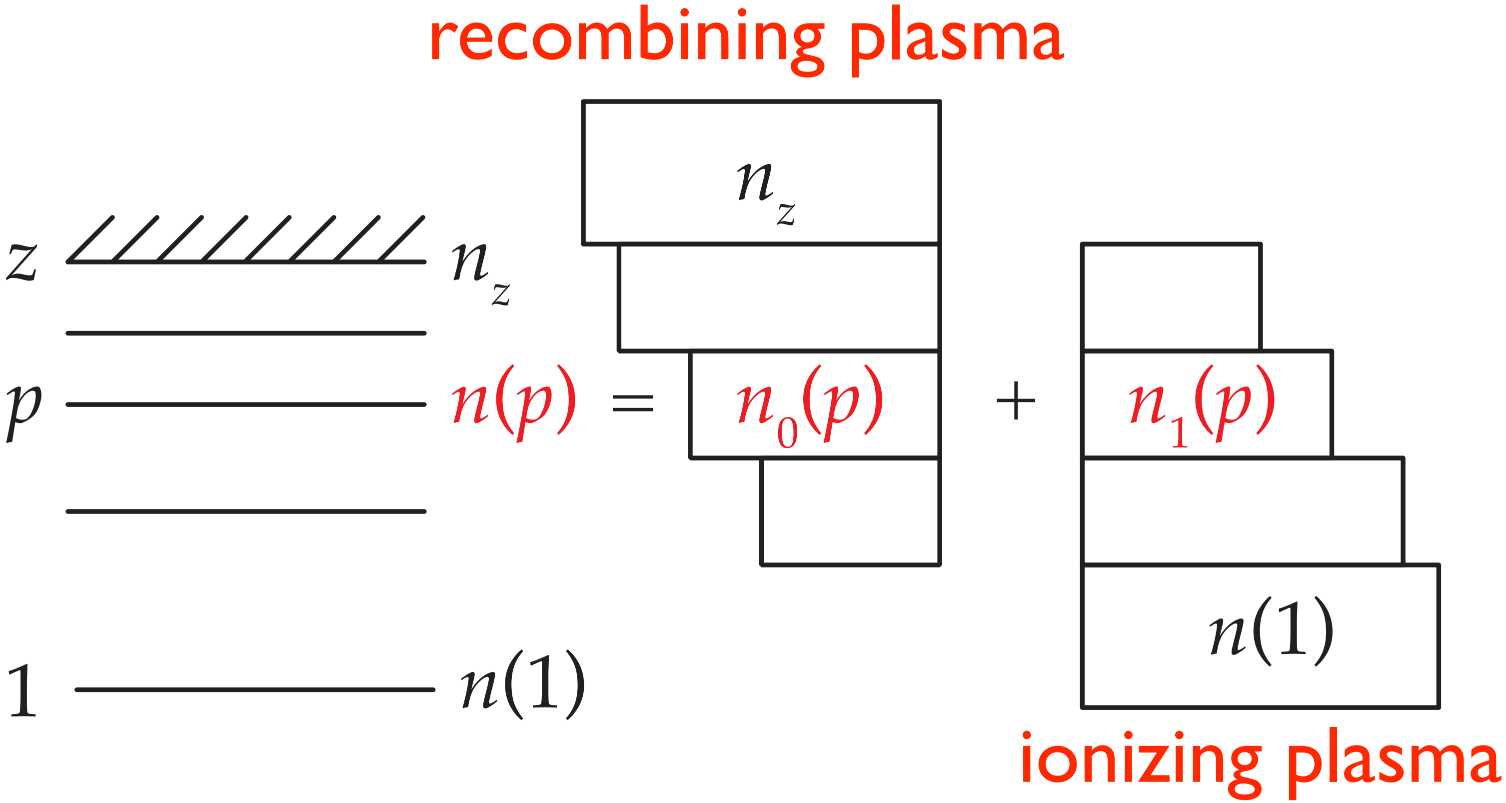
$$\frac{\mathrm{d}}{\mathrm{d}t}n(p) = \Gamma_{\mathrm{in}} - \Gamma_{\mathrm{out}} = 0$$

quasi steady-state approximation

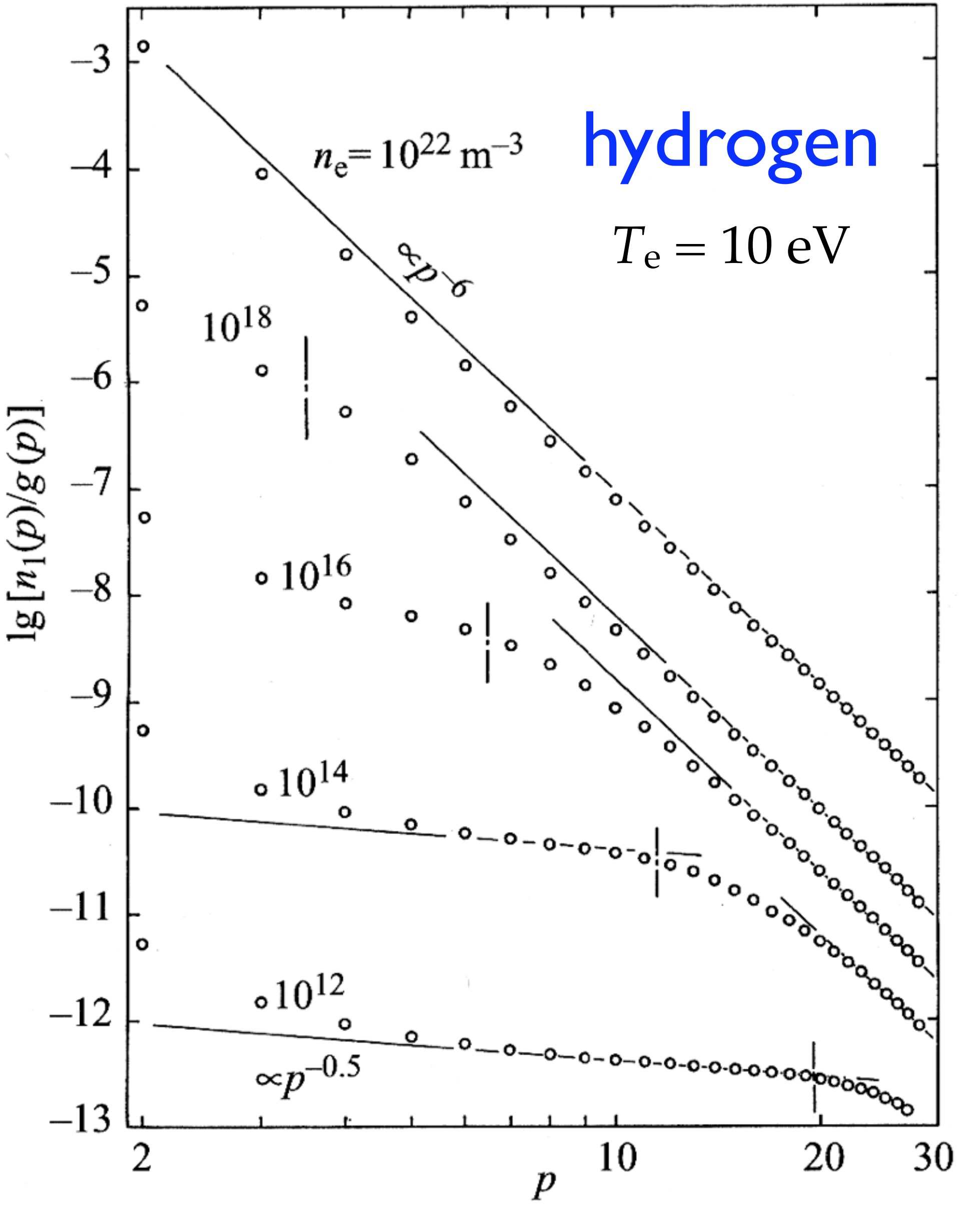
$$\begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} n(2) \\ \cdot \\ n(p) \\ \cdot \end{pmatrix} = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} n(1) + \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} n_{\mathrm{i}}$$

population consists of two independent components

$$\begin{aligned} n(p) &= r_0(p) n_e n_z + r_1(p) n_e n(1) \\ &= n_0(p) + n_1(p) \end{aligned}$$



ionizing plasma



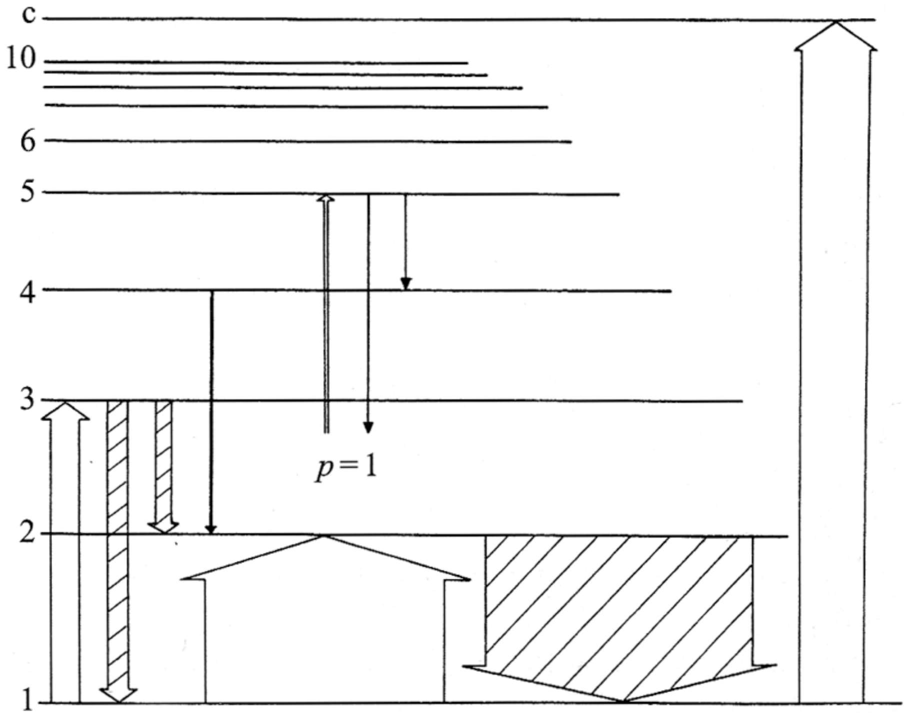
(T. Fujimoto, *Plasma Spectroscopy*)

$$\frac{n(p)}{g(p)} \propto \begin{cases} p^{-0.5} & \text{for low } n_e \\ p^{-6} & \text{for high } n_e \end{cases}$$

T_e dependence on $n(p)$ distribution is small

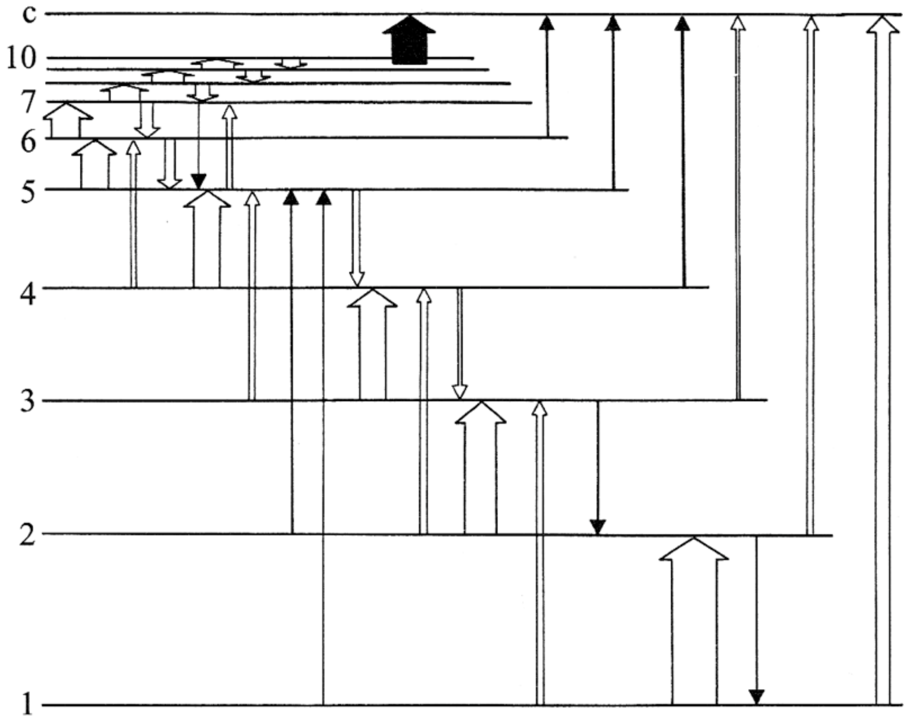
population flows (ionizing plasma)

low n_e case



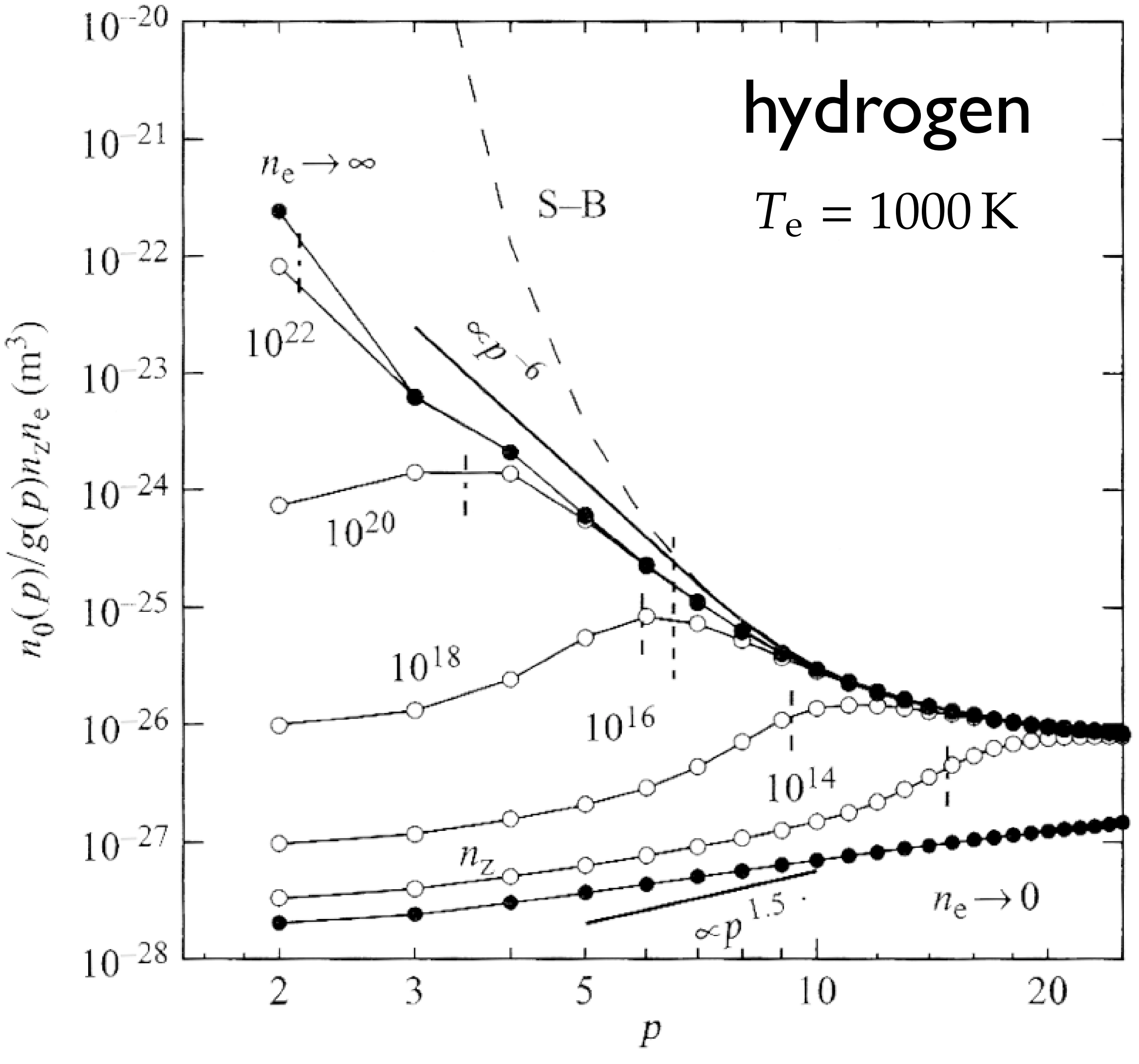
close to corona equilibrium

high n_e case



(T. Fujimoto, *Plasma Spectroscopy*)

recombining plasma



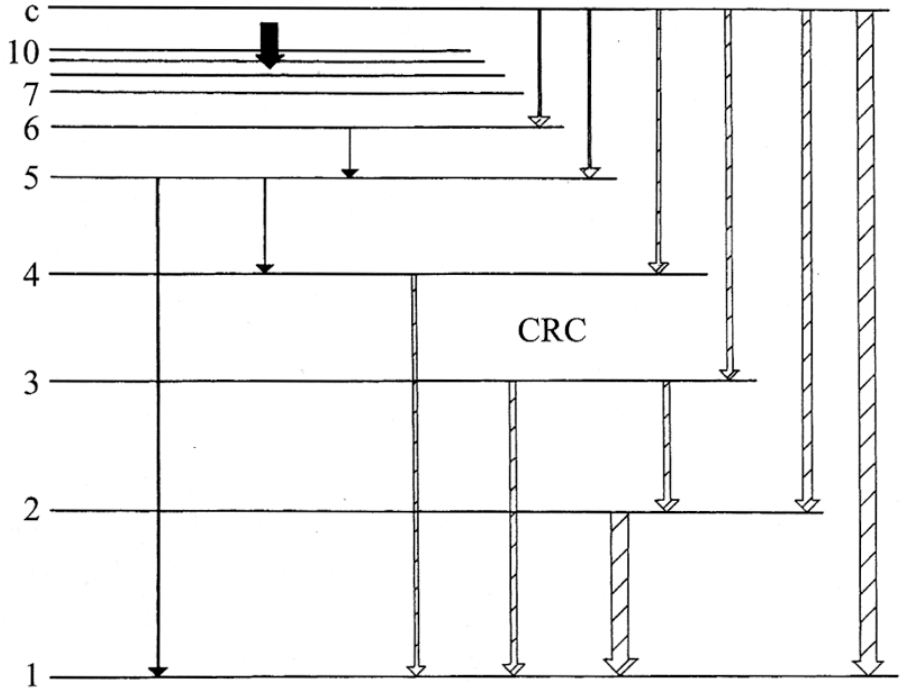
(T. Fujimoto, *Plasma Spectroscopy*)

when n_e is high, $n(p)$ is expressed with Saha-Boltzmann equation

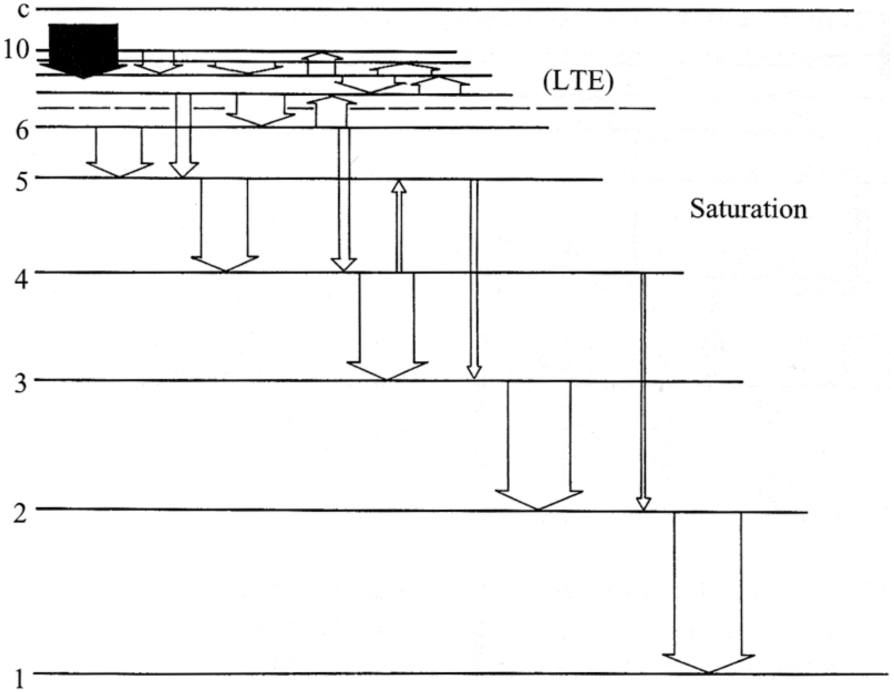
$$n(p) = \frac{g(p)}{2g_z(1)}\left(\frac{h^2}{2\pi m k T_e}\right)^{3/2} \exp\left[\frac{\chi(p)}{kT_e}\right] n_e n_z(1)$$

population flows
(recombining plasma)

low n_{e} case

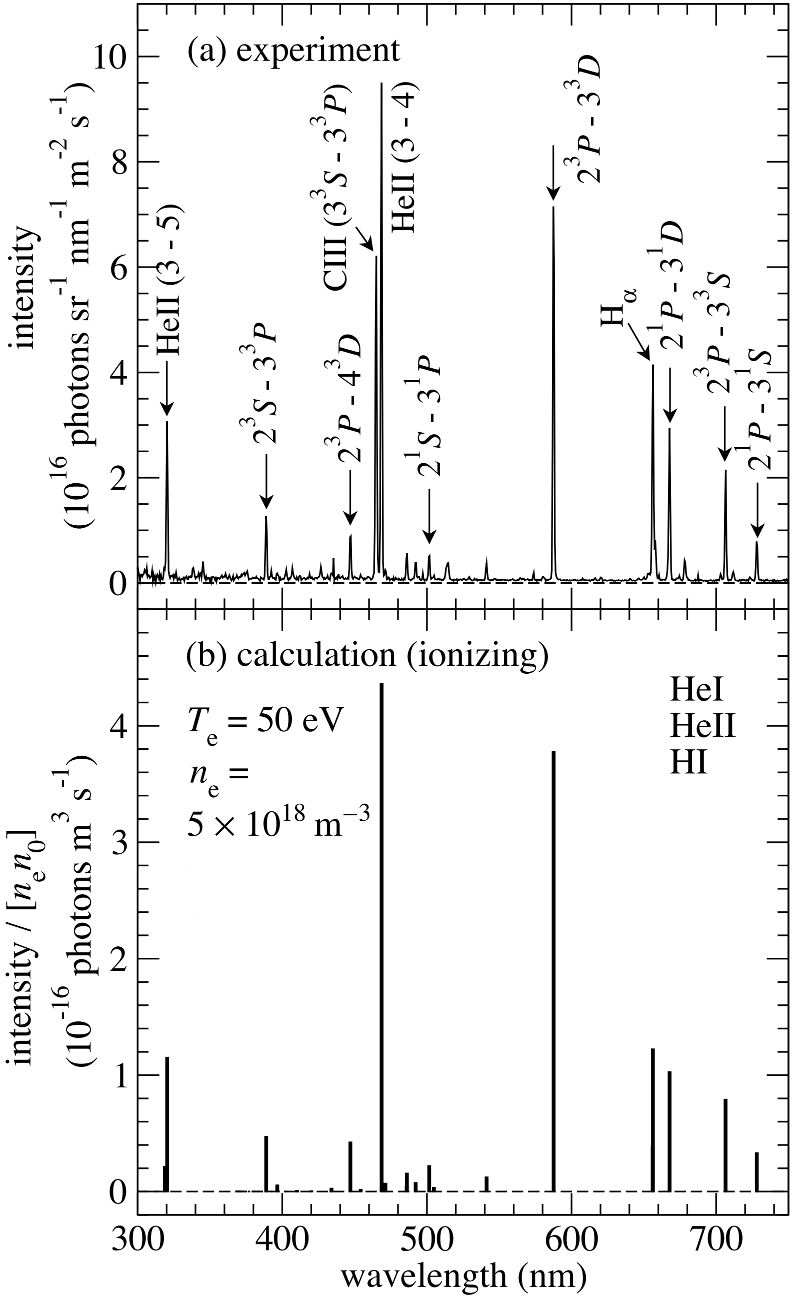


high n_{e} case

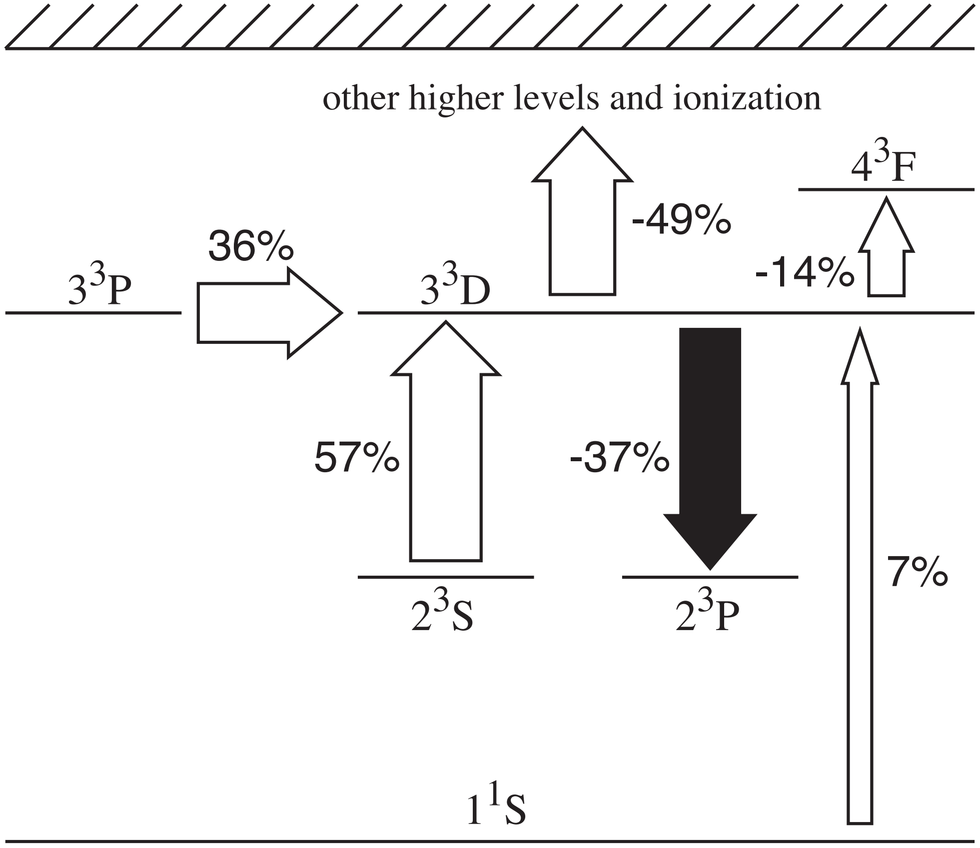


(T. Fujimoto, *Plasma Spectroscopy*)

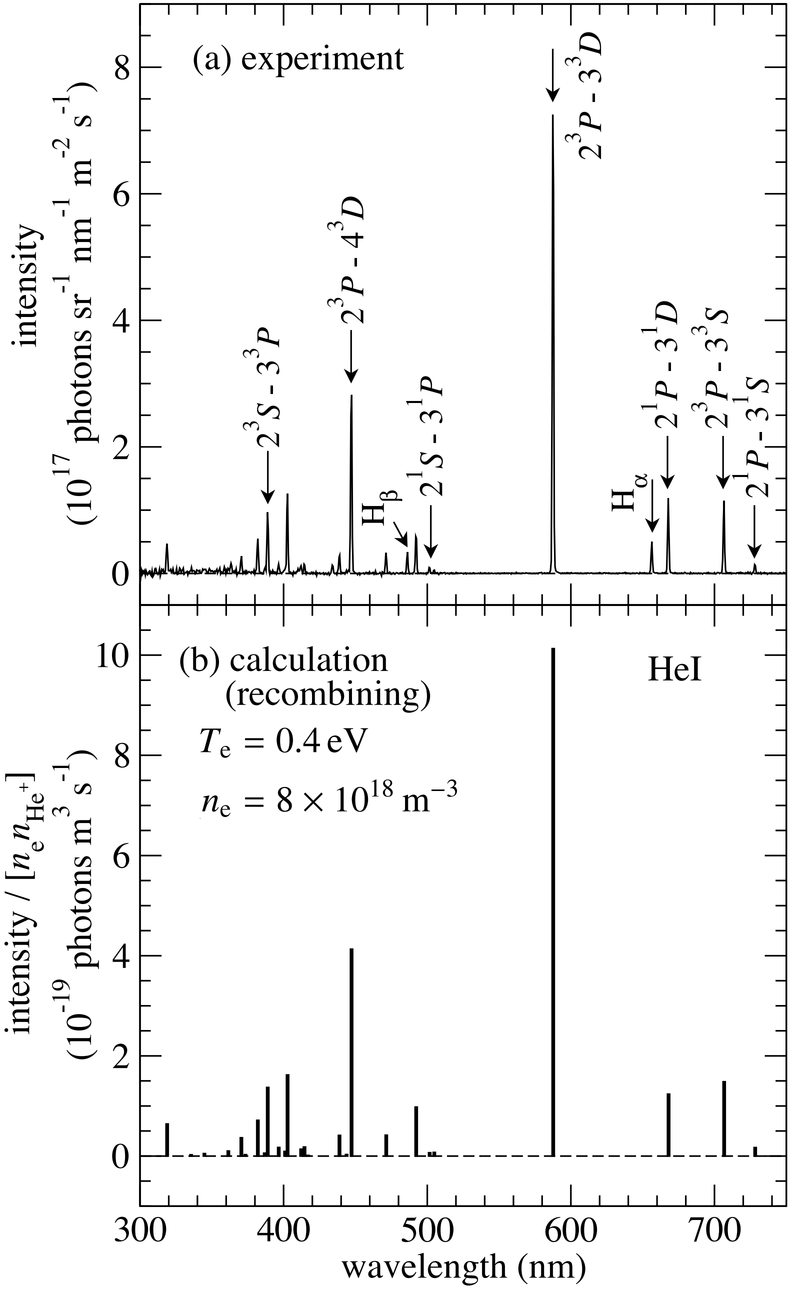
ionizing plasma (I)



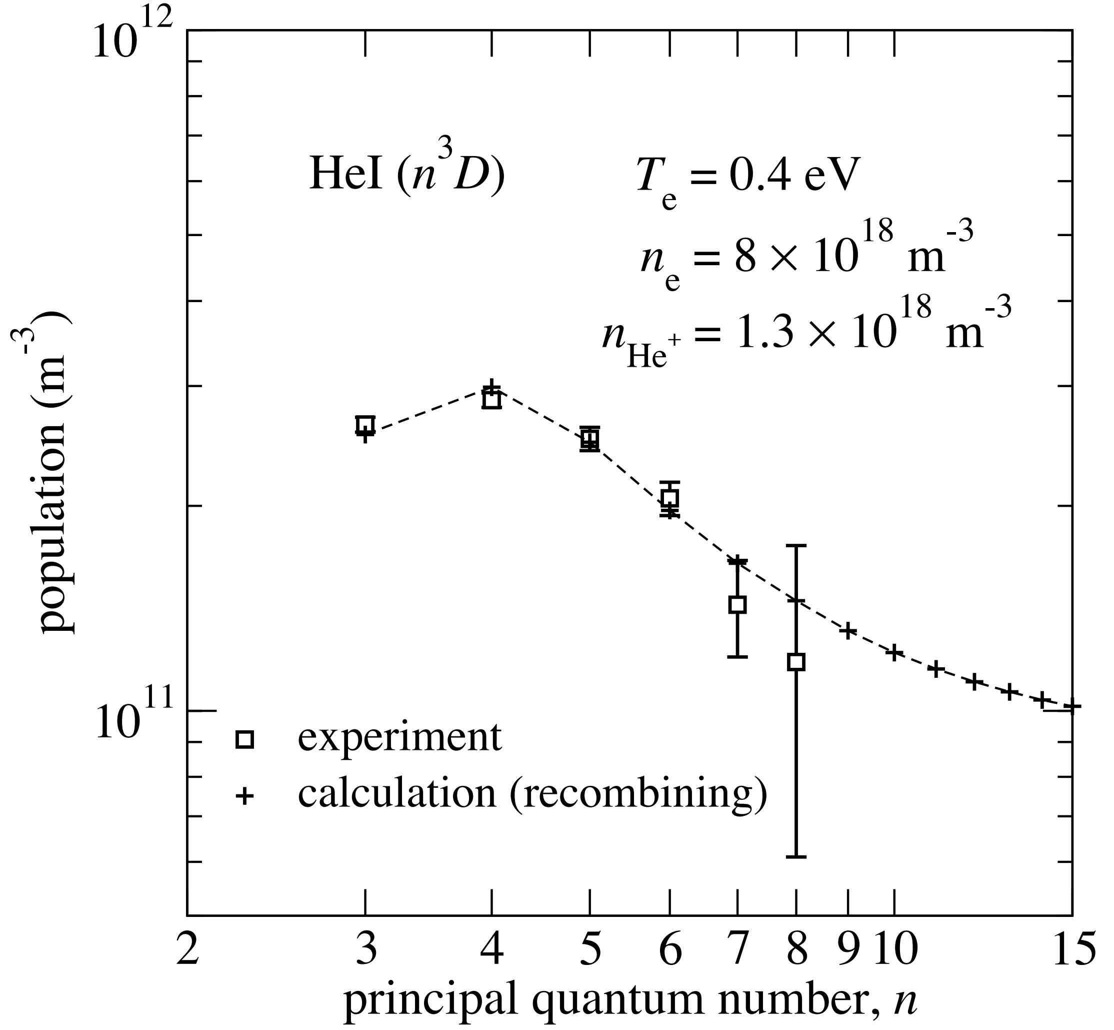
$$n(p) = r_1(p) n_e n(1)$$



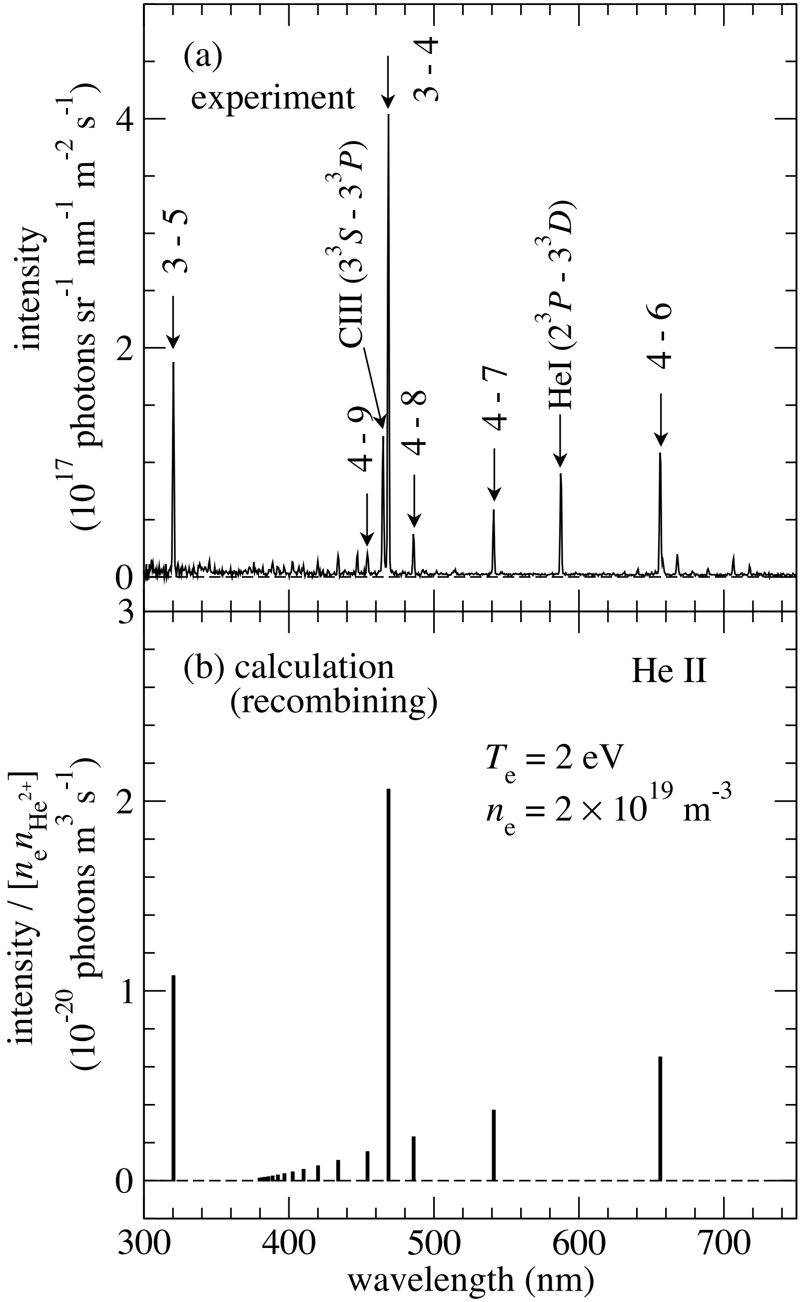
recombining plasma (III)



$$n(p) = r_0(p) n_e n_z$$



recombining plasma (II)



- recombining plasma of HeII appears earlier than HeI
- derived T_e is higher

effect of external fields

splitting, shift, broadening, etc

- Zeeman effect — magnetic field
- Stark effect — electric field
- Stark broadening — electric micro fields

Zeeman effect

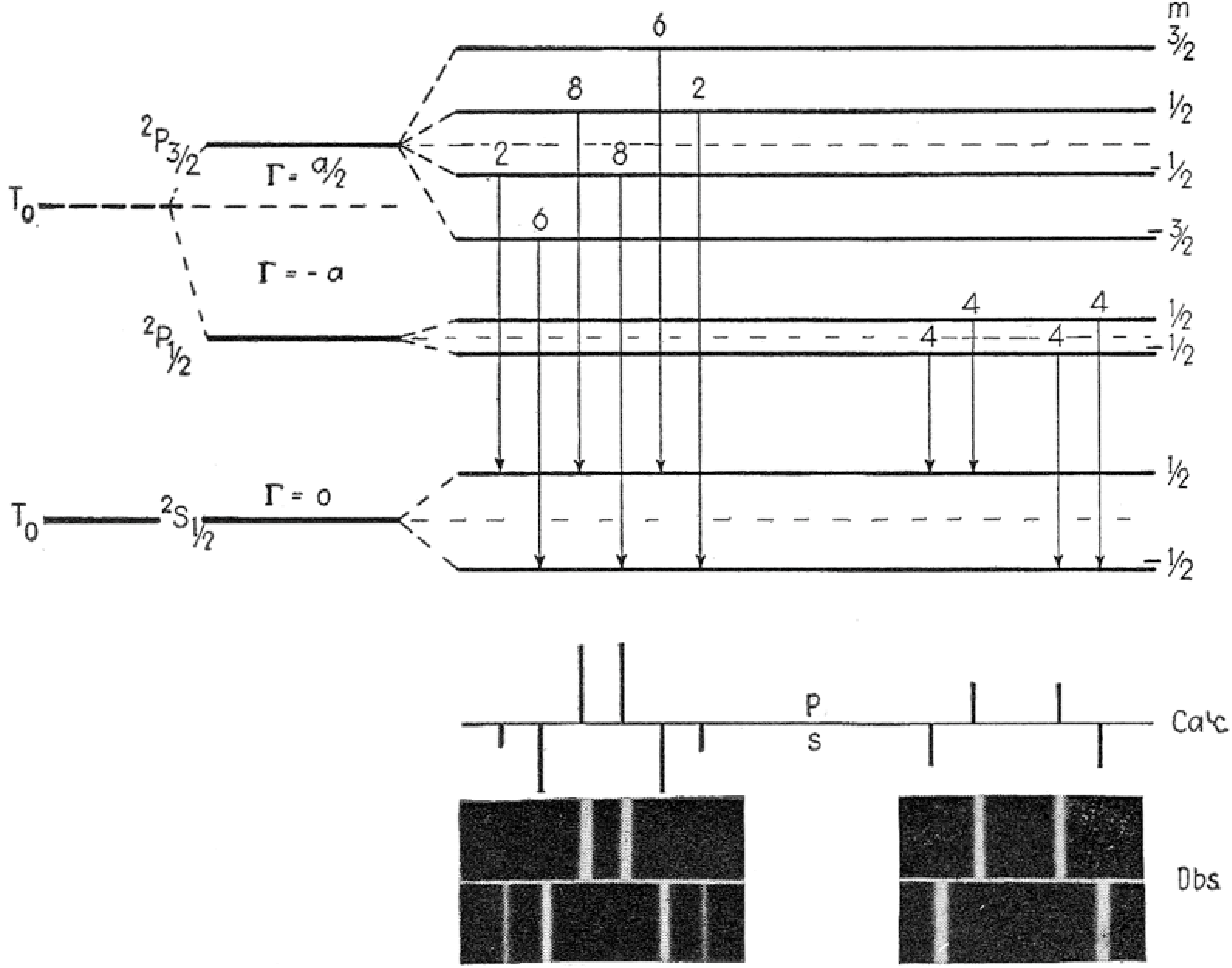


Fig. 10.8.—**Zeeman** effect of a principal-series doublet.

(H. E. White, *Introduction to Atomic Spectra*)

polarization

$\Delta M = \pm 1$ → polarization perpendicular to $\boldsymbol{B}$ (σ-polarization)
$\Delta M = 0$ → polarization parallel to $\boldsymbol{B}$ (π-polarizaiton)

circularly polarized light (σ-polarization)

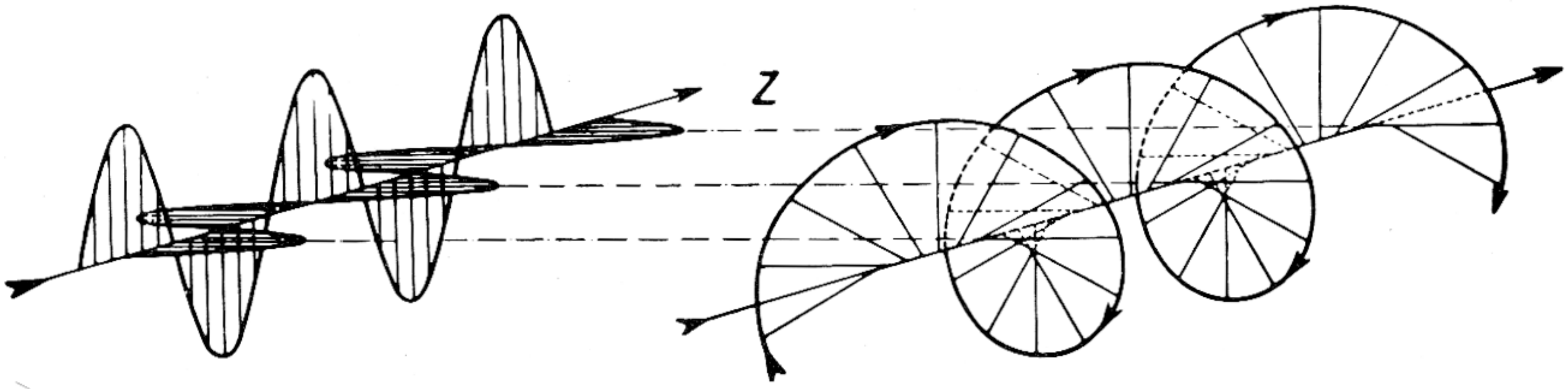


Fig. 10.2. Superposition of two linear-polarized phase-shifted waves (left-hand figure) into a circularly-polarized wave (right-hand figure). The sense of rotation is defined in the text.

(P. H. Heckmann, *Introduction to the Spectroscopy of Atoms*)

π-component is not seen for // B observation

π and σ have the same intensity for ⊥ B observation

perturbation theory (eigensystems)

$$H\psi = E\psi$$

$$\begin{aligned} H &= H^0 + V \\ &= \begin{pmatrix} E_1^0 + V_{11} & V_{12} & \cdots & V_{1n} \\ V_{21} & E_2^0 + V_{22} & \cdots & V_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ V_{n1} & V_{n2} & \cdots & E_n^0 + V_{nn} \end{pmatrix} \end{aligned}$$

perturbation of magnetic field

$|LSJM\rangle$ is taken as base function

$$\langle JM|V|J'M'\rangle$$

$$= -\mu_{\mathrm{B}}B\,\langle JM|\,(g_L L_z + g_S S_z)\,|J'M'\rangle$$

$$= -\mu_{\mathrm{B}}B \sum_{M_L M_S} \langle JM|M_L M_S\rangle\,\langle J'M|M_L M_S\rangle\,(g_L M_L + g_S M_S)$$

$$= -\mu_{\mathrm{B}}B\,\sqrt{2J+1}\,\sqrt{2J'+1}$$

$$\sum_{M_L M_S} \begin{pmatrix} 1 & 1 & J \\ M_L & M_S & -M \end{pmatrix} \begin{pmatrix} 1 & 1 & J' \\ M_L & M_S & -M \end{pmatrix} (g_{\mathrm{L}} M_L + g_{\mathrm{S}} M_S)$$

$$(M = M_L + M_S)$$

1s2p $^3\mathrm{P}_{0,1,2}$ $\langle JM|V|J'M'\rangle = -\mu_\mathrm{B}B\langle JM|\left(g_\mathrm{L}L_z + g_\mathrm{S}S_z\right)|J'M'\rangle$

	J	0	1			2				
J'	M	0	-1	0	1	-2	-1	0	1	2
0	0			-0.8165						
1	-1		-1.5000				-0.5000			
	0	-0.8165						-0.5774		
	1				1.5000				-0.5000	
2	-2					-3.0000				
	-1		-0.5000				-1.5000			
	0			-0.5774						
	1				-0.5000				1.5000	
	2									3.0000

$$\langle JM|V|J'M'\rangle = -\mu_{\mathrm{B}} B \langle JM|\left(g_{\mathrm{L}} L_z + g_{\mathrm{S}} S_z\right)|J'M'\rangle$$

	J	0	1	2	1	2	1	2	2	2
J'	M	0			-1		1		-2	2
0	0	0	-0.8165							
1		-0.8165	0	-0.5774						
2			-0.5774	0						
1	-1				-1.5000	-0.5000				
2					-0.5000	-1.5000				
1	1						1.5000	-0.5000		
2							-0.5000	1.5000		
2	-2								-3.0000	
2	2									3.0000

perturbed eigenstate is expressed with base functions

$$|\psi_m\rangle = \sum_m C_i^m |\psi_i^0\rangle$$

transition probability is written as

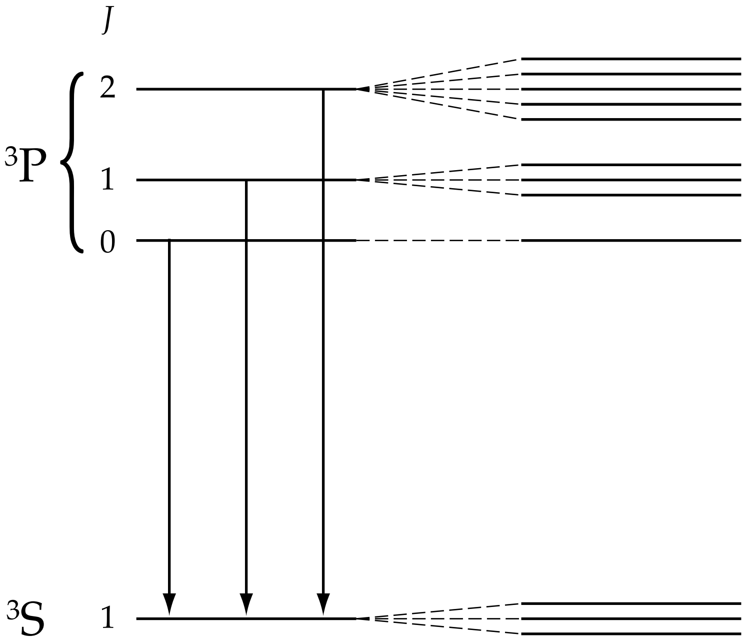
$$A^q_{\psi_m \to \varphi_n} \propto \left|\langle \varphi_n | D_q | \psi_m \rangle\right|^2 = \left| \sum_{i,j} C_j^n C_i^m \langle \varphi_j^0 | D_q | \psi_i^0 \rangle \right|^2$$

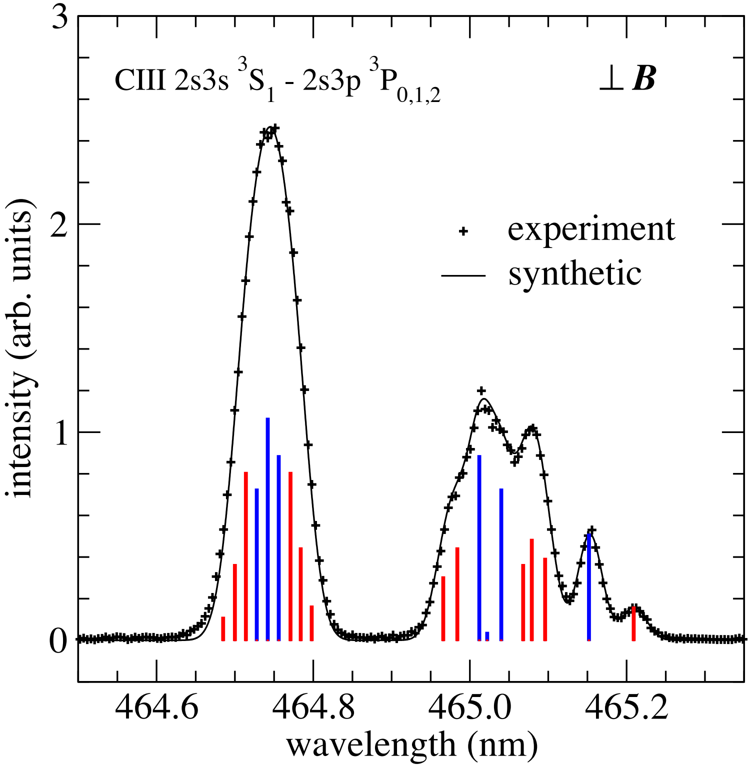
each term is calculated as

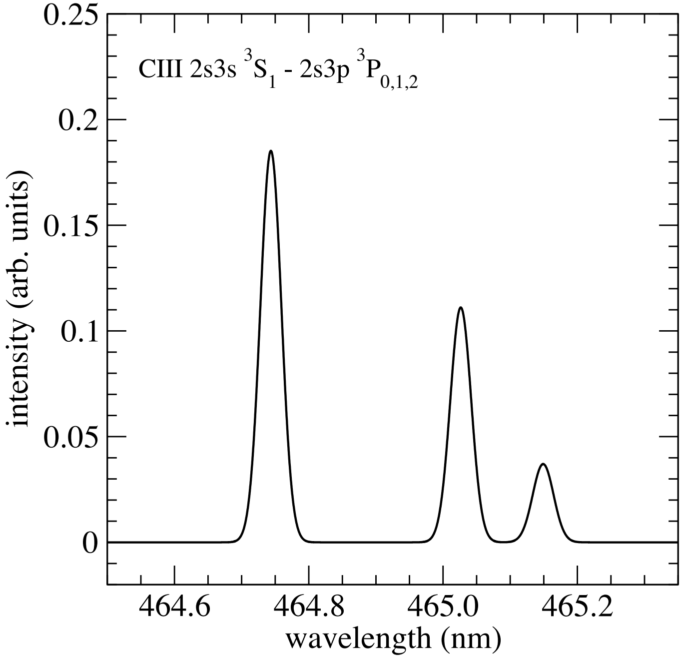
$$|\psi_i^0\rangle = |L_1 S_1 J_1 M_1\rangle$$
$$|\varphi_j^0\rangle = |L_2 S_2 J_2 M_2\rangle$$

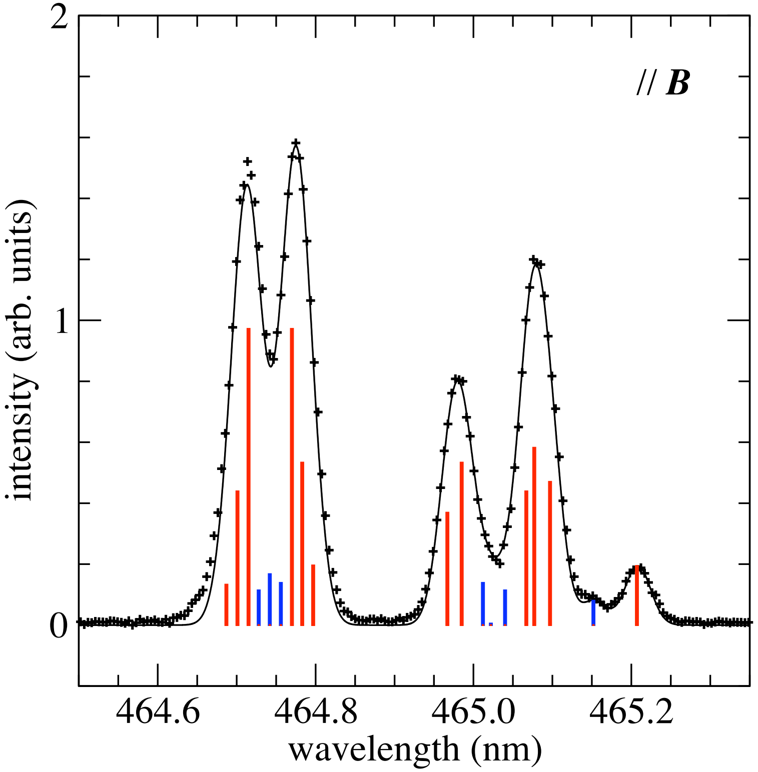
$$\langle \varphi_j^0 | D_q | \psi_i^0 \rangle = (-1)^{(J_1 - M_1) + (S + 1 + L_1 + J_2)} \sqrt{(2J_1 + 1)(2J_2 + 1)} \begin{pmatrix} J_1 & 1 & J_2 \\ -M_1 & q & M_2 \end{pmatrix} \begin{Bmatrix} L_1 & J_1 & S \\ J_2 & L_2 & 1 \end{Bmatrix} \langle L_1 \| D \| L_2 \rangle$$

CIII 2s3s $^3S_1$ – 2s3p $^3P_{0,1,2}$

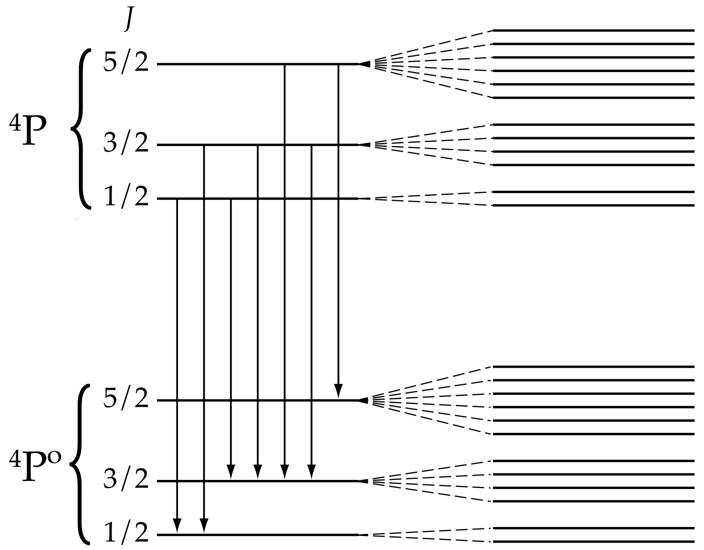


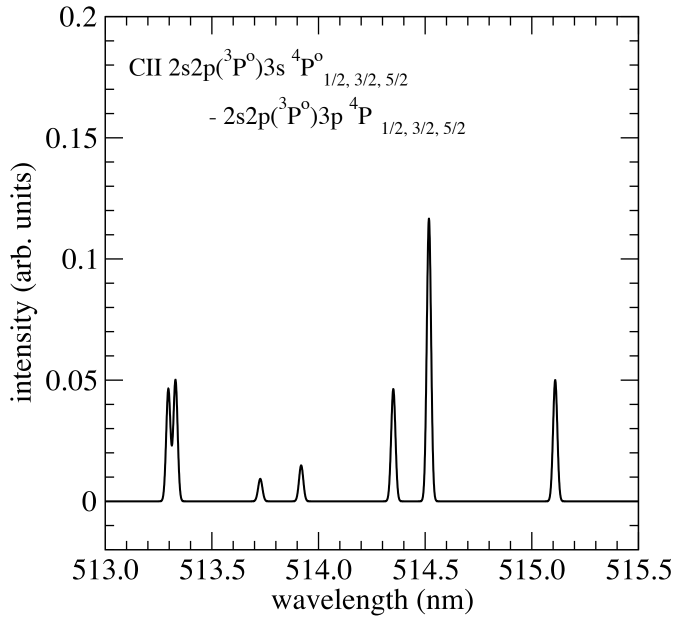


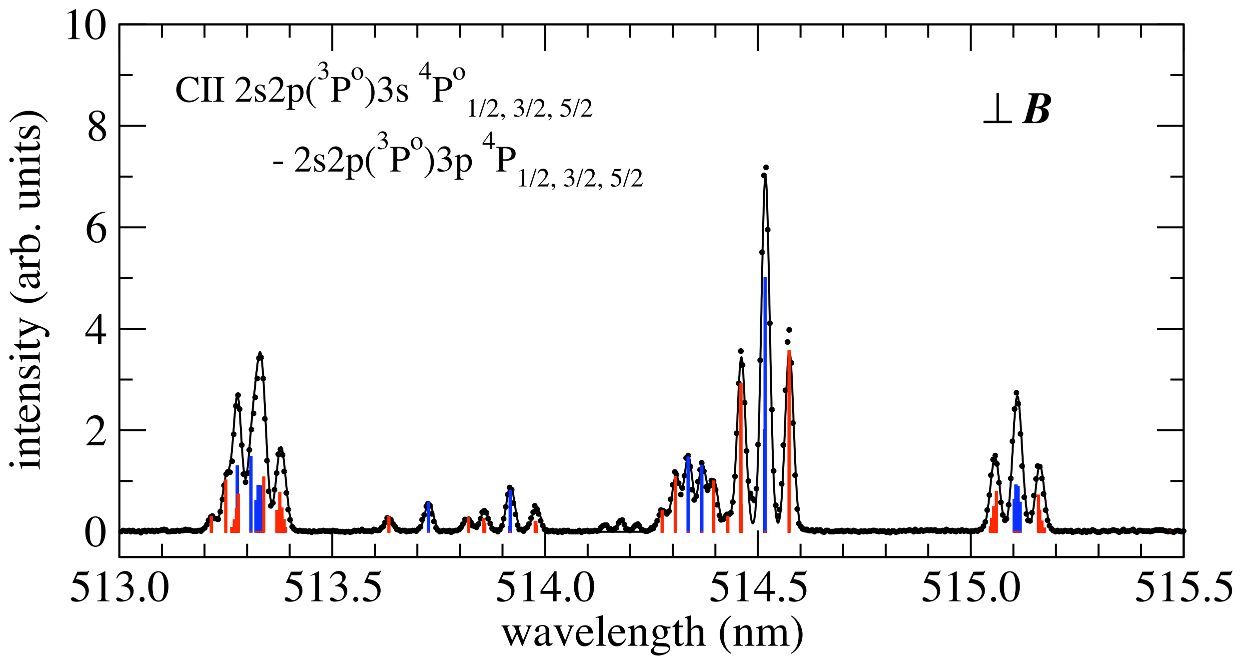


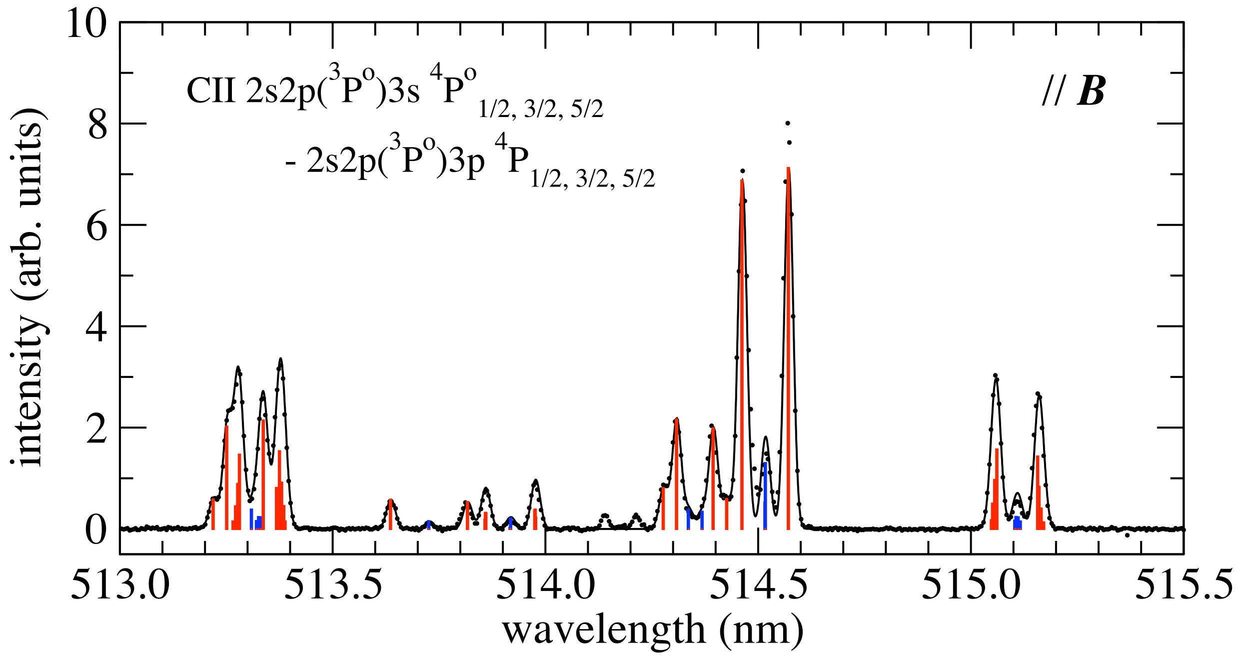


CII $2s2p(^3P^o)3s\ ^4P^o_{1/2,3/2,5/2}$ $-\ 2s2p(^3P^o)3p\ ^4P_{1/2,3/2,5/2}$

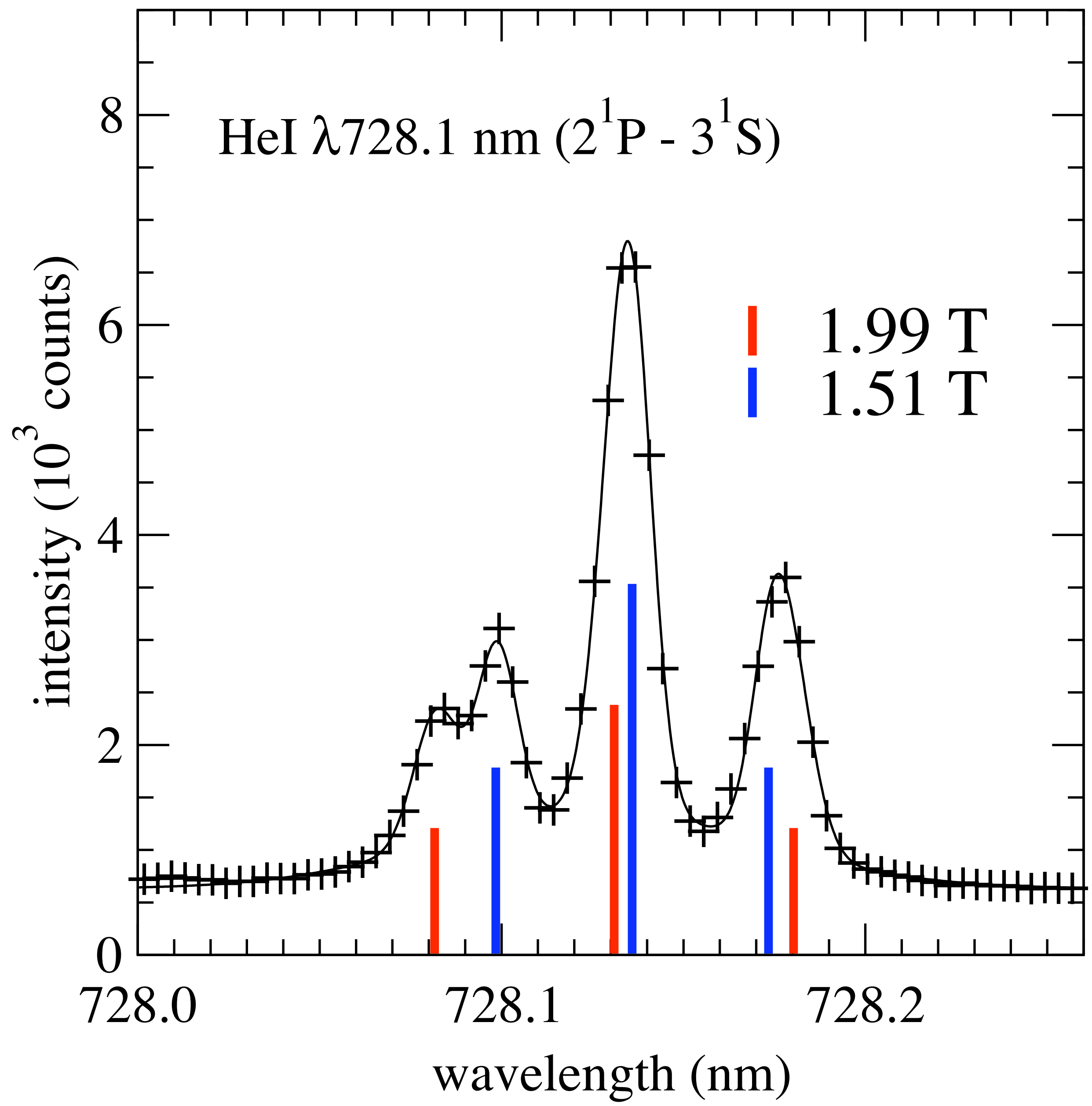




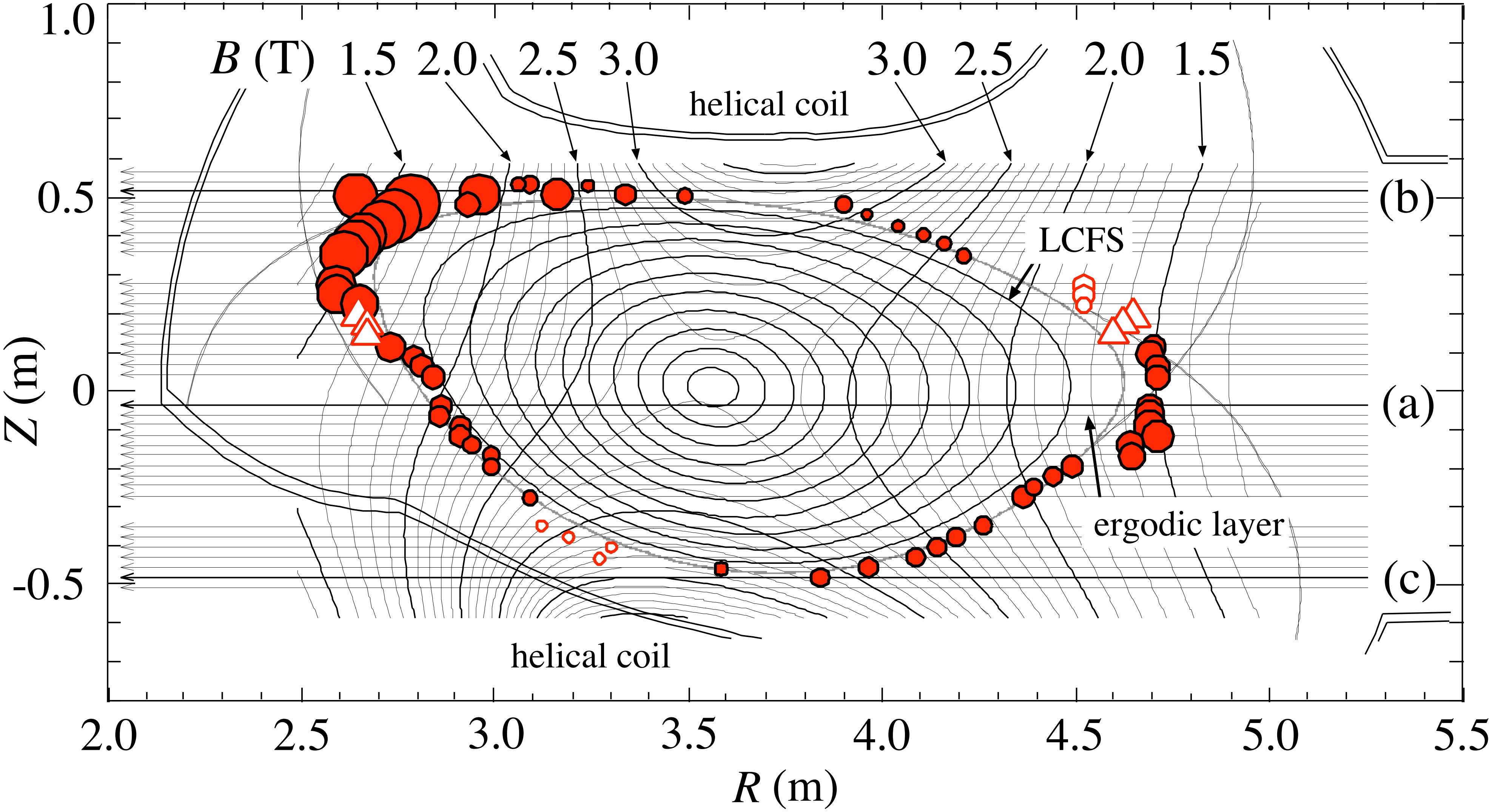




application to fusion plasma



(M. Goto et al., Phys. Rev. E 65, 026401 (2002))



(M. Goto et al., Phys. Rev. E 65, 026401 (2002))